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FIRST QUANTIZATION AND BASIC FOUNDATION OF THE MICROSCOPIC THEORY OF SUPERCONDUCTIVITY

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Abstract: *First quantization approach* has been used for the first time to lay the basic foundation of a general theory of superconductivity applicable to widely different solids. To this effect we first analyze the net Hamiltonian, $H(N)$, of N conduction electrons (ce's) to identify its universal part, $H_o(N)$ (*independent of the specific aspects of a superconductor or a class of superconductors*), and then find the states of $H_o(N)$ to conclude that superconductivity originates, basically, from an inter-play between the zero-point force (f_o) exerted by ce's on the lattice constituents and its opposing force (f_a) originating from inter-particle interactions which decide the lattice structure. While the lattice, in the state of equilibrium between f_o and f_a, assumes a kind of mechanical strain and corresponding energy, E_s, the entire system (N ce's + strained lattice) is left with a net fall in energy by E_g. Obviously, E_g serves as the main source of ce-lattice direct binding and ce-ce indirect binding leading to the formation of (**q**, -**q**) bound pairs of ce's, -finally found to be responsible for the onset of superconductivity below certain temperature T_c. We find a relation for T_c which not only explains its high values observed for non-conventional superconductors but also reveals that superconductivity can occur, in principle, at *room temperature* provided the system meets necessary conditions. Our theory has few similarities with BCS model. It provides microscopic basis for the two well known phenomenologies of superconductivity, *viz.*, the two fluid theory and $\Psi-$theory and corroborates a recent idea that superconducting transition is basically a quantum phase transition. Most significantly, this study demonstrates that microscopic theories of a many body quantum system, such as N ce's in solids, liquid ^{3}He, *etc.*, can be developed by using *first quantization approach*. It also finds reasons for which any approach (*viz.*, second quantization) which uses single particle basis with *plane wave representation of particles* achieved limited success in concluding a *complete, clear and correct* understanding of the low temperature properties (such as superconductivity, superfluidity, *etc.*) of widely different many body quantum systems for the last seven decades.

Keywords: superconductivity; basic foundations; microscopic theory; macro-orbitals; mechanical strain in lattice

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1. Introduction:

The experimental discovery of high T_c superconductivity (HTS) in 1986 [1] came as a great surprise to the physics community, basically, for its challenge to the Bardeen, Cooper and Schrieffer (BCS) model [2] which had emerged as a highly successful theory of superconductors that we knew at that time. Consequently, HTS systems became a subject of intense research activity and thousands of experimental and theoretical papers have been published over the last 25 years. While the important results of various experimental studies on HTS systems are reviewed in [3-13], the status of our present theoretical understanding is elegantly summed up in [3, 12-21].

Several theoretical models based on widely different exotic ideas have been worked out, since no single mechanism could be identified as the basic origin of different properties of HTS systems. We have theories based on Hubbard model [17, 22], spin bag theories [23], anti-ferromagnetic Fermi-liquid theory [24], $d_{x^2-y^2}$ theories [25], anyon theory [26], bipolaron theory [27] and theories based on the proximity effects of quantum phase transition [15] and it may be mentioned that this list is not exhaustive; references to other models and experimental results can be traced from [3, 12-29] and recent articles and reviews [30, 31].

It is evident that, even after a period of nearly 26 years of the discovery of HTS systems, the goal of having a single microscopic theory of superconductivity is far from being achieved. Incidentally, the process of achieving

this goal has been frustrated further by certain experimental results, *viz.* : (i) the coexistence of superconductivity with ferro-magnetism [32], (ii) superconductivity of MgB_2 at $T_c(\approx 39\text{K})$ [33], (iii) pressure/strain induced superconductivity [34], (iv) stripes of charges in a HTS system [35], enhancement of superconductivity by nano-engineered magnetic field in the form of tiny magnetic dots [36], *etc.* as well as by interesting theoretical models which consider two energy gaps [37], formation of Cooper type pairs through spin-spin interaction [38], triplet p-wave pairing and singlet d-wave pairing [39], *etc.* for specific superconductors.

We either have a system specific theory or a class (*i.e.* a set of superconducting solids) specific theory of superconductivity and numerous ideas that have greatly muddled the selection of right idea(s) helpful to develop a unified single microscopic theory (*preferably having basic features of BCS theory*) of the phenomenon. However, we found a way-out when we used *first quantization approach* to lay down the basic foundations of our non-conventional microscopic theory (NCMT) [40] of a system of interacting fermions (SIF) capable of explaining their superconductivity/ superfluidity by using relevant conclusions of our study of the wave mechanics of two hard core identical particles in 1-D box [41]. Since first quantization approach precludes any assumption about the nature of the order parameter (OP) of the superconducting transition or the nature and strength of the interaction responsible for superconductivity, conclusions of our theory are simply drawn from the solutions of the Schrödinger equation of N conduction electrons (ce**s**). It may be noted that different conclusions of [41] also helped us in unifying the physics of widely different many body quantum systems (MBQS) of interacting bosons and fermions [42] and to develop the long awaited microscopic theory of a system of interacting bosons (SIB) by using first quantization [43]. Our approach not only concludes (**q**, -**q**) bound pairs of ce**s** (in several respect different from Cooper pairs) as the origin of superconductivity but also finds some untouched aspects of ce-ce correlation,-mediated by phonons in strained lattice.

We note that ce**s** form a kind of Fermi fluid which flows through the lattice structure of a solid. To a good approximation, each ce can be identified as a freely moving particle which can be represented by a plane wave until it suffers a collision with other ce**s** or constituents of the lattice. It is argued that electrostatic screening effect, which could predominantly be Thomas-Fermi screening and/or quantum mechanical screening, significantly reduces the strength and range of ce-ce repulsion and thereby facilitates the formation of Cooper type pairs which are conventionally believed to be responsible for superconductivity [2]. However, these effects in certain superconductors (*e.g.* in HTS systems) are found to be relatively weak and each theoretical model of such superconductors looks for a possible source of relatively stronger attraction so that the formation of Cooper type pairs of charge carriers becomes possible.

Since the conventional microscopic theories (CMTs) based on single particle basis (SPB) [detailed discussed in Section 2.2] did not achieve desired success to explain superconductivity of widely different superconductors, we decided to find an alternative basis. To this effect we discovered (*cf.* Section 3.0 below) that it is the pair of particles (*not the single particle*) which forms the basic unit of the system and this led us to use pair of particles basis (PPB) to describe the ce fluid in developing our NCMT [40]. Further, as discussed in Appendix-I, we note that SPB used by CMTs does not fit with certain physical realities of ce**s**, particularly, at low temperatures (LTs), while the same realities fit with PPB used in by us.

In this paper we revise our earlier report [40] by adding necessary discussion to: (i) *justify* our approximation in relation to repulsive part of ce-ceinteractions, and (ii) *establish* the analytic nature of a macro-orbital (a kind of pair wave function) (Eqn. 12, Section 3.3) used to describe ce**s** in their states of different angular momentum l (*viz.*, $l = 0$ and $l \neq 0$). The paper has been arranged as follows. The Hamiltonian of the ce fluid in a solid has been analyzed in Section 2.0 to identify its universal component ($H_o(N)$, Eqn. 2, below), -independent of the specific aspects of a superconductor or a class of superconductors, while the wave mechanics of a pair of ce**s** (identified as the basic unit of the fluid), has been examined in Section 3.0 to discover its several important aspects and to conclude that a ce is better represented by a *macro-orbital* (particularly in LT states of the system) rather than a *plane wave*. A wave function ($\Phi_n(N)$, Eqn.28) that represents a general state of the ce fluid is constructed in Section 4.0 by using N macro-orbitals for N ce**s**; the $\Phi_n(N)$ is then used to conclude the ground state configuration of the fluid. The equation of state and free energy of ce fluid are developed in Section 5.0, while important aspects of superconductivity, such as the origin of criticality of ce fluid, onset of lattice strain, energy gap and formation of (**q**, -**q**) bound pairs, transition temperature, *etc.* are discussed in Section 6.0. The paper is summed up by examining the consistency or inconsistency of our model with other well known models such as BCS theory, two fluid theory, $\Psi-$theory, *etc.* in Section 7.0 where we also discuss the agreement of a

macro-orbital state (section 3.4.7) with the states of an electron in electron bubble (a well known experimental reality) which provides unshakable foundation to our theory. Finally important concluding remarks are presented in Section 8.0.

Our theory clearly concludes that a mechanical strain in the lattice, produced by an act of f_o and f_a, is the *main factor* responsible for superconductivity and its stability. This strain is, obviously, different from the electrical strain (*i.e.* electrical polarization of the lattice constituents produced by electrical charge of ce) which is emphasized as the main cause of superconductivity in the framework of BCS theory [2]. However, in what follows from our theory, the electrical strain may have its contribution to the cause of superconductivity only as a secondary factor. Since f_o is a simple mechanical force, the onset of the said mechanical strain below certain T (*cf.* Section 6.2) resulting from its action rightly emerges as the universal factor responsible for the origin of (**q**, -**q**) bound pairs of ces and superconductivity in widely different superconductors. Interestingly, recent experimental studies confirm the occurrence of lattice strain [44] and corroborate the fact that phonons have major role in the mechanism of superconductivity even in HTS systems [45]. While several theoretical studies [46] associate the charge fluctuation, spin fluctuation, phase fluctuation, superconducting density fluctuation, *etc.*, with the onset of superconductivity, the present study finds a role of these fluctuations through their possible coupling with the mechanical strain in the lattice which serves as the basic OP of the transition.

Contrary to a prolonged belief of more than seven decades that microscopic theories of SIFs (such as N ces in solids and liquid ^{3}He) and SIBs (such as liquid ^{4}He) can not be developed by using first quantization, we succeeded in developing the present microscopic theory of superconductivity and similar theory of superfluidity of liquid ^{4}He type SIB [43] by doing so. We also discovered well defined reasons (*cf.*, Appendix-I) for which a many body quantum theory, based on any approach (*viz.*, second quantization) which uses single particle basis with *plane wave representation of particles*, is bound to have limited success in concluding the origin of LT properties such as superconductivity, superfluidity, *etc.* which is commonly observed with all such theories published over the last seven decades.

2. Important Aspects of The Electron Fluid:

2.1. Hamiltonian:

The Hamiltonian of N ces can be expressed, to a good approximation, as

$$H(N) = -\frac{\hbar^2}{2m}\sum_i^N \nabla_i^2 + \sum_{i<j} V(r_{ij}) + V'(N), \tag{1}$$

where m is the mass of a ce, $V(r_{ij})$ is the *central force potential* experienced by two ces and $V'(N)$ stands for the sum of all possible interactions such as ce-phonon, spin-spin, spin-lattice, *etc.* We assume that different components of $V'(N)$ can be treated as perturbation on the states of

$$H_o(N) = H(N) - V'(N). \tag{2}$$

which can be identified as a universal component [*independent of the specific nature of a chosen superconductor or a class of superconductors*] of $H(N)$. This breakup has an advantage that the impact of different (one or more than one) component(s) of $V'(N)$ present in a chosen superconductor (or a class of superconductors) can be examined as a perturbation on the states of $H_o(N)$. To find the states of $H_o(N)$, we consider that the ce fluid is a Fermi fluid where $V(r_{ij})$ is the sum of a short range strong repulsion $V^R(r_{ij})$ and an *indirectly induced* weak attraction $V^A(r_{ij})$ of slightly longer range.

To a good approximation, $V^R(r_{ij})$ can be equated to a *hard core* (HC) interaction $V_{HC}(r_{ij})$ defined by $V_{HC}(r_{ij} < \sigma) = \infty$ and $V_{HC}(r_{ij} \geq \sigma) = 0$ where σ is the HC diameter of a ce. To justify our assumption, $V^R(r_{ij}) \approx V_{HC}(r_{ij})$, we note that the strength and range of inter-ce repulsion in solids is significantly reduced by the *screening effect* described as Thomas-Fermi screening and/or quantum mechanical screening. Consequently, ces can be identified to move randomly as free particles of finite size, -much smaller than the typical size of atoms/ions in a solid; the situation is shown in Fig.1(A) where ces in a channel are depicted by dark color circles (indicator of their finite size σ) attached with small size arrows representing their random motions. A similar situation is shown

in Fig.1(B) where dark circle (representing σ) embedded in light color large circle (depicting the wave packet (WP) size $\lambda/2$ of the ce with λ being its de Broglie wave length) emphasizes that the effective size of a quantum particle should be equal to its WP size if $\lambda/2 > \sigma$. It is obvious that Fig.1(A) corresponds to a high temperature (HT) situation, while Fig.1(B) represents a relatively LT situation. The channel through which ces are assumed to move can be identified as a cylindrical tube of diameter d_c in the crystalline lattice or a 2-D slot of width d_c between two parallel atomic planes. One, obviously, finds that HC size (σ) of a ce satisfies $\sigma < d_c << a$ where a is a lattice constant. One may find that the value of σ is not significant in our theory which assumes that two ces do not occupy a point in normal space simultaneously and this holds true even if ces are considered to have $\delta-$size infinitely repulsive hard core.

The fact, that no ce comes out of a solid unless a definite amount of energy ($\geq$ work function) is supplied from outside, indicates that there are certain factors, such as the presence of $+ve$ charges in the background of moving ces which bind a ce with the entire system (the lattice + other $N-1$ ces). This implies that the ce-lattice attraction (evidently experienced by every ce) indirectly renders an ce-ce attraction which we represent by $V^A(r_{ij})$. To a good approximation, the net attraction for a ce leads to a constant negative potential $-V_o$ whose main role is to keep all ces within the volume of the conductor; in what follows, a ce can be identified as a freely moving HC particle on the surface of a constant $-ve$ potential.

2.2. Basic unit:

Guided by the above discussion, the motion of each ce, to a good approximation, can be expressed by a plane wave,

$$u_{\mathbf{p}}(\mathbf{b}) = A\exp(i\mathbf{p}.\mathbf{b}). \tag{3}$$

Here **p** and **b**, respectively, represent the momentum (in wave number) and position vectors of the ce. However, this motion is modified when the ce collides with other ces or the lattice structure. A collision could either be a *two body collision* (ce-ce collision), or a *many body collision* (a process in which two mutually colliding ces also collide simultaneously with other ces or lattice structure). In the former case two ces (say, e1 and e2) simply exchange their momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ or positions $\mathbf{b}_1$ and $\mathbf{b}_2$ without any difference in the sum of their pre- and post-collision energies. In the latter case, however, e1 and e2 could be seen to jump from their state of $\mathbf{p}_1$ and $\mathbf{p}_2$ to that of different momenta $\mathbf{p}_1'$ and $\mathbf{p}_2'$ (possibly with a different sum of their energies too) but it is clear that even after such collisions e1 and e2 remain in states described by plane waves. Evidently, the complex dynamics of the ce fluid can be described to a good approximation in terms of the simple dynamics of a *pair* of HC particles (discussed in Section-3 below) as its *basic unit*, particularly, if we wish to incorporate the impact of collisions and wave superposition on the state of ces. In variance with this observation, CMTs of superconductivity are found to use SPB to describe the fluid; in a sense this means that: (i) each ce in the fluid can be described by a plane wave, (ii) the momentum and energy of each ce remain good quantum numbers of a state of the fluid, and (iii) a single ce is assumed to represent the basic unit of the fluid. However, as shown in Appendix-I, the said use of SPB does not fit with certain physical realities of the LT states of the fluid. Naturally, all CMTs are bound to encounter serious difficulties in explaining the LT properties such as superconductivity or superfluidity of a SIF.

3. Dynamics of Two HC Particles:

3.1. Schrödinger equation:

The Schrödinger equation of two ces (e1 and e2) as HC impenetrable particles can be described by

$$\left(-\frac{\hbar^2}{2m}\sum_i^2 \nabla_i^2 + V_{HC}(r)\right)\psi(1,2) = E(2)\psi(1,2). \tag{4}$$

Although, the dynamics of e1 and e2, in a many body collision involving lattice structure, can be seen to encounter an interaction different from that involved in a two or many ce collision, however, the fact that the end result of either of the two collisional processes is to take e1 and e2 from their state of $\mathbf{p}_1$ and $\mathbf{p}_2$ to that of $\mathbf{p}_1'$ and $\mathbf{p}_2'$, indicates that the difference is *inconsequential* for the said collisional dynamics. We can, therefore, proceed with our analysis of $\psi(1,2)$ and use its results to find how each ce in $\psi(1,2)$ state in LT states of the system

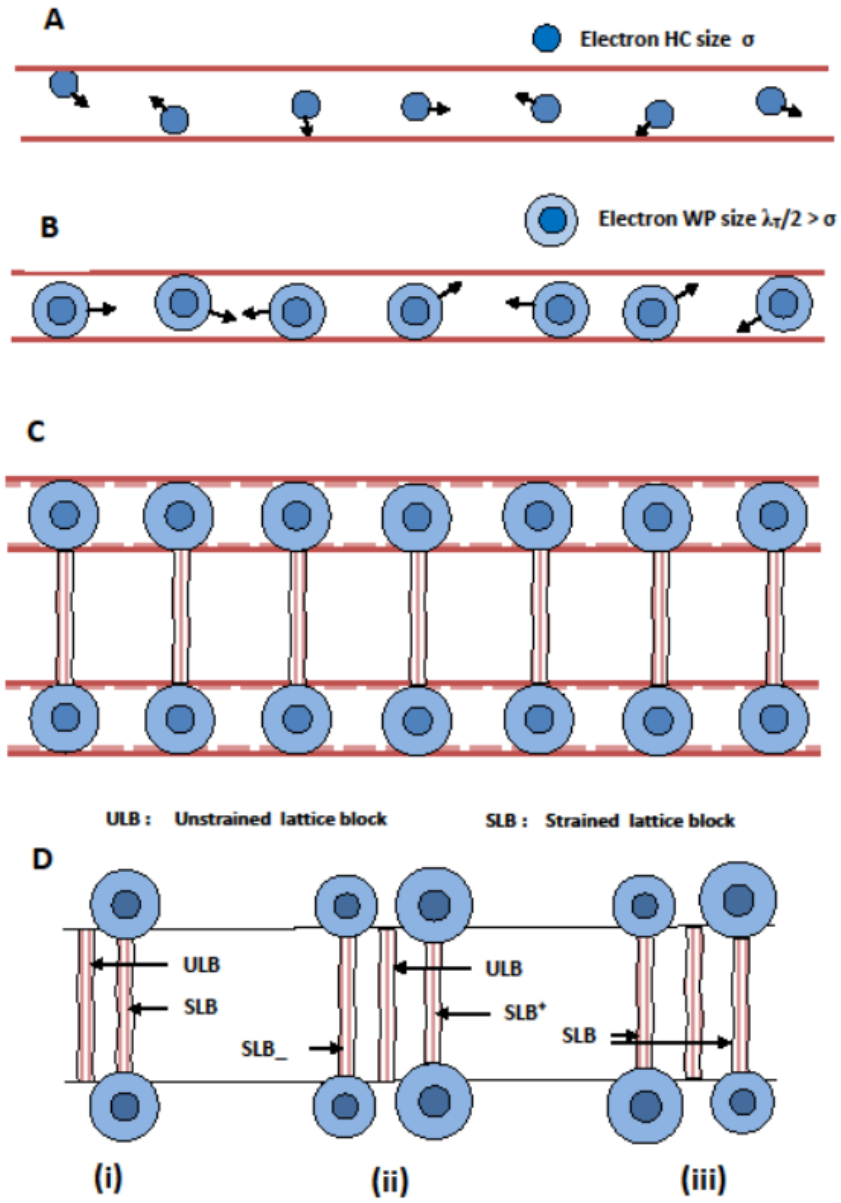

Figure 1: Depiction of ce̊s moving in conduction channels with simplified shape and structure: (A) HT state ($\lambda_T/2 < \sigma$) where ce̊s have random positions and random motions with possibility of mutual collisions and collisions with channel walls, (B) Relatively LT state where average wave packet (WP) size $\lambda_T/2$ satisfies $\sigma < \lambda_T/2 < d_c$; possibilities of collisions are similar to (A), (C) $T < T^*$ state where WP size $\lambda_T/2 = d_c^+$ (slightly $> d_c$) with lattice columns between two channels getting strained by $d_c^+ - d_c$ due to zero-point force f_o operating against f_a responsible for restoring d_c. With fall in T, strain Δd increases from $\Delta d = d_c^+ - d_c \approx 0$ at T^* to $\Delta d = d_c' - d_c$ at $T \leq T_c$. ce̊s in two different strained channels can be seen to have their correlations through strained lattice blocks (SLB), and (D)-(i) SLB has shorter length than unstrained lattice block (ULB); no energy flows when f_o and f_a are in equilibrium, D-(ii) finite energy flows between ce̊s and lattice when WP size of two ce̊s have in-phase oscillations forcing SLB length to oscillate between SLB$_-$ and SLB$^+$, and D-(iii) finite energy flows between two ce̊s when their WP size has out-of-phase oscillations keeping SLB length unchanged. ULB is shown in D-(i), (ii) and (iii) to compare with SLB, SLB$_-$ and SLB$^+$.

assumes a phonon induced inter-ce correlation seemingly essential for the occurrence of superconductivity in widely different superconductors.

The process of solving Eqn.4 is simplified by using : (i) the *center of mass* (CM) coordinate system, and (ii) $V_{HC}(r) \equiv A(r)\delta(r)$ where $A(r)$, representing the strength of Dirac delta function $\delta(r)$, is such that $A(r) \to \infty$ when $r \to 0$. We analyze this equivalence to justify its validity in Section 3.2 where we present its physical basis. It may be noted that this type of equivalence has been mathematically demonstrated by Huang [47]. In the CM coordinate system, we have

$$\mathbf{r} = \mathbf{b}_2 - \mathbf{b}_1 \quad \text{and} \quad \mathbf{k} = \mathbf{p}_2 - \mathbf{p}_1 = 2\mathbf{q}, \tag{5}$$

where $\mathbf{r}$ and $\mathbf{k}$, respectively, represent the relative position and relative momentum of two ces, and

$$\mathbf{R} = (\mathbf{b}_2 + \mathbf{b}_1)/2 \quad \text{and} \quad \mathbf{K} = \mathbf{p}_2 + \mathbf{p}_1, \tag{6}$$

where $\mathbf{R}$ and $\mathbf{K}$, similarly, refer to the position and momentum of their CM. Without loss of generality, Eqns. 5 and 6 also render

$$\mathbf{p}_1 = -\mathbf{q} + \frac{\mathbf{K}}{2} \quad \text{and} \quad \mathbf{p}_2 = \mathbf{q} + \frac{\mathbf{K}}{2}. \tag{7}$$

By using these equations, one may express Eqn. 4 as

$$\left(-\frac{\hbar^2}{4m}\bigtriangledown_R^2 - \frac{\hbar^2}{m}\bigtriangledown_r^2 + A(r)\delta(r)\right)\Psi(r,R) = E(2)\Psi(r,R) \tag{8}$$

with

$$\Psi(r,R) = \psi_k(r)\exp(i\mathbf{K}.\mathbf{R}). \tag{9}$$

It is evident that the HC interaction does not affect the CM motion, $[\exp(i\mathbf{K}.\mathbf{R})]$. It affects only $\psi_k(r)$ (the relative motion of two particles) which represents a solution of

$$\left(-\frac{\hbar^2}{m}\bigtriangledown_r^2 + A\delta(r)\right)\psi_k(r) = E_k\psi_k(r), \tag{10}$$

with $E_k = E(2) - \hbar^2K^2/4m$.

3.2. Basis and related aspects of $V_{HC}(r) \equiv A(r)\delta(r)$:

The physical basis for $V_{HC}(r) \equiv A(r)\delta(r)$ can be understood by examining the possible configuration of two HC particles (say P1 and P2) right at the instant of their collision. When P1 and P2 during a collision have their individual CM located, respectively, at $\mathrm{r}_{CM}(1) = \sigma/2$ and $\mathrm{r}_{CM}(2) = -\sigma/2$ (with r_{CM} being the distance of the CM of a particle (P1/P2) from the CM of the pair (P1 and P2), they register their physical touch at $r = 0$ and their encounter with $V_{HC}(r)$ arises at the point of this touch beyond which two HC particles can not be pushed in. While the process of collision does identify this touch, it fails to register how far are the CM points of individual particles from $r = 0$ at this instant. In other words the rise and fall of the potential energy of P1 and P2 during their collision at $r = 0$ is independent of their σ and this justifies $V_{HC}(r) \equiv A(r)\delta(r)$.

Further since P1 and P2, due to their HC nature, never assume a state where the CM points of individual particles fall on a common point, $r = 0$, it is clear that $\delta-$repulsion should be infinitely strong; evidently, we have to have $A(r) \to \infty$ when $r \to 0$. This should, obviously, be valid for two ce because they too never occupy a common point in space simultaneously. It may, however, be mentioned that this equivalence will not be valid in accounting for certain physical aspects of the system (*e.g.*, the volume occupied by a given number of particles) where the real size of the particle assumes importance.

It is well known that the dynamics of two particles in 3-D space has six independent motions: (i) three components of their CM motion (a linear motion free from the effects of inter-particle interactions), (ii) relative motion with $\mathbf{q}$ || to $\mathbf{r}$ ($s-$wave state), and (iii) two independent rotation like motions around two mutually orthogonal axes passing through their CM points with $\mathbf{q} \perp$ to $\mathbf{r}$ (p, d, f, ... state). Since the interaction operates effectively only in motion-(ii) which clearly represents a 1-D motion, results obtained for the 1-D wave mechanics of two HC particles [41] (see also Appendix-A of Ref.[43]) can be applied to this motion. In the pure rotation type motion

(*i.e.*, motions-(iii) of non-zero l), interaction energy of two particles remains unchanged for which this motion seems to be free from $A(r)\delta(r)$. However, for any probability that P1 and P2 in any motion of $l \neq 0$ happen to have $r = 0$, they would surely encounter the infinity of $\delta-$repulsion. Consequently, the wave function of such states is bound to be zero at $r = 0$.

3.3. State functions:

In order to find $\Psi(r, R)$, -a solution of Eqn. 8, we treat $A(r)\delta(r)$ as a step potential. Since P1 and P2 experience zero interaction in the region $r \neq 0$, they can be represented, independently, by plane waves except around $r = 0$ where $A(r)\delta(r) = \infty$. However, in view of the possible superposition of two waves, the state of P1 and P2 should be described, in principle, by

$$\Psi(1,2)^{\pm} = \frac{1}{\sqrt{2}}[u_{\mathbf{p}_1}(\mathbf{r}_1)u_{\mathbf{p}_2}(\mathbf{r}_2) \pm u_{\mathbf{p}_2}(\mathbf{r}_1)u_{\mathbf{p}_1}(\mathbf{r}_2]. \tag{11}$$

Here we note that $\Psi(1,2)^{+}$ (of $+ve$ symmetry for the exchange of two particles) does not represent the *desired* wave function of two HC particles since, as required, it does not vanish at $\mathbf{r}_1 = \mathbf{r}_2$ where $A(r)\delta(r = 0) = \infty$, while the other function $\Psi(1,2)^{-}$ of $-ve$ symmetry has no such problem. We addressed this problem in our recent analysis of the 1-D analogue of Eqn. 8 in relation to our detailed study of the wave mechanics of two HC impenetrable particles in 1-D box [41]; in what follows, one may easily find that the state of two such particles can be expressed by

$$\zeta(r, R)^{\pm} = \zeta_k(r)^{\pm} \exp(i\mathbf{K}.\mathbf{R}) \tag{12}$$

with

$$\zeta_k(r)^{-} = \sqrt{2}\sin(\mathbf{k}.\mathbf{r}/2) \tag{13}$$

of $-ve$ symmetry, and

$$\zeta_k(r)^{+} = \sqrt{2}\sin(|\mathbf{k}.\mathbf{r}|/2) \tag{14}$$

of $+ve$ symmetry for the exchange of their $\mathbf{r}_1$ and $\mathbf{r}_2$ (or $\mathbf{k}_1$ and $\mathbf{k}_2$). It is obvious that for a state of the pair with given $\mathbf{k}_1$ and $\mathbf{k}_2$, only $\mathbf{r}$ remains its variable; any change in $\mathbf{k}_1$ and $\mathbf{k}_2$ and/or the angle between $\mathbf{k}$ and $\mathbf{r}$ clearly means a change of the quantum state. We note that the second derivative of $\zeta_k(r)^{+}$ with respect to r has $\delta-$like singularity at $r = 0$ but this can be reconciled for the presence of infinitely strong repulsive potential at $r = 0$.

We note that the net wave function (*i.e.*, the product of orbital and spin parts) of a state of two fermions should be anti-symmetric for their exchange. Consequently, for a pair of **ce**s with parallel spins (spin triplet state), the orbital part should be anti-symmetric and it could be represented by $\zeta_k(r, R)^{-}$ (Eqns.12 and 13); similarly, for the pair of anti-parallel spins (spin singlet state), the orbital part should be symmetric and it could be represented by $\zeta_k(r, R)^{+}$ (Eqns.12 and 14).

However, before we proceed with the formulation of our theory further, it is important to speak about the analytic nature of $\zeta_k(r, R)^{-}$ (Eqn.12/13) and $\zeta_k(r, R)^{+}$ (Eqn.12/14) for all possible states of the pair distinguished by different values of angular momentum, $l = 0, 1, 2, 3, ..$, identified, respectively, as s, p, d, ... states. The $l = 0$ state (or the s state), characterised by $\mathbf{q} \,||\, \mathbf{r}$, with lowest $|\mathbf{q}| = q_o$ represents the G-state of the pair because the pair has no motion other than zero point motion, while states with $l \neq 0$ have an additional motion (addition to zero-point motion) represented by $\mathbf{q}_{\perp}$ (component of $\mathbf{q} \perp$ to $\mathbf{r}$) in addition to $\mathbf{q}_{||}$ (component of $\mathbf{q} \,||$ to $\mathbf{r}$) indicating that the net $\mathbf{q} = \mathbf{q}_{||} + \mathbf{q}_{\perp} = \mathbf{q}_o + \mathbf{q}_{\perp}$. Evidently, even for a state of $l \neq 0$, we have $\mathbf{q}.\mathbf{r} = \mathbf{q}_o.\mathbf{r} + \mathbf{q}_{\perp}.\mathbf{r} = q_o r$ which, does not vanish because as concluded in [41-43], q_o has non-zero value for every particle in our system; this inference can also be followed from Eqn.(17) and related discussion (Section 3.4.3- 3.4.5) of this paper. In addition as discussed in Section 7.6, the experimental reality of a system such as electron bubble (*an electron trapped in a spherical cavity in liquid* He) clearly supports the fact that a quantum particle (confined to a cavity formed by neighboring atoms) in its ground state has non-zero q_o. This not only establishes the analytic nature of $\zeta_k(r, R)^{\pm}$ (Eqn.12 with Eqn.13/14) but also indicates that the energy/momentum of a **ce** in its $l = 0$ and $l \neq 0$ states can be related to energy/momentum eigen values of a particle trapped in a spherical cavity.

3.4. Characteristic aspects:

3.4.1. Nature of relative motion:

We note that $\zeta_k(r)^{\pm}$, describing the relative motion of two HC particles, is a kind of *stationary matter wave* (SMW) which modulates the probability, $|\zeta_k(r)^{\pm}|^2$, of finding two particles at their relative phase position $\phi = \mathbf{k}.\mathbf{r}$ in the $\phi-$space. Interestingly, the equality, $|\zeta_k(r,R)^{-}|^2 = |\zeta_k(r,R)^{+}|^2$, concludes a very *important fact* that the relative configuration and relative dynamics of two HC particles in a state of given k is independent of their fermionic or bosonic nature. This implies that the requirement of fermionic symmetry should be enforced on the wave function(s) representing $\mathbf{K}-$motions or spin motions of **ce**s and we use this inference in constructing N **ce**s wave function in Section-4.

A SMW, such as $\zeta_k(r)^{\pm}$, comes into existence when two plane waves (representing identical fields or particles) of equal and opposite momenta have their wave superposition. This implies that two HC particles in $\zeta(r,R)^{\pm}$ state have equal and opposite momenta (**q**,-**q**) in the frame attached to their CM which moves with momentum $\mathbf{K}$ in the laboratory frame and this interpretation is consistent with Eqn.7. One may also find that two particles in their relative motion maintain a center of symmetry at their CM (the point of their collision) which means that

$$\mathbf{r}_{CM}(1) = -\mathbf{r}_{CM}(2) = \frac{\mathbf{r}}{2} \quad \text{and} \quad \mathbf{k}_{CM}(1) = -\mathbf{k}_{CM}(2) = \mathbf{q} \tag{15}$$

where $\mathbf{r}_{CM}$(i) and $\mathbf{k}_{CM}$(i), respectively, refer to the position and momentum of $i-$th particle with respect to the CM of two particles.

3.4.2. MS and SS states:

Since $\zeta(r,R)^{\pm}$ (Eqn.12) is basically an eigenstate of the energy operators of relative and CM motions of P1 and P2 in their wave superposition, it could rightly be identified as a state of their *mutual superposition* (MS). However, one may have an alternative picture by presuming that each of the two particles, *after its collision with other particle*, falls back on its pre-collision side of $r = 0$ (the point of collision) and assumes a kind of *self superposition* (SS) (*i.e,*, the superposition of pre- and post-collision states of one and the same particle). Interestingly, this state is also described by $\zeta(r,R)^{\pm}$ because it too represents a superposition of a plane wave of momentum $\mathbf{p}_1$ (the pre-collision state of P1) and a similar wave of momentum $\mathbf{p}_1' = \mathbf{p}_2$ representing post-collision state of P1 because two particles exchange their momenta during their collision; the same effect can be seen with P2. However, since P1 and P2 are identical particles and there is no means to ascertain whether the two exchanged their positions or bounced back after exchanging their momenta, we can use $\zeta(r,R)^{\pm}$ to identically describe the MS state of P1 and P2 or the SS states of individual particle, P1 or P2. The latter possibility greatly helps in developing the *macro-orbital representation* of each particle in the fluid (*cf.*, point 3.4.7).

3.4.3. Values of $< r >$, $< \phi >$ and $< H(2) >$:

The SMW waveform, $\zeta_k(r)^{\pm}$, has series of anti-nodal regions between different nodal points at $r = \pm n\lambda/(2\cos\theta)$ (with n= 0,1,2,3, ... and θ being the angle between **q** and **r**). This implies that two particles can be trapped on the r line *without disturbing their energy or momenta* by suitably designed cavity of impenetrable infinite potential walls. For example, one may possibly use two pairs of such walls and place them at suitable points perpendicular to $\mathbf{k}_1$ and $\mathbf{k}_2$ or to the corresponding $\mathbf{k}$ and $\mathbf{K}$). In case of $\mathbf{k}||\mathbf{r}$ (representing a $s-$wave state) one can use a cavity of only two such walls placed at the two nodal points located at equal distance on the opposite sides of the point ($r = 0$) of their collision. Using the fact that the shortest size of this cavity can be only λ, we easily find

$$< r >_o \; = \frac{< \zeta_k(r)^{\pm}|r|\zeta_k(r)^{\pm} >}{< \zeta_k(r)^{\pm}||\zeta_k(r)^{\pm} >} = \frac{\lambda}{2} \tag{16}$$

as the shortest possible $< r >$. To this effect, integrals are performed between $r = 0$ (when the two particles are at the center of cavity) to $r = \lambda$ (when one particle reaches at $r = \lambda/2$ and the other at $r = -\lambda/2$ representing the locations of the two walls which reflect the particles back inside the cavity). Following a similar analysis for the general case we identically find $< r > \; = \lambda/(2\cos\theta)$ which not only agrees with Eqn.16 but also reveals that the two particles assume $< r >=< r >_o$ only when they have head-on collision. Evidently, from an experimental view point, two HC particles never reach closer than $\lambda/2 = \pi/q$ and in this situation their individual locations (*cf.* Eqn. 15) are given by $< \mathbf{r}_{CM}(1) >_o = - < \mathbf{r}_{CM}(2) >_o = \lambda/4$. Note that this result is consistent with the WP representation of a quantum particle because the representative WPs of two HC particles are not expected to have any overlap in the real space, since such particles do not occupy any space point simultaneously. Finding

similar result for their shortest possible distance in $\phi-$space and $< V_{HC}(r) >$, *etc.* we note that $\zeta_k(r)^{\pm}$ state is characterized by

$$< \zeta_k(r)^{\pm}|r|\zeta_k(r)^{\pm} > \geq \lambda/2 \quad \text{and} \quad < \psi_k(r)^{\pm}|\phi|\psi_k(r)^{\pm} > \geq 2\pi, \tag{17}$$

$$< \zeta_k(r)^{\pm}|V_{HC}(r)|\zeta_k(r)^{\pm} > = < \zeta_k(r)^{\pm}|A\delta(r)|\zeta_k(r)^{\pm} > = 0, \tag{18}$$

$$E(2) = < \zeta(r,R)^{\pm}|H(2)|\zeta(r,R)^{\pm} > = \left[\frac{\hbar^2 k^2}{4m} + \frac{\hbar^2 K^2}{4m}\right]. \tag{19}$$

While Eqn.(19), which reveals that two particles in $\zeta(r,R)^{\pm}$ states have only kinetic energy, agrees with the experimental fact that ce s behave like free particles, at the same time our other result $< r > \geq \lambda/2$ [Eqn.(17)] concludes that two HC particles in $\zeta(r,R)^{\pm}$ states are restricted to have $q \geq q_o = \pi/d$. This, evidently, shows that the nature and magnitude of the energy of the relative motion of a pair as expressed by Eqn.(19) is not free from inter-particle interactions. While we address this aspect again in Sections 3.4.4 and 3.4.5, here we have two important facts to be noted.

(a). Eqn.(18) (as analyzed in Appendix-A of [43]) is valid for *all physically relevant situations* of two particles.

(b) $\zeta(r,R)^{\pm}$ is not an eigenstate of the momentum/ energy operators of individual particle. In stead, it is the eigenstate of only the energy operator of the pair indicating that the momentum of individual particle or of the pair does not remain a good quantum number.

3.4.4. quantum size:

In what follows from Eqns.17 and 18, a HC particle of momentum q exclusively occupies $\lambda/2$ space if $\lambda/2 > \sigma$ because only then the two particles maintain $< r > \geq \lambda/2$. We call $\lambda/2$ as *quantum size* of the particle. In a pair state, $\zeta_k(r)^{\pm}$, one may identify quantum size as the size of a particle (say P1) as seen by the other particle (say P2) or *vice versa*. To this effect P1 may be considered as an object to be probed and P2 as a probe (or *vice versa*) and apply the well known principle of image resolution. We find that P2 can not resolve the σ size of P1 if its $\lambda/2 > \sigma$ and the effective size of P1 as seen by P2 (or *vice versa*) would be limited to $\lambda/2$. But the situation is different for the particles of $\lambda/2 \leq \sigma$ because here they can resolve the σ size of each other. Naturally, in all states of $q \geq \pi/\sigma$, P1 and P2 would see each other as particle of size σ. This concludes that the effective size of low momentum particles ($q < \pi/\sigma$) is $q-$dependent, while the same in case of high momentum particles ($q \geq \pi/\sigma$) is $q-$independent and this explains why a MBQS exhibits the impact of wave nature only at LTs.

On the qualitative scale our meaning of *quantum size* seems to be closer to what Huang [47] refers as quantum spread but on the quantitative scale, while we relate *quantum size* of a particle with its momentum by a definite relation $\lambda/2 = \pi/q$, quantum spread has not been so related in [47]. The fact, that no particle can be accommodated in a space shorter than $\lambda/2$, implies that *quantum size* of a particle can be identified as the maximum value of its quantum spread or as the minimum possible size of $r-$space, -it occupies exclusively. It may also be mentioned that our meaning of quantum size word differs from the meaning it has in *quantum size* effects on the properties of thin films, small clusters [48], *etc.*

3.4.5. zero-point force:

The fact that each HC particle exclusively occupies a minimum space of size $\lambda/2$ whose average value for a particle in a SIF can be identified with $\lambda_T/2$ with $\lambda_T = h/\sqrt{2\pi m k_B T}$ being the thermal de Broglie wavelength; here $h =$ Planck constant and $k_B =$ Boltzmann constant. We first use this observation to make our conclusions in relation to the zero-point force exerted by a particle in its ground state in a system of interacting fermions (SIF) like ce fluid in a conductor or liquid ^{3}He. To this effect we ignore their $K-$motions which retain certain amount of energy at all T including $T = 0$ due to their fermionic nature; how $K-$motions and their energy affect the relative configuration of particles and related properties of the system would be addressed in Section 6.1.

Since λ_T increases with decreasing T, each particle at certain $T = T_o$ (at which $\lambda_T/2$ becomes equal to *average nearest neighbor distance* d) assumes its maximum possible quantum spread [47] and finds itself trapped in a box of size d (a cavity formed by neighboring atoms); this means that almost all particles at $T = T_o$ have maximum

possible $\lambda/2(=d)$ and minimum possible $q = q_o = \pi/d$ which represents the ground state of each particle. Using $\lambda_T/2 = d$ at $T = T_o$, we have

$$T_o = \frac{h^2}{8\pi m k_B d^2}. \tag{20}$$

Evidently, when a SIF like liquid ^{3}He is cooled through T_o, each particle tries to have $\lambda/2 > d$ for its natural tendency to have lowest possible energy and to this effect it expands the cavity size d by exerting its zero-point force $f_o = h^2/4md^3$ against another force f_a (originating from inter-particle interactions) which tries to restore the cavity size.

Similar physical situation exists with CEs constrained to move through narrow channels of diameter/ width d_c which, obviously, represents space size of their confinement. Consequently, they too reach a state where they all have $\lambda_T/2 = d_c$ at T_o (Eqn.20, with $d = d_c$) and exert their $f_o = h^2/4md_c^3$ on the walls of the channel if the conductor is cooled through this T_o; obviously f_o is opposed by f_a originating from inter-particle interactions which decide d_c. In the state of equilibrium between f_o and f_a, the lattice assumes a non-zero mechanical strain in terms of a small increase in d_c (*cf.*, Fig.1(C) which depicts this increase as a shift of channel walls from dashed lines to solid lines) which plays a crucial role for the onset of superconductivity (*cf.* Section 6.0).

We note that d_c is much smaller than inter-CE distance d and the unit cell size a; to this effect a rough estimate of the shortest $d = (\mathrm{v}/n)^{1/3}$ (with v = unit cell volume and n = number of CEs in the cell) reveals $d = a/2$ if we use $n = 8$, -the maximum possible n contributed to an unit cell presumed to have one atom). Since this d is larger than expected d_c because a finite portion of v is also occupied by the atom in the unit cell, it is clear that d_c is more relevant than d or a in deciding lowest possible $q = q_o$ for a CE and in determining the ground state properties of CE fluid in a superconductor.

3.4.6. Phase correlation:

In our recent paper [43] related to the microscopic theory of a system of interacting bosons, we obtained a relation for the quantum correlation potential [49, 50] between two HC bosons which also occupy a state identical to $\zeta_k(r)^{\pm}$. Hence, following the same procedure, we determine the quantum correlation potential $U(\phi)$ between two CEs. We have,

$$U(\phi) = -k_B T \ln |\zeta_k(r)^{\pm}|^2 = -k_B T \ln [2\sin^2(\phi/2)] \quad \text{with} \quad T = T_o, \tag{21}$$

where $\phi = \mathbf{k}.\mathbf{r}$ is the relative position of two CEs in phase space. It may be mentioned that T in Eqn.21 should be replaced by T_o (Eqn.20 with $d = d_c$) representing T equivalent of $\varepsilon_o = h^2/8md_c^2$ because $q-$motion energy of each CE at $T \leq T_o$ gets frozen at ε_o (*cf.* Section 4.2). We note that $U(\phi)$ at a series of points, $\phi = (2s+1)\pi$ with s being an integer, has its minimum value (= $-k_B T_o \ln 2$) and at other points, $\phi = 2s\pi$, has its maximum value (= ∞). Evidently, two CEs in the states of their wave superposition $\zeta_k(r)^{\pm}$ prefer to have their phase positions separated by $\Delta\phi = 2n\pi$ (with $n = 1, 2, 3, ...$) representing the distance between two points of $U(\phi) = -k_B T_o \ln 2$. This inference is strongly supported by the experimentally observed coherence in the motion of CEs, particularly in the superconducting state. In addition, the $-ve$ value of $U(\phi)$ indicates that two CEs develop a kind of binding in the phase space.

3.4.7. Macro-orbital representation:

We note that in spite of their binding in the $\phi-$space, as concluded above, two HC particles in the real space experience a kind of mutual repulsion, if they happen to have $< r > < \lambda/2$ or no force, if $< r > \geq \lambda/2$. This implies that each particle in $\zeta(r, R)^{\pm}$ state can be identified as independent particle and be represented by its self superposition (*cf.* point 3.4.2) described by a kind of pair waveform $\xi \equiv \zeta(r, R)^{\pm}$ proposed to be known as macro-orbital and expressed as,

$$\xi_i = \sqrt{2}\sin[(\mathbf{q}_i.\mathbf{r}_i)]\exp(\mathbf{K}_i.\mathbf{R}_i), \tag{22}$$

where i (i = 1 or 2) refers to one of the two particles; here r_i could be identified with $r_{CM}(i)$ (*cf.* Eqn. 15) which changes from $r_i = 0$ to $r_i = \lambda/2$, while R_i refers to the CM point of $i-$th particle. Although, two particles in $\zeta(r, R)^{\pm}$ state are independent but it is clear that each of them represents a (**q**, -**q**) pair whose CM moves with momentum **K** in the lab frame. This implies that each particle in its macro-orbital representation has two motions: (i) the plane wave $K-$motion which remains unaffected by inter-particle interactions, and (ii) the $q-$motion which is, obviously, affected by the inter-particle interaction as evident from Eqn.(10). In other words a macro-orbital

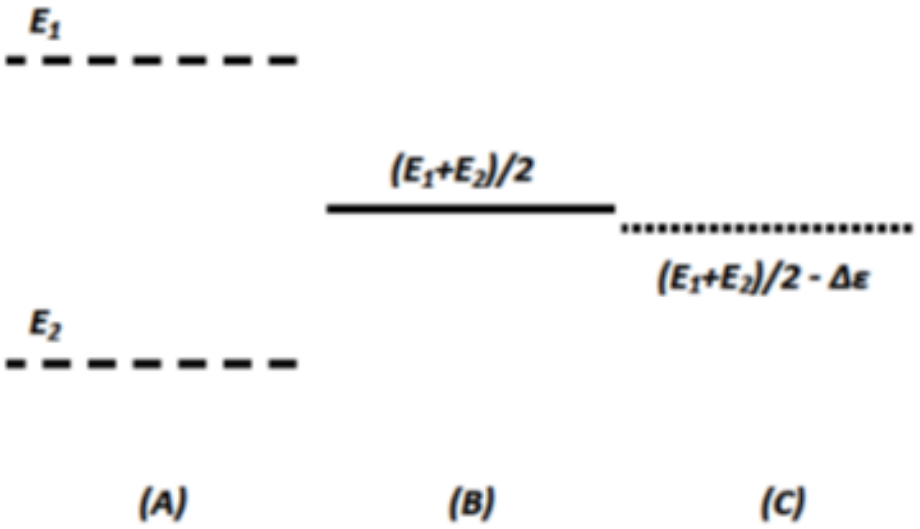

Figure 2: Two ce s with energies E_1 and E_2 in three different situations: (A) when they do not have their wave superposition, (B) when they have their wave superposition, and (C) when their states of wave superposition is perturbed by forces leading to lattice strain and net fall the energy of their relative motion by $\Delta\epsilon = 2\epsilon_g$ (see text).

identifies each ce as a WP of effective size $\lambda/2$ moving with momentum K and this gives due importance to the WP manifestation and the quantum size of a quantum particle as invoked by wave mechanics. We find that this picture is consistent with two fluid phenomenology of superconductivity (*cf.* Section 7.2). Since $\zeta(r,R)^{\pm}$ is neither an eigenfunction of the energy operator nor of the momentum operator of a *single particle*, each particle shares the pair energy $E(2)$ equally. We have,

$$\varepsilon_1 = \varepsilon_2 = \frac{E(2)}{2} = \frac{\hbar^2 q^2}{2m} + \frac{\hbar^2 K^2}{8m}. \tag{23}$$

It is interesting to note that two particles, having different momenta ($\mathbf{p}_1$ and $\mathbf{p}_2$) and corresponding energies E_1 and E_2 before their superposition (*cf.*, Fig.2(A)) have equal energies $\varepsilon_1 = \varepsilon_2$ (Eq.23 and Fig.2(B)) in $\zeta(r,R)^{\pm}$ state. This clearly indicates that wave superposition of two ce s take them into a kind of *degenerate state* which tends to happen with all ce s when the system is cooled through certain $T = T^* < T_o$ (*cf.* Section 6.1).

In order to show that ξ_i fits as a solution of Eqn.4 [with $V_{HC}(r) \equiv A\delta(r)$], we recast the two particle Hamiltonian $H_o(2) = -\sum_i^2 (\hbar^2/2m)\,\nabla_i^2 + A\delta(r)$ as $H_o'(2) = \sum_i^2 h(i) + A\delta(r)$ by defining

$$h_i = -\frac{\hbar^2}{2m}\nabla_i^2 \quad \text{and} \quad h(i) = \frac{h_i + h_{i+1}}{2} = -\frac{\hbar^2}{8m}\nabla_{R_i}^2 - \frac{\hbar^2}{2m}\nabla_{r_i}^2 \tag{24}$$

with $h_{N+1} = h_1$ for a system of N particles. While ξ_i is, evidently, an eigenfunction of $h(i)$ with $< h(i) >= (\hbar^2 q_i^2/2m + \hbar^2 K_i^2/8m)$, the two particle wave function, $\Phi(2) = \xi_1\xi_2$ (or with added permuted terms), is an eigenfunction of $H_o'(2)$ with $< H_o'(2) >= E(2)$ (*cf.* Eqn. 19) because $< A(r)\delta(r) >= A(r)|\xi_1|^2_{r_1=0}|\xi_2|^2_{r_2=0} = A(r)\sin^2 q_1 r_1|_{r_1=0}\sin^2 q_2 r_2|_{r_2=0} = 0$; to this effect it is noted that $r = 0$ implies $r_1 = r_2 = 0$ (*cf.* Eqn. 15, with $r_i \equiv r_{CM}(i)$). We prove the validity of $< A(r)\delta(r) >= 0$ for all physically relevant situations in Appendix-A of Ref.[43].

3.4.8. Accuracy and relevance of macro-orbitals:

While the fact, that the fall of a ce into its SS state (*cf.*, Section 3.4.2) is independent of the details of its collision (*i.e.*, two body collision, many body collision or the collision with the lattice structure), justifies its representation by ξ_i in general, we also find that the functional nature of ξ_i matches almost exactly with

$$\eta_{q,K}(s,Z) = A\sin[(\mathbf{q}.\mathbf{s})]\exp(\mathbf{K}.\mathbf{Z}) \tag{25}$$

representing a state of a particle in a cylindrical channel with **s** being the 2-D space vector perpendicular to $z-$axis (the axis of the channel) and,

$$\eta_{q,K}(z,S) = B\sin[(\mathbf{q}.\mathbf{z})]\exp(\mathbf{K}.\mathbf{S}) \tag{26}$$

which represents a similar state of a particle trapped between two parallel impenetrable potential sheets. Interestingly, since superconductivity is a behavior of low energy **Ce**s and a **Ce** in a solid can be visualized, to a good approximation, as a particle moving along the axis of cylindrical channel (*e.g.* in a conventional superconductor) or that moving between two parallel atomic sheets (*e.g.* in HTS systems), the accuracy and relevance of macro-orbitals in representing the **Ce**s in their low energy states is well evident. Most importantly, as discussed in Section 7.6, it is supported strongly by the experimental reality of the existence of an electron bubble.

4. States of $N-$Electron Fluid:

4.1. General state:

Using N macro-orbitals for N **Ce**s and following standard method, we have

$$\Psi_n^j(N) = \Pi_i^N \zeta_{q_i}(r_i)\sum_P^{N!}(\pm 1)^P \Pi_i^N \exp\left[i(P\mathbf{K}_i\mathbf{R}_i)\right] \tag{27}$$

for one of the $N!$ micro-states of the system of energy E_n (*cf.* Eqn. 29, below). Here $\sum_P^{N!}$ represents the sum of $N!$ product terms obtainable by permuting N particles on different $\mathbf{K}_i$ states with $(+1)^P$ and $(-1)^P$, respectively, used for selecting a symmetric and anti-symmetric wave function for an exchange of two particles. In principle, the permutation of N particles on different $\mathbf{q}_i$ states renders $N!$ different $\Psi_n^j(N)$ and we have

$$\Phi_n(N) = \frac{1}{\sqrt{N!}}\sum_j^{N!}\Psi_n^j(N) \tag{28}$$

as the complete wave function of a possible quantum state of energy E_n given by

$$E_n = \sum_i^N\left[\frac{\hbar^2 q_i^2}{2m} + \frac{\hbar^2 K_i^2}{8m}\right] \tag{29}$$

where q_i and K_i can have different values depending on the type channel/cavity/box in which **Ce**s are free to move in a given conductor. For *all practical purposes*, while K_i can be considered to have any value between 0 and ∞, low values of $\mathbf{q}_i$ are expected to be discrete depending on the geometry of the channel; to a good approximation these could be taken as integer multiple of π/d_c. To follow Eqn.29, one may use Eqn.24 to recast $H_o(N) \approx \sum_i^N h_i + \sum_{i>j}^N A\delta(r_{ij})$ as

$$H_o(N) = \sum_i^N h(i) + \sum_{i>j}^N A\delta(r_{ij}) \tag{30}$$

Here we may mention that: (i) our result $< V_{HC}(r_{ij}) >= 0$ (*cf.* Eqn. 18) agrees with the fact that two **Ce**s do not occupy common point in real space, (ii) the energy of **Ce**s (Eqn.19) is basically kinetic in nature, and (iii) **Ce**s are restricted to have $< r > \geq \lambda/2$ (Eqn. 17) and $q \geq q_o$ which clearly indicates that $V_{HC}(r_{ij})$ plays an important role in deciding the relative configuration (*i.e.* the allowed values of $< r >$, $< \phi >$ and q) of **Ce**s, particularly, when **Ce** fluid tends to assume the ground state of the $q-$motions of all **Ce**s by having $q = q_o$.

4.2. Ground state:

We note that each **Ce** has two motions q and K. While the $q-$motions are constrained to have $q \geq q_o(= \pi/d_c)$ representing the lowest possible q of a **Ce** restricted to move through channels of size d_c, the $K-$motions are guided by the Pauli exclusion principle. Consequently, the ground state of the fluid is defined by all $q_i = q_o$ and different K_i ranging between $K = 0$ to $K = K_F$ (the Fermi wave vector). This renders

$$E_{GSE} = N\varepsilon_o + \bar{E_K} = N\frac{h^2}{8md_c^2} + \frac{1}{4}\frac{3}{5}NE_F \tag{31}$$

as the *ground state energy* of the fluid. Here $\varepsilon_o = h^2/8md_c^2$ represents lowest possible energy of the $q-$motion of a ce and $\bar{E_K}$ being the net $K-$motion energy of N ces with E_F being the Fermi energy; the factor 1/4 in the last term represents the fact that each ce in its macro-orbital representation behaves like a particle of mass $4m$ for its $K-$motions. In order to understand how different components of inter-ce interactions enter our formulation to control the ground state energy of ce fluid, it is important to note that d_c in a given system is decided by all such interactions. Naturally, all these interactions indirectly control the ground state momentum through $q_o = \pi/d_c$ and hence the ground state energy ε_o. Expressing E_{GSE} (Eqn. 31) in terms of its temperature equivalent, we have

$$T_{GSE} = T_o + T(\bar{E}_K) \approx T_o + 0.15T_o \approx 1.15T_o \tag{32}$$

where we use $T_o \equiv \varepsilon_o$ and $T(\bar{E}_K) \equiv 3E_F/20)$. In writing $T(\bar{E}_K) = 0.15T_o$ we approximated $E_F(\approx h^2/8md^2)$ to $\varepsilon_o(= h^2/8md_c^2)$ by using d_c for $d = (\mathrm{V}/N)^{1/3}$ where V is the net volume of the solid containing N ce. Since d is always expected to be larger than d_c, $1.15T_o$ (Eqn. 32) can be identified as the upper bound of T_{GSE}, while T_o being the lower bound.

In what follows, since the the macro-orbital state $\sin(q_o r)\exp i(\mathbf{K}.\mathbf{R})$ of a ce also represents a pair of ces with relative momentum $k = 2q_o$, we can easily infer that: (i) have $< k >=< -i\hbar\partial_r >= 0$ by using the fact that r varies between $r = 0$ to $r = d_c$, and (ii) $< r >$ of each ce lies on the axis of the cylindrical tube (Fig.1(C)) (a channel through which they move). While inference-(i) implies that two ces, for all practical purposes, cease to have relative momentum indicating loss of collisional motion or scattering with other ces, inferences-(i and ii) reveal that ces can move (*if they are set to move*) only in the order of their locations along the axis of the channel(s), obviously, with identically equal K, -a characteristic of coherent motion. In addition since the WP size of each ce fits exactly with the channel size [*cf.*, Fig.1(C)], ces also have no collision with channel walls or the lattice.

5. Equation of State:

What follows from Eqn. 29, the energy of a particle in our system can be express as

$$\epsilon = \varepsilon(K) + \varepsilon(k) = \frac{\hbar^2K^2}{8m} + \frac{\hbar^2k^2}{8m}. \tag{33}$$

However, since the lowest $k = 2q$ is restricted to $2q_o$ for the condition, $q \geq q_o$, ϵ can have any value between $\varepsilon_o = \hbar^2q_o^2/8m$ and ∞. Interestingly, this possibility exists even if $\hbar^2k^2/8m$ in Eqn. 33 is replaced by the lowest energy ε_o since K can have any value between 0 and ∞. In other words, we can use

$$\epsilon = \frac{\hbar^2K^2}{8m} + \varepsilon_o \tag{34}$$

which is valid, to a very good approximation, at LTs where we intend to study the system. Using Eqn. 34 in the starting expressions of the standard theory of a system of fermions [51, Ch.8] we obtain

$$\frac{PV}{k_BT} = -\Sigma_{\varepsilon(K)}\ln\left[1 + z\exp\left(-\beta[\varepsilon(K) + \varepsilon_o]\right)\right] \tag{35}$$

and

$$N = \Sigma_{\varepsilon(K)}\frac{1}{z^{-1}\exp\left(\beta[\varepsilon(K) + \varepsilon_o]\right) + 1} \tag{36}$$

with $\beta = \frac{1}{k_BT}$ and fugacity

$$z = \exp(\beta\mu) \qquad (\mu = \text{chemical} \quad \text{potential}). \tag{37}$$

Once again, by following the steps of the standard theory [51] and redefining the fugacity by

$$z' = z\exp(-\beta\varepsilon_o) = \exp\left[\beta(\mu - \varepsilon_o)\right] = \exp\left[\beta\mu'\right] \qquad \text{with} \quad \mu' = \mu - \varepsilon_o \tag{38}$$

we easily have

$$\frac{P}{k_BT} = -\frac{2\pi(8mk_BT)^{3/2}}{h^3}\int_0^\infty x^{1/2}\ln(1 - z'e^{-x})dx = \frac{g}{{\lambda_T}^3}f_{5/2}(z') \tag{39}$$

and

$$\frac{N}{V} = \frac{2\pi(8mk_BT)^{3/2}}{h^3}\int_0^\infty \frac{x^{1/2}dx}{z'^{-1}e^x - 1} = \frac{g}{{\lambda_T}^3} f_{3/2}(z') \tag{40}$$

where g is the weight factor that arises from inherent character such as spin of particles, $x = \beta\varepsilon(K)$, $\lambda_T = h/(2\pi(4m)k_BT)^{1/2}$ and $f_n(z')$ has its usual expression. This reduces our problem of HC particles to that of non-interacting fermions but with a difference. We have m replaced by $4m$ and z by z' (or μ by $\mu' = \mu - \varepsilon_o$). The range of z and z' remain unchanged. In other words if μ and z are, respectively, replaced by μ' and z', system of HC fermions can be treated statistically as a system of non-interacting fermions. As such we can use Eqns. 35 and 36 and Eqns. 39 and 40 to evaluate different thermodynamic properties of our system. For example, the internal energy $U = -\frac{\partial}{\partial\beta}(\frac{PV}{k_BT})|_{z,V}$ of our system can be expressed as,

$$U = \frac{3}{2}k_BT\frac{gV}{{\lambda_T}^3}f_{5/2}(z') + N\varepsilon_o = U' + N\varepsilon_o \tag{41}$$

with $U' = -\frac{\partial}{\partial\beta}(\frac{PV}{k_BT})|_{z',V}$ being the internal energy contribution of non-interacting quasi-particle fermions representing $K-$motions and $N\varepsilon_o$ being the added contribution from $k-$motions. Similarly, we have

$$A = N\mu - PV = N\varepsilon_o + (N\mu' - PV) = N\varepsilon_o + A' \tag{42}$$

as the Helmholtz free energy of fermionic fluid with A' being the Helmholtz free energy of non-interacting fermions. In the following Section, we analyze A for the physical conditions for which it becomes critical leading to superconductivity.

6. Important Aspects of Superconductivity:

6.1. Free energy and its criticality:

Since the free energy component A' in Eqn.42 represents $K-$motions, -free from any involvement of $V(r_{ij})$, it can be attributed to a *system of non-interacting fermions* (SNIF) known to exhibit no phase transition [51]. Naturally, the origin of any possible transition in a SIF should rest with $N\varepsilon_o$, -the remaining part of free energy A (Eqn.42). This agrees with the fact that $N\varepsilon_o$ represents $q-$motions which are controlled by $V(r_{ij})$.

As $N\varepsilon_o$ has no explicit dependence on physical parameters such as T, P, *etc.*, it provides no mathematical solution for T_c, P_c, *etc.* at which it may become critical. In fact, as we find from the following discussion, system becomes critical at certain $T = T_c$ because particles in wave mechanics behave like WPs with average size $\lambda_T/2$ which changes with T as $T^{-1/2}$. In addition each particle exclusively occupies a space of size $\lambda/2$ of its WP. Consequently, we examine our SIF for its criticality by analyzing the evolution of its states with decrease in T which causes average WP size of particles $(\lambda_T/2)$ to assume equality with d_c in superconductor or to d liquid ^{3}He type SIF. Since **ce**s in a channel of size d_c are constrained to have $q \geq q_o(= \pi/d_c)$, the system is expected become critical when it is cooled through a $T = T^*$ at which all **ce**s try to have $q < q_o$ (*if they can*) after assuming $q = q_o$.

In principle, nearly all **ce**s are expected to have $q = q_o$ at a $T \approx T_o$ (Eqn. 20 with $d = d_c$). However, due to Pauli exclusion, fermions can have identically equal q (say q_o), if they have different values K or equal K and different q. The latter possibility implies that a state with all fermions having $q = q_o$ would not assume stability unless their $K-$motions have their least possible energy $\approx 0.15T_o$ (Eqns. 31 and 32). This indicates that the lower and upper bound of T^* should be $0.15T_o$ and T_o, respectively. Once all **ce**s have $q = q_o$ at T^*, they tend to have $q < q_o$ by expanding the channel size by exerting their f_o on the walls of the channel; however, this action of f_o calls for an opposing force f_a representing the internal stress of the channel. In the following section we analyze how f_o prepares the system for a criticality at $T = T_c$ leading to superconductivity.

6.2. Onset of lattice strain:

Analyzing the system for the state of equilibrium between f_o and f_a, one naturally finds that the channel size increases by $\Delta d = d_c' - d_c$ as a strain in the lattice structure with corresponding increase in the volume of the entire system when it is cooled through T^*. The experimental fact, that liquid ^3He on its cooling through 0.6K

(matching closely with T^* [$0.15T_o < T^* < T_o$ for $T_o \approx 1.4$K]) exhibits volume expansion (characterized by $-ve$ volume expansion coefficient [52]), proves that f_o (expected to operate around $T^* \leq T_o$) undoubtedly produces strain (expansion) in ^3He $-^3$ He bonds. Similar effect is, naturally, expected from the f_o exerted by the ce**s** in superconductors. In fact the recent experimental studies [44, 45] have confirmed the presence of mechanical strain in HTS systems.

6.3. Energy gap and (q, -q) bound pairs:

With the onset of lattice strain Δd, the $q-$motion energy of a ce falls below ε_o by

$$\Delta\epsilon = \varepsilon_o - \varepsilon_o' = \frac{h^2}{8md_c^2} - \frac{h^2}{8m(d_c+\Delta d)^2} \approx \frac{h^2}{4md_c^3}(\Delta d). \tag{43}$$

This naturally corresponds to fall in q from $q_o = \pi/d_c$ to $q_o' = \pi/d_c' = \pi/(d_c + \Delta d)$. As reported in Appendix-II, a simple analysis of the equilibrium between f_a and f_o concludes that, to a good approximation, half of $\Delta\epsilon$ is *stored with the lattice as its strain energy*, ϵ_s, and the rest half

$$\epsilon_g = \frac{h^2}{8md_c^3}(\Delta d). \tag{44}$$

moves out of the system as the net fall in the ground state energy of a ce in the solid. This is depicted in Fig.2(C) where net fall in pair energy is depicted by $\Delta\epsilon = 2\epsilon_g$; it is easy to understand that ϵ_g depends on T and P. Derivation of similar results can also be found in Section 5.1(ii) of Ref.[43] and Section 4.3 of Ref.[53].

A detailed study [53] of a simple representative of trapped quantum particle(s) interacting with oscillating particle(s) also reveals that q of a ce confined to move through a channel oscillates with the frequencies of lattice oscillations (*i.e.* phonons). To understand this inference, without going through the details available in [53], it may be noted that ε_o and q_o of such a ce depends on d_c. Naturally, when d_c oscillates with the frequency of a phonon, ε_o and q_o would also oscillate at the same frequency and in this process, the said ce and lattice can be seen to exchange energy/ momentum from each other (See Section 4.4 of [53]). Since a ce remains in this state unless it receives ϵ_g energy from outside, ϵ_g can be identified as an *energy gap* between its state with strained lattice and that with zero-strained lattice. Further since each ce in our theoretical framework represents (**q**, -**q**) pair, the existence of this gap implies that ce**s** are in a state of (**q**, -**q**) bound pairs and the effective free energy of $q-$motions can be expressed by

$$N\varepsilon_o' = N\varepsilon_o - N\epsilon_g(T) = N\varepsilon_o - E_g(T) \tag{45}$$

where $E_g(T)$ is the net decrease in the free energy of all the N ce**s**. Since, as discussed in Section 2.0, each ce binds with the lattice and $N-1$ other such ce**s**, $E_g(T)$ could be identified as an added *collective binding* of all ce**s** in the solid; however, it does not imply that ce**s** form units like a diatomic molecule of O_2, N_2, H_2, *etc.* It only means that each ce is a part or a representative of (**q**, -**q**) bound pair since even a single ce can represent such a pair when it occupies an energy state represented by a macro-orbital, *viz.* the state depicted in Fig.2(C).

6.4. Transition temperature:

In what follows from Sections 6.2 and 6.3 the formation of (**q**, -**q**) bound pairs (with $q = q_o$) starts at T^* with the onset of lattice strain/ volume expansion. However, the limited number of such pairs does not influence the collective behavior of the ce**s** because these pairs have the possibility to jump into a state of unbound pairs with $q > q_o$; this possibility arises because two fermions can have either different K and equal q or equal K and different q indicating that ce**s** can have different q ($> q_o$) at the cost of their $K-$motion energy. Evidently, the said bound pairs assume stability only when the system is cooled to $T \leq T_c(\equiv \epsilon_g)$ where thermal energy of each ce is lower than the its binding energy ϵ_g with the entire system. This renders

$$T_c = \frac{h^2}{8\pi mk_Bd_c^2}\frac{\Delta d}{d_c} = T_o\frac{\Delta d}{d_c} = T_o\frac{\beta l}{d_c} \tag{46}$$

with $T_o = h^2/(8\pi mk_Bd_c^2)$, and $l = a$ (representing the *inter-atomic separation* in conventional superconductors) or $l = c$ (*the lattice parameter perpendicular to the conduction plane of ce**s** in HTS systems*).

In Eqn. 46, we have $\Delta d = \beta l$ because Δd should be proportional to l with proportionality constant β representing a kind of the elastic property of *inter-ionic bonds* in conventional systems or *lattice parameter* c in HTS systems. Since ce**s** in their bound pair state cease to have relative motion, they move in order of their positions without any collision (*not even with lattice*) or scattering (Section 4.2). This not only indicates that the LT phase is left with no source of resistance to the flow of ce**s** but also reveals that they have correlated motion without disturbing their relative positions in $r-$ and $\phi-$spaces which represents another characteristic of superconducting phase known as coherence in ce motions.

The fact that the stability of LT phase is not disturbed by a low energy ($< \epsilon_g$) perturbation such as the application of weak external magnetic field, flow of low density electric current *etc.*, indicates that *the long range* ce-ce *correlations* and related properties such as superconductivity, coherence, persistence of currents, *etc.*) are not disturbed unless the energy of these perturbations crosses ϵ_g. We note that this inference is supported by the experimental observation of critical magnetic field(s), critical currents, *etc.*

6.5. Nature of transition:

As discussed in Section 6.1, A' is not expected to have any change at T_c. In addition one finds that changes in $N\varepsilon_o$, arising due to fall in energy of each ce by ϵ_g for its transition from (**q**, -**q**) unbound pair state (*cf.*, Fig.2(B)) to (**q**, -**q**) bound pair state (*cf.*, Fig.2(C)), start at T^* and ends at $T = 0$; the experimental evidence to this effect (*viz.*, the lattice strain observed in superconductors [44,45] and the volume expansion of liquid ^{3}He [52]) have been discussed in Section 6.2. It is clear that the net fall in $N\varepsilon_o$ by $N\epsilon_g$ occurs over a wide range of T from T^* to $T = 0$ indicating that $N\varepsilon_o$ passes smoothly from $N\varepsilon_o(T_c^+)$ to $N\varepsilon_o(T_c^-)$. Evidently the transformation of the ce fluid into its superconducting state at T_c is a *second order transition*.

Since the $\phi-$positions of two ce**s** in a state of (**q**, -**q**) bound pairs are separated $\Delta\phi = 2\pi$, it is clear that the transition of the system from its normal to superconducting state move ce**s** from their disordered positions $\Delta\phi > 2n\pi$ in phase space to ordered positions $\Delta\phi = 2n\pi$. This shows that the said transition can also be identified as an order-disorder transition of ce**s** in respect of their $\phi-$ positions. This agrees with the experimental fact that ce**s** in superconducting state maintain a definite phase correlation or the coherence of their motion and exhibit quantized vortices or quantized magnetic field but the same is not observed in normal state of ce**s**.

6.6. Typical estimates of T_c:

The universal component of the Hamiltonian $H_o(N)$ (Eqn.2) of ce fluid in a solid does not differ from $H_o(N)$ of liquid ^3He, if spin-spin interaction and spin-orbital interactions are also excluded from its $H(N)$. Evidently, superfluid T_c for both fluids can be obtained by Eqn.46. Since experimental d and Δd of reasonably high accuracy are available for liquid ^3He, it is instructive to determine its T_c from Eqn.46 and compare it with experimental T_c to have an idea of its accuracy. Accordingly, we use the density data available from [52] to determine (i) $d = 3.935718$ Å at $T = 0.6$K at which the volume expansion (or onset of $He - He$ bond strain is observed), (ii) $d = 3.939336$Å at $T = 0.1$K and (iii) $\Delta d = 0.003618$ Å to find $T_c = 1.497$mK which agrees closely with experimental $T_c \approx 1.0$mK [54, 55]. The fact that no other theory [56] has predicted a T_c for liquid ^3He that falls so close to the experimental value, demonstrates the accuracy of Eqn.46.

Although, crystal structural data for widely different superconducting solids are available in the literature, and one can use these data to determine the inter-particle distance but what we need are the accurate values of d_c and Δd_c which, however, are not available. Consequently, we use Eqn.46 for the ce fluid only to estimate the range of typical values of T_c by using typical numbers for d_c and Δd_c. To this effect we first find that the force constant $C_o = 2.735$ dyne/cm (estimated from $C_o = 3h^2/4md^4$) related to f_o for liquid ^3He matches closely with $He - He$ single bond force constant ≈ 2.0 dyne/cm estimated from zero wave vector phonon velocity 182 m/sec [52]. A similar estimate of C_o for the f_o of ce**s** can be made by using (i) $d_c = 3.935718$ Å (*i.e.* as large as d_{He-He}) and (ii) as short as $d_c = 1.0$ Å which is expected to represent the typical d_c for superconducting solids. Using the standard value of electron mass $m_e = 0.9109\text{x}10^{-27}$ gm, we, respectively, find $C_o = 15\text{x}10^3$ dyne/cm and $C_o = 36.0\text{x}10^5$ dyne/cm which compares well with the typical force constants for a bond between two nearest neighbors in widely different solids. In view of this observation, we assume that the strain factor $\Delta d/d$ in superconducting solids approximately has the same value (= $9.1897\text{x}10^{-4}$) that we observe experimentally for liquid ^3He and use Eqn.46 to find $T_c = 8.23$ K for $d_c = 3.935718$ Å and $T_c = 124$ K for $d_c = 1.0$ Å which

closely fall in the range of experimentally observed, T_c ranging from 0 to $\approx$ 135 K under normal pressure. This not only shows the accuracy of Eqn.46 but also demonstrates its potential to explain the experimental T_c which does not differ significantly in its order of magnitude from 135 K.

6.7. Factors affecting T_c:

Since ce**s** in a solid move in an interacting environment, m appearing in Eqn.46 could be replaced by m^* (the effective mass of a ce). Evidently, T_c depends on channel size d_c, strain factor βl, and m^* which means that one may, *in principle*, change T_c at will if there is a method by which these parameters for a given solid can be suitably manipulated. However, any controlled change in these parameters does not seem to be simple.

For example we may apply pressure to decrease d_c in order to increase T_c but the compression produced by pressure may increase ce-lattice interactions in such a way that an increase in m^* may overcompensate the expected increase in T_c and one may find that T_c decreases with increase in pressure. Evidently, though T_c is normally expected to increase with pressure, its pressure dependence, for some superconductors, may show an opposite trend or a complex nature.

Similarly, we can take the example of a change in T_c with Δd which equals βc for a HTS system and βa for a conventional superconductor. Since βc is much larger than βa, lattice strain could be one factor which may increase T_c of a HTS system by a factor of c/a, if d_c, β, m^*, *etc.* for two types of systems do not differ. As analyzed by Leggett [57], T_c increases with the number of conducting planes (n_{cp}) per unit cell for certain groups of HTS systems indicating that T_c really increases with c, since c increases with n_{cp}. However, T_c does not increase always with n_{cp} [57] which means that the dependence of T_c on d_c, β, m^* and Δd is not simple. What we need is a comprehensive study of different possible mechanisms which may help in manipulating d_c, β, and m^* and increase T_c.

Our theory does not rule out the possibility of achieving *room temperature* (RT) superconductivity since increase of T_c from 124K to 300K (in the light of Eqn.46) simply requires a system where $(1/m^* d_c^2)(\Delta d/d_c)$ is increased from 1.0 to 2.5 which can achieved if m^* alone decreases from m to $0.4m$ or $\Delta d/d_c$ changes from 0.001 to 0.0025 or d_c is reduced by a factor of 1.6. Our theory also indicates that, as a matter of principle, any change or perturbation, which adds (*removes*) KE to (*from*) $q-$motions, will decrease (*increase*) T_c.

6.8. Strain energy of lattice:

The strain in lattice produced by (*say*) $i-$th ce is a local effect. Its magnitude depends on the quantum size $\lambda_i/2$ (*i.e.*, q_i) of the ce which renders $\epsilon_s = \epsilon_s(q_i)$. However, since identical local strains are produced by all ce**s** distributed uniformly in the solid, a collective long range impact of these strains can be observed due to strong inter-atomic forces, and the net strain energy of the lattice can be expressed as $E_s = E_s(q_1, q_2, q_3, ...)$. Evidently, a sustained exchange of energy between e1 and e2 through strained lattice (*i.e.*, by exchange of phonons) and an e1/e2 and lattice is expected when the channel size oscillating with a phonon frequency causes the quantum size of different ce**s** to oscillate with the same frequency.

The above stated phenomenon can be visualized by considering two ce**s** e1 and e2 separated by a small lattice block between them, as depicted in Fig.1(D-i) by two light color circles embedded dark color circles separated by a rectangular block. It may be noted that e1 and e2 gain (*lose*) energy from (*to*) the strained block when it performs a kind of breathing oscillation with an expansion (*contraction*) in its length leading to decrease (*increase*) in its strain (*cf.*, Fig.1(D-ii) and corresponding strain energy. This will also render a decrease (*increase*) in the size of two channels occupied by e1 and e2 causing corresponding increase (*decrease*) in ε_o; a ce with an increased (*decreased*) energy is depicted by smaller (*larger*) size circle. However, if the position of the said block oscillates around its CM without any change in its size (Fig.1(D-iii), e1 and e2 exchange energy with each other. If the block moves towards e1, it decreases d_c for e1 and increases d_c for e2, and in this process ε_o(e1) increases at the cost of ε_o(e2) and *vice versa*; the necessary energy flows from e2 to e1 and *vice versa*, obviously, through an appropriate mode of phonon in the lattice block.

The dynamics of atoms in a solid is far more complex than the two motions that we considered in the above examples. However, the said examples clearly explain how ce**s** exchange energy with strained lattice or how two

ce̊s exchange energy through phonons that propagate in the lattice block separating them. While the observation of superconductivity at $T = 0$, at which no phonon exists in the system, seems to question the phonon mediated correlation between two ce̊s, our theory finds that the strain energy E_s, which stays with the entire lattice even at $T = 0$, can serve as a source of necessary phonons to mediate correlated motion of ce̊s by an energy exchange between them at all $T \leq T_c$ including $T = 0$.

6.9. Order parameter(s):

The ce̊s in their superconducting state are in the ground state of their $q-$motions with free energy $N\varepsilon_o - E_g(T)$. Since $N\varepsilon_o$ is a constant value, only $E_g(T)$ is crucial for different aspects of superconducting state. Evidently, $E_g(T)$ (or its equivalent $E_s(T)$ representing lattice strain) can be identified as the basic OP of the transition. We note that the ce̊s below T_c assume a configuration characterized by : (i) some sort of localization in their positions in the real space unless they are set to move in order of their locations, (ii) an ordered structure in $\phi-$space defined by $\Delta\phi = 2n\pi$ with $n = 1, 2, 3, ...$, (iii) definite momentum $q = q'_o$, (iv) definite orientation of their spins *as preferred by different interactions involving spins* (*cf.* Section 6.11), (v) definite amount of superfluid density ρ_s (*cf.* Sections 7.2 and 7.3), etc. Naturally, ce̊s at $T \approx T_c$ must have large amplitude fluctuations in their positions, which can obviously lead to charge density fluctuation, $\phi-$fluctuation, momentum fluctuation, spin fluctuation, ρ_s-fluctuation, *etc.* which can be easily visualized to have a coupling with the lattice strain which our theory concludes as the basic OP of the transition. However, the nature and strength of coupling may differ from system to system. Evidently, it is not surprising if different people underline different aspects of the ce̊ fluid as the OP of superconducting transition in different systems.

6.10. Comparison with normal state:

In what follows from the above discussion and Eqn.(17), the relative configuration of two ce̊s in the normal phase of the ce̊ fluid (*i.e.*, at $T > T_c$) can have $< r > \geq \lambda/2$ implying $q \geq \pi/d_c$ and $< \phi > \geq 2n\pi$ (with n = 1,2,3, ...). This means that ce̊s in general have random distribution in $r-$, $q-$ and $\phi - spaces$. They have relative motions, mutual collisions and collisions with lattice (the walls of the channels through which they move) and no phase relationship in their motions. Naturally, they have all reasons to be incoherent in their motions and encounter resistance for their flow.

On the other hand every two ce̊s in LT phase (*i.e.*, at $T \leq T_c$), have $< r >= \lambda/2$ (which means $q = q_o = \pi/d_c$) and $< \phi >= 2n\pi$. They, obviously, cease to have relative motions and mutual collisions. They do not collide even with lattice because their quantum size fits exactly with the size of the channels through which they flow. They keep definite phase relation in their motions. If they are made to move, they move coherently in order of their locations without any change in their relative positions. Naturally, they have no reason to encounter resistance for their flow for which the system is found to exhibit superconductivity. One may identify this difference in the states of ce̊s in normal and superconducting phases with the difference in the random positions and random movements of people in crowd and the ordered positions and orderly movement of parading soldiers of an army platoon.

6.11. Co-existence with other properties:

The fact, that ce̊s in their LT phase have an orderly arrangement in their positions and they cease to have mutual collisions and collisions with lattice (*cf.* Section 6.10), clearly shows that ce̊s in the superconducting state have right environment for definite orientations of their spins for which they can have well defined magnetic state (*viz.*, diamagnetic or ferro-magnetic or anti-ferromagnetic) as decided by the different interactions such as spin-spin interaction of ce̊s, spin-lattice interaction, *etc.* Evidently, our theoretical framework finds no compelling reason for the superconducting state to be only diamagnetic, as concluded by BCS theory. In fact the magnetic nature of the superconducting state of a particular solid should be governed by the condition of minimum free energy with respect to an appropriate order-parameter. The diamagnetism found with most of the superconductors and the co-existence of ferro-magnetism or anti-ferro-magnetism with fewer superconductors, should be a simple consequence of this condition. For the similar reasons, we may argue that pairing of ce̊s can also occur in triplet $p-$state or singlet $d-$state.

6.12. Principles of superconductivity:

Recently, Mourachkine [58] analyzed general principles of superconductivity from the standpoint of practical realization of RT superconductivity. He observes that : (i) RT superconductivity, if ever realized, would not be BCS type, (ii) the quasi-particle pairing which takes place in momentum space could possibly take place in real space and if it happens BCS theory and future theory of unconventional superconductors can hardly be unified, (iii) the mechanism of ce pair formation in all superconductors differs from the mechanism of Cooper pair condensation, (iv) the process of ce pairing precedes the process of Cooper pair condensation, *etc.* In this context our theory reveals the following:

(a) The main factor, which induces an indirect attraction between two ces necessary for the formation of their bound pairs, is a kind of mechanical strain in the lattice produced by the zero-point force of ces; while this fact supplements the BCS model in limited respect, at the same time, it underlines the fact that the real mechanism of pairing of ces responsible for superconductivity of widely different solids differs from BCS theory.

(b) The quasi-particle ce pairing takes place not only in momentum space as envisaged by BCS model but in certain sense it occurs in $r-$ and $\phi-$spaces (*cf.* Section 6.10).

(c) The conditions, in which ce pair formation is possible, exist at $T \leq T_o$, however, the conditions in which (**q**, -**q**) bound pairs have their stability exist only at $T \leq T_c$ (orders of magnitude lower than T_o). This clearly shows that the process of bound pair formation precedes the process of pair condensation.

As such these points indicate that our inferences agree to a good extent with the basic principles of superconductivity as envisaged by Mourachkine [58]. However, in variance with some of his observations, ces occupying (**q**, -**q**) bound pair state (with a binding induced by an act of f_o) and a phonon assisted process of energy exchange between them are unquestionably found to be universal and basic aspects of superconductivity and related properties of widely different superconductors. The BCS model suffers for its weakness arising due to its use of SPB to describe the ce fluid (*cf.* Appendix-I).

7. Consistency with Other Theories and Existence of Electron Bubble:

7.1. BCS theory:

Although, our theory based on first quantization reinforces two basic inferences of the BCS theory [2], *viz.* : (i) the formation of (**q**, -**q**) bound pairs of ces and their condensation as the origin of superconductivity, and (ii) phonons as a means of an energy exchange between two ces, it clearly differs from this picture on several points. For example, while BCS theory identifies (**q**, -**q**) bound pair as a unit of two freely moving electrons with momenta **q** and -**q** with non-zero binding only in momentum space, our theory identifies that each of the two electrons is in a quantum state represented by a SMW (resulting from a superposition of two waves of momenta **q** and -**q**) and they have non-zero binding in $r-$, $q-$ and $\phi-$spaces. Similarly, while BCS theory identifies the electrical polarization (a kind of electrical strain produced the charge of ce) of the lattice constituents as a main source of binding, our theory finds that the mechanical strain in the lattice produced by an act of f_o (Sections 3.4.5 and 6.2) is the main factor responsible for ce-lattice direct binding and ce-ce indirect binding through strained lattice. Since phonons are basically mechanical waves, they have direct relation with the said strain and corresponding energy stored with the lattice for their involvement in the process of energy exchange between two ces. However, this inference does not exclude other possible mechanisms from contributing to the binding energy of ce pairs; the electrical polarization emphasized by BCS model can also contribute to the said binding. Interestingly, it may be noted that the mechanical strain alone predicts a $T_c \approx 124$K (*cf.* Section 6.6), while the electrical strain in BCS picture accounts for a $T_c < \approx 25$ K only. Assuming that both strains contribute in all systems in ratio of 25 : 124, it becomes clear that electrical strain contributes only around 16% which means that mechanical strain plays the primary role. Our theory further finds the following:

(i) The lattice in the superconducting phase stores an additional potential energy E_s (*cf.* Section 6.8) as its strain energy but the net energy of the system (ces + lattice) falls with the onset of (**q**, -**q**) bound pair formation. This implies that each ce in superconducting state assumes additional binding with rest of the system (strained lattice + $(N-1)-$ces) and $E_g(T) = N\epsilon_g$ is a kind of collective binding of all ces + strained lattice.

(ii) While two ces do not form a kind of diatomic molecule such as O_2, they certainly occupy states labeled by two different macro-orbitals distinguished by different $K-$values.

(iii) The strain energy E_s readily serves as a source of phonons which mediate the correlated motion between two ce's at all $T \leq T_C$ including $T = 0$ at which no phonon exists in the system. Evidently, our theory does not need a postulate that the said correlation is mediated by exchange of a *virtual phonon* between two ce's.

(iv) Our theory has the potential to explain superconductivity of widely different superconductors (conventional as well as non-conventional), while BCS theory does not.

(v) Our theory does not need a postulate that two ce's in their state of (**q**, -**q**) bound pair have a dynamics similar to a ball room dance as advocated by BCS theory to explain the state of Cooper pairs in momentum space. In stead it finds that each ce occupies a quantum state represented be the superposition of two plane waves of momenta (**q**, -**q**) pair; its direct binding with strained lattice or indirect binding with another ce in similar state is a simple consequence of the equilibrium between f_o and f_a as explained in Appendix=II and Ref. [40, 42, 43].

(vi). While BCS theory concludes that only those ce's which occupy states near Fermi surface fall in Cooper pair state, our theory concludes that the state of (**q**, -**q**) bound pairs is assumed by all ce's and they all participate in the phenomenon. In addition the ce-ce binding occurs not only in $q-$space (as advocated by BCS theory) but also in $r-$ and $\phi-$ spaces.

Since our theory too finds an energy gap $E_g(T)$ between superconducting and normal phases as a source of superconductivity and related properties, different aspects of superconducting phase such as coherence length, critical current, critical magnetic field, persistence of current, *etc.* can be understood by using their relations with $E_g(T)$ available in [43].

7.2. Two fluid theory:

We note that: (i) each ce represented by a macro-orbital has two motions, q and K, (ii) they have separate free energy contributions, $N\varepsilon_o$ and A' (Eqn.42) and (iii) the onset of superconductivity locks the $q-$motions of all ce's at $q = q_o$ with an energy gap $E_g(T)$ which isolates them from $K-$motions. Evidently, the superconducting state of the fluid at $T \leq T_c$ can be described by

$$\Psi^S(N) = \Pi_i^N \zeta_{q_o}(r_i) \sum_P^{N!} (\pm 1)^P \Pi_i^N \exp\left[i(P\mathbf{K}_i.\mathbf{R}_i)\right] \tag{47}$$

which has been obtained by using all $q_i = q_o$ in Eqn.(27); interesting as soon as we do so, all $N!$ micro-states $\Psi_n^j(N)$ appearing in Eqn.28 merge into one. We note that $\Psi^S(N)$ (Eqn. 47) can be expressed as $\Psi^S(N) = \Psi_K(N)\Psi_q(N)$ which is a product of two separate functions,

$$\Psi_K(N) = \sum_P^{N!} (\pm 1)^P \Pi_i^N \exp\left[i(P\mathbf{K}_i.\mathbf{R}_i)\right] \tag{48}$$

and

$$\Psi_{q_o}(N) = \Pi_i^N \zeta_{q_o}(r_i) \tag{49}$$

This implies that the ce fluid at $T \leq T_c$ can be identified as a homogeneous mixture of two fluids: (F1) described by $\Psi_K(N)$ where ce's represent some sort of quasi-particles described by plane waves of momentum K and (F2) described by $\Psi_{q=0}(N)$ where each ce represents a kind of localized particle in (**q**, -**q**) bound pair state (with $q = q_o$) where it ceases to have collisional motion. With all ce's having $q = q_o$, F2 represents their ground state in respect of their $q-$motions; evidently, each ce in this state has no thermal energy (*i.e.*, no energy above the zero-point energy, ε_o). Because the number of possible configurations with all ce's having $q = q_o$ counts only 1, F2 has *zero entropy*. Further since the ce's in F2 are basically localized, they move (*if they are set to move*) in order of their locations with no relative motion, no collision or scattering. Naturally, they find no reason to lose their flow energy which concludes that their flow should be resistance free implying that F2 represents the superconducting component of ce fluid.

Since each ce in the superconducting state has an energy gap (ϵ_g) with respect to that in normal state at T_c^+ (just above T_c), the former is stable against any perturbation (such as external magnetic field, flow of ce's at velocities

above certain values, *etc.*) of energy $< \epsilon_g$. Naturally, when this fact is clubbed with the coherent motion of macroscopically large number of ce s it becomes evident that the source of resistance should be strong enough to reduce the velocity of all such ce s in a single event which however is an impossible task when N is of the order of 10^{23}; this explains why super current persists for very long times.

As such we find that F1 and F2 at all $T \leq T_c$ have all properties that have been envisaged in [59] in the normal fluid and superfluid components of the ce fluid which implies that our theory provides microscopic foundations for the two fluid phenomenology. We note that Bardeen [60] also analyzed BCS theory [2] as the microscopic basis of two fluid theory.

Two fluid theory assumes that superfluid density $\rho_s(T)$ and normal fluid density $\rho_n(T)$ (with total density $\rho(T) = \rho_s(T) + \rho_n(T)$) under any cause such as temperature difference between two regions flow in opposite directions. So far no microscopic theory has provided a clear reason for it. However, since the strain energy $E_s(T)$ of the lattice increases with decrease in T from its minimum value $E_s(T_c) = |E_g(T_c)| = 0$ to maximum value $E_s(0) = |E_g(0)| = N|\epsilon_g(0)|$, it is clear that while thermal excitation energy representing $\rho_n(T)$ flows from high T region to low T region, $\rho_s(T)$ represented by $E_s(T) = |E_g(T)|$ flows in opposite direction; note that $\rho_s(T)$ can be correlated with $E_s(T) = |E_g(T)|$ because it is assumed to increase from its minimum value $\rho_s(T_c) = 0$ to a maximum value $\rho_s(0) = \rho$. Further since $E_s(T)$ depends on $q_1, q_2, q_3 ... q_N$ of N ce, it can serve as the origion of phonon like waves of the **o**scillations of these **m**omenta around q_o which are named as omon. Similar waves are also sustained in superfluid state of liquid ^{4}He [43] and they are discussed in detail in [50]. While phonons serving as the carriers of KE in the system flow from high T region to low T region, omons serving as the carriers of $E_s(T)$ a kind of its potential energy flow from low T region to high T region. In what follows our theory provides clear reasons for the flow of ρ_n and ρ_s in opposite directions.

7.3. $\Psi-$ theory:

In what follows from Eqn.(49), F2 can be described by

$$\Psi_{q_o}(N) = \Pi_i^N \zeta_{q_o}(r_i) = \sqrt{\mathrm{n}} \tag{50}$$

(with $\mathrm{n} = N/\mathrm{V}$ being the ce number density). We note that each ce in $(\mathbf{q} - \mathbf{q})$ configuration under the influence of a perturbation that makes it move with a momentum say $\Delta\mathbf{K}$ assumes ($\mathbf{q}$ + $\Delta\mathbf{K}$, -$\mathbf{q}$ + $\Delta\mathbf{K}$) configuration which is described by

$$\zeta(r, R) = \zeta_{q_o}(r) \exp(i\mathbf{Q}.\mathbf{R}) \tag{51}$$

with $\mathbf{Q} = 2\Delta\mathbf{K}$. Evidently, superconducting state under such a perturbation would be described by

$$\Psi'_{q_o}(N) = \Pi_i^N \zeta_{q_o}(r_i) \exp(i\Phi) = \sqrt{\mathrm{n}} \exp(i\Phi) \tag{52}$$

with its phase $\Phi = \sum_i^N \mathbf{Q}_i.\mathbf{R}_i$ and $\mathbf{Q}_i = 2\Delta\mathbf{K}_i$. However, for the phenomenological reasons (*viz.* the number density of superconducting electrons (n_s) need not be equal to n) we replace Φ by $\Phi + i\Phi'$ and recast $\Psi'_o(N)$ as

$$\Psi'_o(N) = \sqrt{n_s} \exp(i\Phi) \tag{53}$$

which renders $\mathrm{n}_s = \mathrm{n} \exp(-2\Phi')$. We note $\Psi'_o(N)$ clearly has the structure of $\Psi-$function that forms the basis of the well known $\Psi-$theory of superfluidity. This shows that our theory provides microscopic foundation to the highly successful $\Psi-$theory [20].

7.4. Theory based on the proximity of a QPT:

In view of Sections 7.2 and 7.3, superconductivity is a property of F2 in its $T = 0$ state. This implies that superconducting transition is, basically, a *quantum phase transition* that occurs in F2 exactly at $T = 0$. However, the stability of F2 against small energy perturbation and its proximity with F1 makes it appear at non-zero T in the real system which represents a homogeneous mixture of F1 and F2. Our theory finds that each particle participate in F1 and F2 simultaneously; it does not support the view that some particles participate in F1 and rest in F2. F1 and F2 manifest as two separated fluids at all $T \leq T_c$ for the presence of the energy gap; as soon as the gap vanishes at $T > T_c$, the said separation too ceases to exist. Evidently, our theory is also consistent with the idea which relates superconductivity with the proximity effect of a quantum phase transition [15].

7.5. Theories of other SIFs such as liquid ^{3}He:

The present theory can be applied to any other system of HC fermions with weak inter-particle attraction (*viz.* liquid ^3He) by simply assigning the role of *lattice structure* to the atomic arrangement of neighboring ^3He atoms around a chosen ^3He atom whose $q_o = \pi/d$ is decided by $d = (V/N)^{1/3}$. In this context, it may be mentioned that no other theory has been able to obtain superfluid $T_c \approx 2$mK for liquid ^3He which agrees closely with its experimental value $\approx$ 1mK (*cf.* Section 6.6). In addition, we also explained [61] the experimentally observed $P-$dependence of superfluid T_c of liquid ^{3}He by using Eqn.20 and in a forthcoming paper we would to discuss the application of this theory to liquid ^{3}He in detail.

7.6. Existence of electron bubble:

An excess electron in liquid helium exclusively occupies a self created spherical cavity (known as electron bubble) of certain radius when it assumes its *ground state* in the cavity; to create the said cavity, electron exerts its zero-point force on helium atoms in its surroundings and works against the forces originating from inter-atomic interactions and external pressure on the liquid [62,63]. We note that the state of the electron in a drifting bubble is represented by a waveform which exactly matches with a macro-orbital ξ_i (Eqn.22) since the electron for its localization in the spherical cavity has zero-point motion identified by its zero-point momentum q_i and its drifting motion identified with K_i. Similarly, while its position in the bubble is identified by r_i and that with the drifting bubble is represented by R_i. Thus the reality of the existence of electron bubble, where two different motions of the electron are clearly identified, provides strongest experimental proof for a quantum state described by macro-orbital ξ_i (Eqn.22). This is, particularly, significant when a ce occupies its ground state in a conductor where it experiences short range strong repulsion with its surrounding atoms/ions. The way, an electron uses its f_o to displace the He atoms to create a bubble, the same way a ce strains the lattice around its location. As such the basic foundation of our theory is strongly supported by the existence of electron bubble.

8. Concluding Remarks:

Following the fundamental principles of wave mechanics, we note that first quantization approach renders a theory of unquestionable accuracy (*certainly the degree of accuracy depends on the order of approximation used in dealing with the interactions*) if the solutions of the Schrödinger equation of the system are correct. To this effect we find that in the present case even the microscopic structure (*i.e.*, a macro-orbital representing the wave function of the state of a ce) of $N-$body wave functions obtained as the solutions of the Schrödinger equation (Eqn.2) of N ces, is supported by experimental observation of electron bubble (*cf.*, Section 7.6). In addition, as discussed in Sections 6.0 and 7.0, our theory agrees closely with experiments in respect of different properties of superconductors and liquid ^{3}He. Guided by these facts, we hope that this theory would find its place as a viable theory of superconductivity and fermionic superfluidity.

1. For the first time, first quantization approach has been used to lay the basic foundation for the microscopic understanding of superconductivity. Accordingly, each ce (particularly, in low energy states of $N-$ces) is more accurately represented by a macro-orbital (*cf.*, Section 3.4.7) (*not by a plane wave*). The origin of superconductivity lies with the condensation of $(\mathbf{q}, -\mathbf{q})$ bound pairs of ces having limited resemblance with Cooper pairs in BCS theory [2]. The formation of the said pairs is, unequivocally, a game of two opposing forces, f_o and f_a (Section 6.0) which lead to a mechanical strain in the lattice and a ce-ce indirect binding mediated by phonons or omons (Section 7.2) in strained lattice.

2. In principle, our approach does not exclude any component of $V'(N)$ (Eqn.1) from contributing to the process of bound pair formation. Naturally, the electrical polarization (a kind of electrical strain produced by the ce charge) of lattice constituents, spin-spin interaction, spin-lattice interaction, *etc.* can, obviously, have their contributions to this process. However, a study of these contributions of $V'(N)$ would be reported in our future publications.

3. As inferred by BCS theory [2], our theory also concludes an energy gap $(E_g(T)$ between superconducting and normal states of the ce fluid. Consequently, $E_g(T)$ related properties of a superconductor can be explained by using relevant relations available in [2]; we also obtained such relations in context of the superfluidity of He-II [43]. However, it should be noted that our relation for $E_g(T)$ (Eqns.44 and 45) differs from that inferred by BCS theory and for this reason T_c (*cf.* Eqn.46) concluded by our theory finds no upper bound. Eqn.46 not only

accounts for the highest $T_c \approx 135$ K (under zero pressure) that we know to-day but also reveals a possibility of observing superconductivity at RT provided $\Delta d/m^* d_c^3$ factor for a material is higher than the corresponding values in known superconductors. This inference is supported by the fact that T_c increases, in general, by increase in pressure (expected to decrease d_c) on a superconductor. It appears that materials of higher T_c can be designed if we understand how to increase strain factor $\Delta d/d_c$ or decrease m^* and d_c.

4. The process through which current carrying particles, electrons/holes, come into existence at a $T \geq T_c$ is unimportant for superconducting behavior of a system; what is important is that free charge carriers exist at $T \geq T_c$. This indicates that our approach is also applicable to the systems with holes as charge carriers. In fact the flow of holes is nothing but the flow of electrons (*once again through the narrow channels*) by way of hopping between successive electron vacancies.

5. Our theory finds that superconductivity is basically a property of the ground state of $N-$**Ce**s where each **Ce** is identified as a *part* or *representative* of $(\mathbf{q}, -\mathbf{q})$ bound pair with q having its ground state value, $q_o' = \pi/d_c'$. Excess energy of a thermally excited **Ce** due to non-zero T of superconducting phase corresponds to a flucuation in $\mathbf{q}$ by $\Delta\mathbf{q}$ around $|\mathbf{q}| = q_o'$ or to its $K-$motion with $\varepsilon_K \geq E_F + \epsilon_g$. Following the argument behind Eqn.(51), the said fluctuation in $\mathbf{q}$ also appears as a change in $\mathbf{K}$ by $\mathbf{Q} = 2\Delta\mathbf{q}$. The said excess energy can propagate from one **Ce** to another **Ce** in the superconductor through a phonon which is produced by the former **Ce** by losing its state of excitation and is abosrbed by the latter which moves to its excited state; this effectively means that the excitation moves from the location of the former to that of the latter by using phonon as a carrier of this energy. We note that these events are made possible by the fact that the energy of **Ce**s and lattice depends on strain as a common factor as demonstrated by the simplest possible analogy (where common factor x also represents a strain) discussed briefly in Appendix-II and in details at Sections 4.3 and 4.4 of Ref.[53]. The system specific or class specific properties of the superconducting state, obviously, depend on how $V'(N)$ affects the superconductivity that we conclude in this paper. It is clear that our theory does not forbid: (i) pair formation in triplet $p-$state and singlet $d-$state as well as (ii) the coexistence of superconductivity with ferro-magnetism or anti-ferromagnetism.

6. Although, pseudo-gap and charge stripes observed experimentally in HTS systems are not analyzed in this paper, however, we note that these observations could be related to some of the basic conclusions of this study. While the pseudo-gap appears to have its relation with the conclusion that the formation of bound pairs of **Ce**s comes into existence at $T^* > T_c$, the observation of charge stripes seems to find its origin with our inferences that (i) electric charges of superconducting electrons assume a kind of ordered and localised arrangement which allows them to move coherently in order of their locations, (ii) in HTS systems such electrons move in 2-D conducting channels having well defined separation c representing a unit cell size $\perp$ to the conduction plane, and (iii) during such a motion they can exchange energy with mechanically strained lattice as well as with other **Ce** through phonons. Interestingly, these points are consistent with similar suggestions reported in [64].

7. While our theory assumes that **Ce**s (representing HC particles) flow through narrow channels, it makes no presumption about the nature of the microscopic mechanism of superconductivity. Its all inferences are drawn from a systematic analysis of the solutions of the Schrödinger equation corresponding to an universal part of Hamiltonian, $H_o(N)$. In general our approach finds [42] that a SIB or SIF described by $H_o(N)$ exhibits superfluidity/ superconductivity if its particles have inherent or induced inter-particle attraction and the system retains its fluidity at $T \leq T_o$. The mathematical formulation of our theoretical framework is simple and it has great potential for developing equally simple understanding of different aspects of superconductivity and related behavior of widely different superconductors.

8. Over the last three decades, one of the major thrusts of researches in the field of superconductivity has been to find the basic mechanism which can account for the experimentally observed high T_c. Interestingly, the present work has succeeded in achieving this objective; it not only provides a clear picture of the ground state configuration of **Ce**s but also helps in finding the origin of inter-**Ce** correlations in $q-$, $\phi-$ and $r-$spaces required to understand the transport properties of superconducting phase. As evident from Sections 5.0 and 6.0, the present study also reveals that **Ce** fluid in solids should behave like: (**i**) a system of non-interacting fermions at $T > T^*$ at which **Ce**s can be represented by plane waves, (**ii**) a Landau-Fermi liquid (with quasi-particle mass $\approx 4m$ which may, however, be modified due to interacting environment seen by **Ce**s) at $T^* > T > T_c$ when they are better represented by macro-orbitals, and (iii) a singular Fermi liquid at $T \leq T_c$ when the system becomes a superconductor and **Ce**s assume a state of stable (**q**, -**q**) bound pairs. Varma *et.al.* [56] have elegantly introduced the subject related to these three phases of different properties, -a SIF is found to have.

9. The macro-orbital representation of a particle, which finds unquestionable experimental support for its accuracy (*cf.*, Section 7.6), not only renders a simple method of finding the solutions of $N-$body Schrödinger equation of a system such as ce fluid (studied in this paper) and liquid ^{4}He [43] but also helps in developing its complete microscopic theory with clarity of physical arguments, accuracy of results and unparalleled simplicity of mathematical formulation which represent the merits that a theory should have.

10. Recently Anderson [65] has strongly argued against the Cooper type pairs of ces, having phonon induced binding in momentum space, as a source of superconductivity of HTS systems. In addition, guided by the most recent experimental observation of the existence of real space localized Cooper pairs by stewart et al [66] in different solids, Huang [67] not only argues that real space ce-ce interactions can play an important role for pairing ces in HTS but also emphasises that BCS theory is *fundamentally wrong*. To this effect, our theory concludes that it is the real space interaction clubbed with WP manifestation of ces which produces ce-ce correlations in all the three spaces (real, momentum and phase) and renders superconductivity below T_c. Very recently Eagles [68] has summed up the claims of observing superconductivity at room temperature (RT) and even at higher T. Interestingly, these claims (if true) are certainly consistent with our theory and to this effect it is, particularly, significant that the experimentally observed T_c is found to depend on the length of $c-axis$ [57] and a related parameter named as partial weight ratio [69] which seems to have qualitative agreement with our relation (Eqn.20). However, the observed dependence is not as simple as it appears from Eqn.20. This could be because the difference in c values between two HTS are likely to follow differences in other parameters such as β, d_c, m^*, *etc.*

11. Ever since the experimental discovery of superconductivity on April 8, 1911 by Onnes [70], a long time of more than 105 years is lapsed but a microscopic theory which explains the phenomenon has been awaited for so long. In what follows from Appendix-I, the reason for this situation lies with the use of SPB with plane wave representation of particles in developing the desired theory. To this effect it has, somehow, been argued that second quantization approach greatly simplifies the problem and first quantization approach makes the task impossible. However, the said SPB (Appendix-I) used as an integral part of second quantization approach is inconsistent with LT physical realities of the system. This is evident from the fact that none of such theories (including BCS theory) of superconductivity could emerge as a viable theory of the phenomenon. Interestingly, *contrary to the said argument of the users of second quantization*, our approach of macro-orbital representation of a particle helps: (i) in finding the first quantization solutions of $N-$particle Schrödinger equation, (ii) in concluding the basic origin of superconductivity (reported here), and (iii) in discovering the long awaited theory of superfluidity of a SIB like liquid ^{4}He [43]. System specific or class specific modifications in the present theory can be determined by using $V'(N)$ (after identifying its appropriate details in a given case) as perturbation on the states of $H_o(N)$ (Eqn.2). Thus our theory provides necessary foundation for an accurate and simplified microscopic understanding of different superconductors and other MBQS(s). Further since our theory makes far less assumptions than other theories, it is consistent with the well known philosophical principle, -the Occam's razor, which states that the explanation of a phenomenon should make as few assumptions as possible or the simplest solution to a problem is preferable to more complicated solutions.

12. This paper clearly demonstrates that first quantization approach is suitably equipped to conclude the physics of a MBQS. A theory developed by using this approach has highly simplified mathematical formulation, clarity of physics and accuracy of results. This can be observed with the theory of superconductivity reported here and the theory of superfluidity of a SIB like liquid ^{4}He reported in [43]. Interestingly, we also find several reasons (Appendix-I) for which a many body quantum theory, based on any approach (*viz.*, second quantization) which uses SPB with *plane wave representation of particles*, is bound to have *limited success* in concluding the origin of LT properties such as superconductivity, superfluidity, *etc.*; this is corroborated by the fact that this observation holds true with all such theories developed over the last seven decades. We hope that this study would help in finding the right direction for developing superconducting materials of higher and higher T_c.

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Appendix-I:

Plane wave representation of particles and single particle basis (SPB):

Formulation of microscopic theories of widely different many body quantum systems (MBQS) such as ce fluid in solids, liquid ^{4}He, *etc.* use SPB with plane wave (Eqn.3) representation of particles. In other words, each particle in the system is basically considered to be a free particle and its momentum (**p**) and corresponding energy ($\epsilon = \hbar^2 p^2/2m$ with p being expressed in wave number) are assumed to be good quantum numbers in every state of the system with a possibility that p and ϵ can have any value between 0 and ∞. However, a critical examination of a MBQS (*as reported below*) reveals that the plane wave representation of a particle is inconsistent with two physical realities pertaining to its state in the system at LTs; in addition it finds reasons for which such theories could not achieve desired success in explaining the origin of LT properties (such as superconductivity, superfluidity and related aspects) of different MBQS.

(Reality-1): As evident from the experimental observations, it is amply clear that the LT behavior of a MBQS below certain temperature is dominated by the wave nature of its constituent particles and this arises when their de Broglie wave length becomes larger than their inter-particle distance. Since particles in such a situation are bound to have their *wave superposition* as a natural consequence of *wave particle duality*, their quantum states, to a good approximation, are described by $\psi(1,2)^{\pm}$ (Eqn. 11) *not be plane waves* (Eqn.3). In what follows $\psi(1,2)^{\pm}$ (Eqn. 11) [reformulated as $\zeta(r,R)^{\pm}$ (Eqn.12)] is not an eigen function of momentum operator (-$i\hbar\partial_{r_i}$) or energy operator (-$(\hbar^2/2m)\partial^2_{r_i}$) of any individual particle ($i = 1$ or 2). In stead $\zeta(r,R)^{\pm}$ is an eigen state of the energy operator of a pair of particles (*cf.*, Section 3.4.3). Evidently, particles in the LT states of a MBQS unquestionably occupy $\zeta(r,R)^{\pm}$ state where momentum and energy of individual particle are not good quantum numbers. This physical reality of LT states is, evidently, ignored by all theories of different MBQS (such as BCS theory of superconductivity) using SPB with plane wave representation of particles.

(Reality-2): When particles of a MBQS lose their kinetic energy (KE) with falling T, their behavior at LTs is, obviously, dominated by $V(r_{ij})$; even the weakest component of $V(r_{ij})$ is expected to demonstrate its presence when they tend to have $T = 0$. Not merely a matter of argument or speculation, it is established by experimental observations. For example, it is a widely accepted fact that: (i) liquids ^{4}He and ^{3}He which exhibit superfluidity, respectively, at $T < T_\lambda = 2.17$ K and $T < T_c(\approx 1$ mK) do not become solid due to zero-point repulsion $f_o = h^2/4md^3$ between two nearest neighbor particles arising from their zero-point energy, $\varepsilon_o = h^2/8md^2$, and (ii) both these liquids exhibit volume expansion on their cooling through T_λ^+ (slightly above T_λ) and $\approx$ 0.6K [52] and this behavior is undoubtedly forced by none other than f_o. Evidently, f_o dominates the physical behavior of these systems over the entire range of T in which they exhibit superfluidity. In addition, the physical reality of the existence of electron bubbles in helium liquids [62,63] establishes how a quantum particle (electron) behaves when it occupies its ground state in a system whose particles have short range repulsion with it. The electron occupies maximum possible space by exerting its f_o on its nearest neighbors and this action calls for an opposing force f_a originating from $V(r_{ij})$ between the said neighbors. It is evident (*cf.*, Section 7.6) that the state of such an electron is represented by a macro-orbital ξ_i (a pair waveform, Eqn.22) not by a plane wave. Interestingly, it is clear that the plane wave representation of a particle renders no clue to the reality that particles in their LT states exert f_o on their neighbors because the energy of a free particle is not expected to depend on d. All these observations not only establish the inconsistency of SPB with the physical reality that f_o (a kind of two body repulsion) dominates the natural behavior of a MBQS in its LT states but also suggest the use of pair of particle basis (PPB) for the correct understanding of such systems or to convert SPB results into PPB by using appropriate relations and conditions as demonstrated in [71].

In principle, though the use of SPB with plane wave representation of particles in theories of different MBQS is mathematically valid, however, it is also noted that something which sounds mathematically correct is not always accepted in physics. For example, it is well known that mathematically sound solutions of the Schrödinger equation of several systems are accepted only when they are subjected to appropriate boundary conditions. As argued rightly in [72], the plane wave representation of particles is not always a useful starting point. For atomic structure, where electrons move around a positively charged point size nucleus, hydrogenic eigenstates are more useful basis functions, while for electrons moving in a constant magnetic field, Landau orbitals are more suitable [72]. Evidently, the use of SPB with plane wave representation of particles which appears to be reasonably suitable to describe the HT states of MBQS, does not remain equally appropriate for LT states where particles have their

wave superposition. Considering the well understood case of vibrational dynamics of a polyatomic molecule

which can be described, in principle, in terms of the oscillations of Cartesian coordinates (r_i) of atoms or internal coordinates (q_i) of the molecule (representing inter-atomic bonds, bond angles, *etc.*), or normal coordinates (Q_i), we note that a complete and clear description (consistent with experiments) is obtained only in terms of Q_i [*not in terms of* r_i, *or* q_i]. The reason lies with the fact that only Q_i represent the eigen states of the Hamiltonian H of the molecule or the $H-$matrix assumes its diagonal form only when Q_i (not r_i or q_i) are used as its basis vectors. By analogy since single particle states described plane waves do not represent the eigen states of the H of a MBQS or the $H-$matrix does not assume its diagonal form when plane waves form its basis vectors, theories using SPB with plane representation of particles are not expected to render complete and clear microscopic understanding that agrees with experiments. Interestingly, this is corroborated by the fact that such theories of superconductivity or superfluidity achieved only limited success in accounting for the experimentally observed LT properties of widely different MBQS in spite of numerous efforts made over the last seven decades.

In what follows, this analysis renders a *general principle* that any theory, such as BCS theory, developed by using SPB with plane wave representation of particles would not provide a complete, clear and correct microscopic understanding (*having close agreement with experiments*) of the LT properties, such as, superconductivity or superfluidity and related aspects of a MBQS. The results of such a theory can be made physically meaningful only when they are transformed to basis vectors (such as macro-orbitals) for which $H-$matrix of the system assumes its diagonal form. This has been demonstrated for liquid ^{4}He and similar systems in [71].

Appendix-II:

Electron-lattice and electron-electron binding and zero-point force:

Consider a system of (i) a quantum particle (QP) of mass m trapped in a box of size $d = l - a$ (see box on the left size in Fig.3) and (ii) a 1-D quantum oscillator (QO) [a particle of mass M attached to a spring S of force constant C and length a] placed side by side in a common 1-D box (*cf.* Fig.3) of infinitely rigid size of length l, partitioned, presumably, by a virtual wall at 1. Assuming that the WP of the QP (shown by a single loop of standing matter wave of size $\lambda/2 = d$) and the QO do not share any space with each other simultaneously and they are in their ground state, the sum of their energies is

$$E_o = \frac{h^2}{8md^2} + \frac{1}{2}\hbar\omega \qquad (II-1)$$

with $\omega = \sqrt{C/M}$ is the fundamental frequency of QO. However, if this system is left to itself, the QP can be seen to exert its zero-point force $f_o = h^2/4m(d+x)^3$ on the partition in its natural bid to have the least possible energy. In the process it tends to compress the spring S by x and calls for an opposing $f_a = Cx$. In the state of equilibrium between f_o and f_a, the partition is shifted from 1 to 3 (Fig.3(B)) with $x = \Delta d$ and we have,

$$\frac{h^2}{4m(d+\Delta d)^3} = \frac{h^2}{4md'^3} = C\Delta d \qquad (II-2)$$

Under the changed situation where the box length is increased from d to $d' = d + \Delta d$ and the spring S is compressed by $\Delta d = d' - d$, E_o changes to E'_o given by

$$E'_o = \frac{h^2}{8md'^2} + \frac{1}{2}\hbar\omega + \frac{C}{2}\Delta d^2 \qquad (II-3)$$

Using Eqns.(II-1), Eqn.(II-2) and Eqn.(II-3), we have

$$\epsilon_g = E'_o - E_o \approx \frac{h^2}{8m}\left[\frac{1}{d'^2} - \frac{1}{d^2}\right] + \frac{C}{2}\Delta d^2 \approx -\frac{h^2}{8md^2}\left[\frac{\Delta d}{d}\right] \approx -\frac{h^2}{8md'^2}\left[\frac{\Delta d}{d'}\right]. \qquad (II-4)$$

Evidently, ϵ_o of QP falls by $(h^2/8m)[d'^{-2} - d^{-2}] = -(h^2/4md^2)(\Delta d/d)$, while the strain energy in the spring goes up by $(C/2)\Delta d^2$ = $-(h^2/8md^2)(\Delta d/d)$. This indicates that the action of f_o not only makes energy of QP and that of QO to have inter-dependence through a common variable x but also reveals that the net ground state energy of the two falls by ϵ_g which implies that QP and QO have a state of mutual binding.

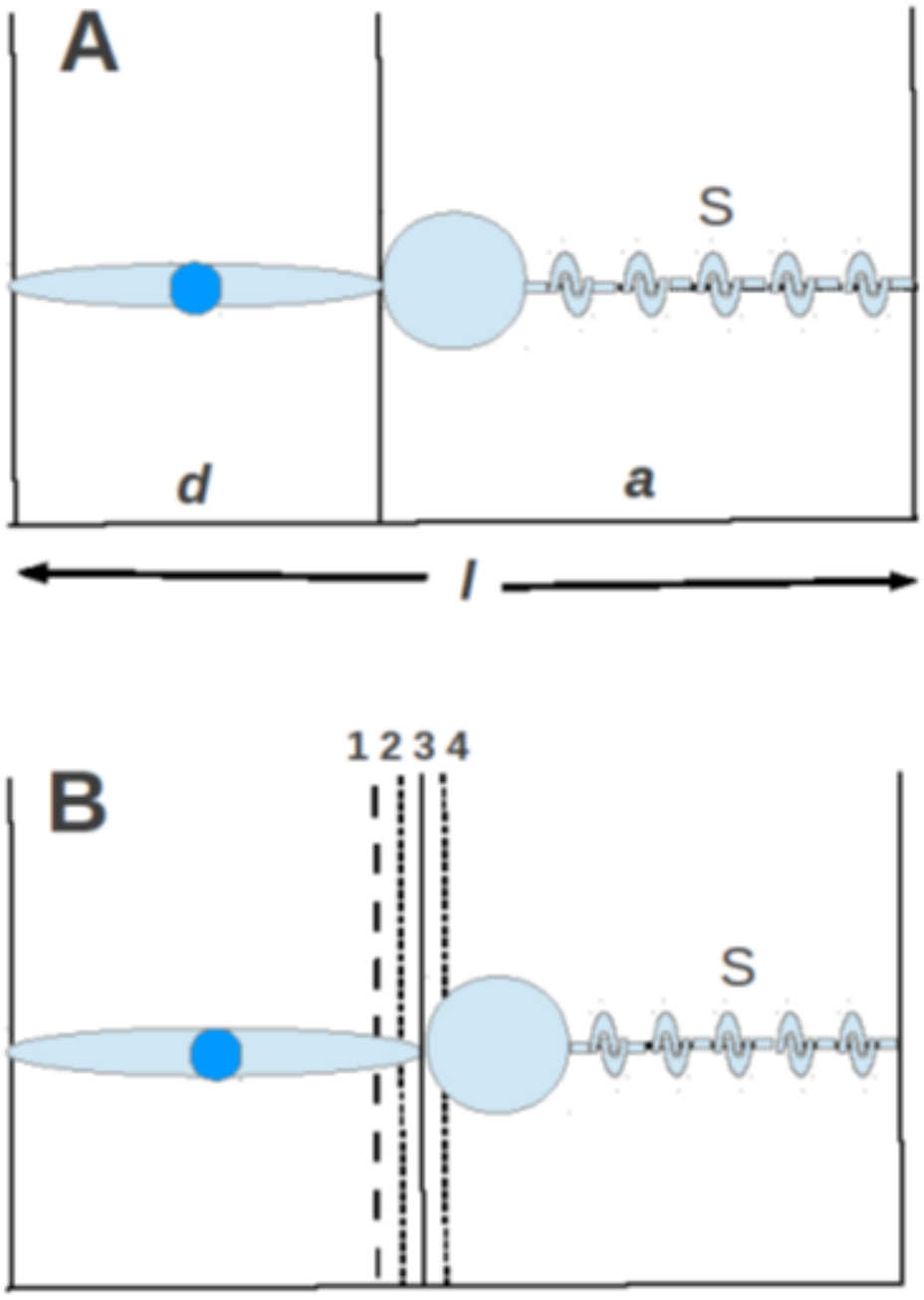

Figure 3: (A) A quantum oscillator (QO : a particle of mass M connected to a spring S) is placed in a box of size a on the right side of another box (size $d = l - a$) where a quantum particle (QP) of mass m is trapped in its ground state and (B) the QP exerts its zero-point force f_o and shifts the divider from position 1 to 3 when $f_o = h^2/4m(d+x)^3$ assumes the state of equilibrium with $f_a = Cx$ by which spring (S) opposes this action; in the process, the box size d increases to $d' = d + \Delta d$. When the system is made to oscillate around this equilibrium in such a manner that the position of divider wall oscillates between 2 and 4, the QP can be seen to gain (lose) energy when divider moves from 3 towards 2 (4). (For more details see Appendix-II).

Applying the above results to many ce's in their ground state representing the QP and oscillating atoms representing the QO, it can be easily understood that all ce's in their ground state assume a binding with all atoms in the solid which also implies a mutual binding [with an energy $2\epsilon_g$] between every two ce's [occupying (**q**, -**q**) pair state] indirectly induced by the act of f_o. Naturally, two ce's in this state of binding, keep their (energy, q, r and/or ϕ) correlations through different modes of phonons in the lattice block connecting the two. To this effect, it may be noted that if the partition (Fig.3(B)) is displaced by small $x < \Delta d$ and left to achieve its equilibrium, QP and QO can be seen to oscillate at a frequency close to ω and, in this process, QP (representing ce) will keep exchanging energy with QO (representing oscillations of the strained lattice). When ce loses a part of its ε_o, lattice absorbs it as strain energy and *vice versa* and this helps in visualising energy exchange between ce's and lattice/phonons.

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