THEORY OF SUPERFLUIDITY IN HELIUM- A REVIEW

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Abstract: In nature the inert gas helium exists in two forms, isotopes, with fundamentally different properties. The study of superfluidity and its effect on the property of helium liquid is considered to be principle interest of the researcher from the last century. In this paper a brief account of the property of helium isotopes and the physics involve in the superfluidity of ^{3}He and ^{4}He are being discussed. Also listed in this paper are different methods suggested and work done by various research groups to explain superfluidity and its related property in liquid helium.

Key words: ^{3}He; ^{4}He; superfluidity

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1. Introduction:

A superfluid is a state of matter in which the matter behaves like a fluid with zero viscosity and zero entropy. The substance, which looks like a normal liquid, will flow without friction past any surface, which allows it to continue to circulate over obstructions and through pores in containers which hold it, subject only to its own inertia. Fundamental understanding of the properties of such a liquid requires an advanced form of quantum physics, and these very cold liquids are therefore termed quantum liquids. By studying the properties of quantum liquids in detail and comparing these with the predictions of quantum physics low-temperature, researchers are contributing valuable knowledge of the bases for describing matter at the microscopic level [1,2].

2. Discovery of Superfluidity:

In nature the inert gas helium exists in two isotopes, ^{4}He and ^{3}He with former behaving as boson and later as fermion and thus having fundamentally different properties. The superfluidity in ^{4}He was known much before than ^{3}He. The superfluity in ^{4}He and ^{3}He is separately discussed below:

Helium-4:

The discovery of superfluidity in liquid ^{4}He was announced to the scientific world on 8 January 1938, when two short papers were published back to back in Nature. One was by Peter Kapitza [3] the director of the Institute for Physical Problems in Moscow, and the other was by two young Canadian physicists, Jack Allen and Don Misener [4], both working at the Laboratory at the University of Cambridge in the U.K. Both studies reported that liquid helium flowed with almost no measurable viscosity below the transition temperature of 2.18 K. Very soon afterwards, theoretical work by Lev Landau [5], Fritz London [6] and Laszlo Tisza [7] showed that this zero viscosity was evidence for a new superfluid phase of matter. We now understand that superfluidity is associated with the motion of a Bose–Einstein condensate. As a result, the quantum liquid exhibits macroscopic quantum effects that are visible to the eye, such as the ability of the liquid to flow up and out of a container and the famous fountain effect. Superfluid helium also became the testing ground for theories about collective behaviour in quantum many-body systems.

Helium-3:

David M. Lee, Douglas D. Osheroff and Robert C. Richardson [8] discovered at the beginning of the 1970s, in the low-temperature laboratory at Cornell University, that the helium isotope ^{3}He can be made superfluid at a temperature only about two thousandths of a degree above absolute zero. This superfluid quantum liquid differs greatly from the one already discovered in the 1930s and studied at about two degrees (i.e. a thousand times) higher temperature in the normal helium isotope ^4He. The new quantum liquid ^3He has very special characteristics. One thing these show is that the quantum laws of microphysics sometimes directly govern the behaviour of macroscopic bodies also.

Figure 1 (a) is the phase diagram of ^4He. It is a P-T diagram indicating the solid and liquid regions separated by the melting curve (between the liquid and solid state) and the liquid and gas region, separated by the vapor-pressure line. Later ends in the critical point where the difference between gas and liquid disappears. The diagram shows the remarkable property that ^4He is liquid even at absolute zero. ^4He is only solid at pressures above 25 bar.

Figure 1(a) also shows the λ-line. This is the line that separates two fluid regions in the phase diagram indicated by He-I and He-II. In the He-I region it behaves like a normal fluid; in the He-II region it is superfluid. The name lambda-line comes from the specific heat – temperature plot which has the shape of the Greek letter λ. figure 1(a), which shows a peak at 2.172 K, is the λ-point of ^4He. Below the lambda line the liquid can be described by the two-fluid model. It behaves as if it consists of two components: a normal component, which behaves like a normal fluid, and a superfluid component with zero viscosity and zero entropy.

The phase diagram of ^3He at low temperatures is shown in the figure 1(b). ^3He remains liquid if the pressure is less than approximately 34 atmospheres (3.4 MPa). ^3He enters into superfluid phase at temperatures below 0.0025 K. There are two superfluid phases, A and B, which both show very unusual properties. It was possible to identify altogether three distinct stable superfluid phases of bulk ^3He; these are referred to as the A, B and A1 phases. In zero magnetic fields only A and B phases are stable. In particular, in zero fields A phase only exists within a finite range of temperatures, above a critical pressure of about 21 bar. Hence its region of stability in the pressure temperature phase diagram has a roughly triangular shape as shown in Fig. 1(b). The B phase, on the other hand, occupies the largest part of this phase diagram and is found to be stable down to the lowest temperatures attained so far. Application of an external magnetic field has a strong influence on this phase diagram. First of all, the A phase is now stabilized down to zero pressure. Secondly, an entirely new phase, the A1 phase, appears as a narrow wedge between the normal state and the A and B phases. Although the three superfluid phases all have very different properties, they have one important thing in common: the Cooper pairs in all three phases are in a state with parallel spin (S = 1) and relative orbital angular momentum l = 1. This kind of pairing is referred to as “spin-triplet p-wave pairing”. They may be represented as $|\uparrow\uparrow\rangle$ with $Sz = +1$. $2^{-\frac{1}{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$ with $Sz = 0$, $|\downarrow\downarrow\rangle$ with $Sz = -1$

It should be noted that Cooper pairs in a superconductor and in superfluid ^3He are therefore very different entities: in the former case pairs are formed by point like, structureless electrons and are spherically symmetric, while in the case of ^3He Cooper pairs are made of actual atoms (or rather of quasi-particles involving ^3He atoms) and have an internal structure themselves [1, 2, 9].

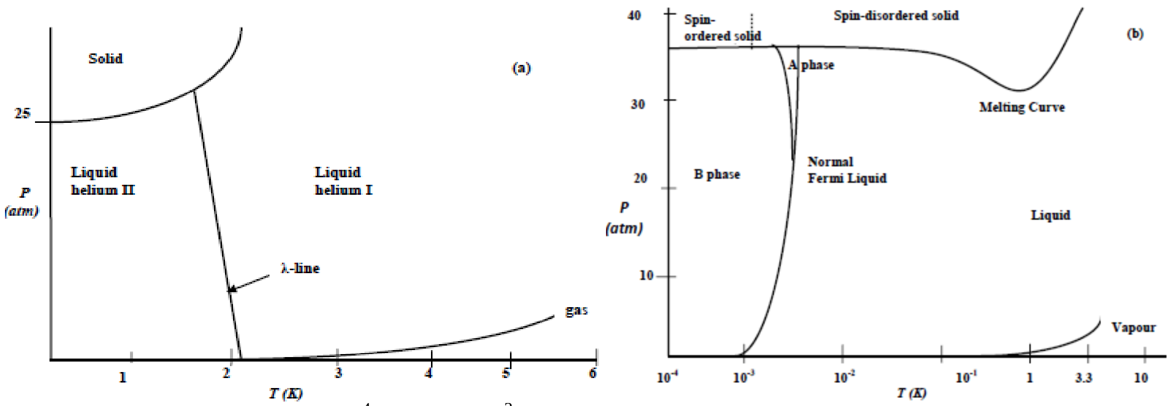

Figure 1: Phase diagram of (a) ^4He and (b) ^3He showing superfluidity and normal liquid state [a schematic representation adopted from 1].

3. Theoretical studies:

Owing to its unusual behavior, many theories of liquid helium have been profound and developed. An attempt to explain the λ-transition as a special kind of order-disorder transition was made by Frohlich [10] but the idea was aborted by London [6] who considered the phenomenon as a manifestation of BEC of ideal Bose gas distorted by inter-molecular forces.

The famous "two fluid theory" of Tisza [7] was a phenomenological theory developed to explain some of the important experimental results. According to the two fluid model of Tisza helium can be considered as a mixture of two fluids "superfluid" and "normal". Liquid helium is purely superfluid i.e the atoms are in the lowest energy state at zero temperature. With the increase of temperatures excited molecules are formed and these constitute the normal component. Tisza considered the motion of helium as some kind of turbulent motion because the velocity pressure dependence is more similar to turbulent than to laminar motion. The most important success of Tisza theory was the prediction of second sound in helium.

Based on London's idea Landau [5, 6] also developed another two fluid model. Landau's two fluid theories [6] also assume existence of two independent motions in helium. These two motions occur without momentum transfer from one to another and treated the superfluid flow by expressing the macroscopic hydrodynamical variables like density and velocity, as quantum mechanical operators. He further showed that the equations of motion for these operators implied the continuity equation and Euler's equation for an ideal fluid. In this two fluid model there are two kinds of excited molecules-phonons or quanta of longitudinal compressible waves and rotons. These molecules needed a minimum energy Δ to excite them. Although Landau's two fluid model can explained a very large fraction of the exotic behavior of helium without invoking the notion of BEC, it has several shortcomings, inconsistencies and quantitative disagreements with certain experimental results which were pointed out by Putterman [11].

Next significant contribution to the development of microscopic theory of helium came in 1947 from Bogoliubov [12]. Analysing a system of weakly interacting bosons he tried to overcome the basic objection to the London's idea of associating BEC to the unique behavior of helium-II. He considered liquid ^4He as a system of degenerate non-perfect Bose gas and used the method of second quantization to show that in the presence of small interactions between the atoms, the low excited state of the gas can be described as a perfect Bose gas of certain "quasi-particles" (elementary excitation). In this theory the fraction of atoms which condense into the p=0 state does not assume value 1 even at absolute zero. Furthermore these atoms can move without any friction with respect to the elementary excitation. This theory can reproduced Landau's "phonon-roton" spectrum of low Q. Bogoliubov's theory [12] was based on the assumption that liquid ^4He in a system of weakly interacting Bose gas but the inter-atomic interactions between helium atom to a good approximation are strong.

In 1953 Feynman [13] using the quantum path integral method showed that LHe-4 should exhibit a transition analogous to the transition of ideal Bose gas regardless of the strong inter-atomic interaction.

Different treatments were developed by different authors viz. De Boer [14], Chester [15], Miller et. al. [16], Lee and Mohling [17, 18] etc. In Chester theory of liquid ^4He, free energy was expanded in powers of a coupling constant g. The first term of the series gave the free energy of the London theory and lead to all the usual properties. According to this theory the transition at the lambda temperature is of third order but the second term in the expansion of free energy raises the transition to one of second order. Different pictures of roton are given by these authors. While de Boer considers roton as short wavelength longitudinal elastic mode, Chester proposed roton as quasi-particles with modified mass associated with the motion of ^4He atom. Lee and Mohling after examining the experimental data of the total cross section for the inelastic scattering of cold neutrons in helium II concluded that the projection of the angular momentum on the direction of the momentum, p of the roton is zero. This implies that the roton excitation has zero angular momentum and it loses rotational character. The idea of vortex line and quantization of superfluid circulation which was first introduced by Onsager [19] was further developed by Feynman.

A generalized mathematical description of BEC was developed by Penrose and Onsager [19]. Based on the first principle, Penrose and Onsager indicated that liquid helium-II in equilibrium shows BEC. The hard core repulsive part of the He-He interaction and the macroscopic occupation of a particular state pose serious difficulties in developing a viable microscopic theory of helium-II. Efforts were made by Beliaev [20], Hugenholtz and Pines [21] to overcome these difficulties. It was followed by the classic work of Bogoliubov on the dilute weakly interacting gas. However, since He atoms experiences strong repulsive which forbids any two helium atoms from occupying same point in real space, Bogoliubov theory is found to be inconsistent with the known experimental results.

Brueckner and Sawada [22] tried to overcome these difficulties by considering the system to be dilute one. The method was used by Goble and Trainer [23] to show that the condensate fraction varies from 0.37 to 0.79 as the hard core radius altered from 3.0 to 1.0^{o}A. Similar calculations were carried out by Bycking [24] but he allowed for all partial waves of Lennard-Jonnes potentials with a hard core radius of 2.6^{o}A. The numerical value obtained for excitation curve agrees qualitatively with those of the experimental results. Liu et. al. [25] showed that if two-body pseudo-potential is used then one can get a qualitative correct excitation spectrum.

Another alternative treatment to study strongly interacting bosons such as LHe-4 is the Correlated Basis Function (CBF) introduced by Feenberg and collaborators [26].

Various Monte Carlo methods such as Variational Monte Carlo used by Masserini et. al. [27], diffusion Monte Carlo used by Caperly et. al. [28, 29] and Moroni et. al. [30], Path Integral Monte Carlo used by Caperley and Pollock [31] and Green-function Monte-Carlo used by Whitlock and Panoff [32] to calculate the properties of LHe-4

4. Conclusion:

From the above discussion one can arrived to the conclusion that there is no single microscopic theory of liquid helium which can explained all its observed properties, different theories are needed to explain the various properties of liquid helium. So we in our future course of study we will further elaborate on these aspects which will be reported later [33, 34].

References:

[1] R. K Patheria, Statistical Mechanics, Pergamon, Oxford, 1979.
[2] L. D Landau and E. M Lifshitz, Course of Theoretical Physics, Series in Statistical Mechanics part 2, Pergamon Press, Oxford, 1980.
[3] P. Kapitza, Nature, 141, 74, 1938.
[4] J. Allen and D. Misener, Nature, 141, 75, 1938.
[5] L. D Landau, J. Phys. U.S.S.R, 11, 91, 1947.
[6] F. London, Nature, 141, 644, 1938
[7] L. Tisza, Phys. Rev., 72, 838, 1947.
[8] D. D Osheroff, Review of Modern Physics, 69, 667, 1997.
[9] A. J. Legget, Review of Modern Physics, 76, 999, 2003.
[10] H. Frohlich, Physica, 4, 639, 1937.
[11] S. J Putterman, Superfluid Hydrodynamics, North Holland/ American Elsevier, Amsterdam, 1974.
[12] Z. M. Galasiewicz, Helium-4, Pergamon Press, Oxford, 1971.
[13] R. P Feynman, Phys. Rev., 94, 262, 1954.
[14] D. Boer, In G. Careri, editor, liquid Helium, Academic Press, 1963.
[15] G. V Chester, In G. Caereri, editor, Liquid Helium, Academic Press 1963.
[16] A. Miller, D. Pines and P. Nozieres, Phys. Rev., 127, 1452, 1965.
[17] T. D. Lee and F. Mohling, Phys. Rev. Lett., 2, 284, 1959.
[18] D. K. Lee, Phys. Rev., 162, 134, 1967.
[19] L. Onsager and O. Penrose, Phys. Rev., 104, 576, 1956.
[20] S.T. Beliaev, Soviet Phys., JEPT, 7, 289, 1958.
[21] N.M. Hugenholtz and D. Pines, Phys. Rev., 116, 489, 1959.
[22] K.A. Brueckner and K. Sawada, Phys. Rev., 106, 1128, 1957.
[23] D.F. Goble and L.E.H. Trainor, Can. J. Phys., 46, 839, 1968.
[24] E. Bycking, Phys. Rev., 145, 71, 1966.
[25] L. Liu and K.W. Wong, Phys. Rev., 132, 1349, 1963.
[26] H.W. Jackson and E. Feenberg, Rev. Mod. Phys., 34, 686, 1962.
[27] G.L. Masserini, L. Reatto and S.A. Vitiello, Phys. Rev. Lett., 69, 2098, 1992.
[28] P.J. Reynolds, D.M. Caperley, B.J. Alder and W.A. Lester, J.Chem. Phys., 77, 5593, 1982.
[29] D.M. Caperley, J. Stat. Phys., 63, 1237,1991

[30] S. Moroni, S. Fantoni and G. Senatore, Phys. Rev. B, 55, 1040, 1997.
[31] D.M. Caperley and E.L. Pollock, Phys. Rev. Lett., 56, 351, 1986.
[32] P.A. Whitlock and R. Panoff, Can. J. Phys., 65, 1409, 1987.
[33] S. Dey, J. P Gewali, A. K Jha, L. Chhangte and Y. S Jain, Indian Journal of Physics, 85, 1309, 2011.
[34] Y. S. Jain, L. Chhangte, S. Chutia and S. Dey, Current Science, 101, 769, 2011.