MAXWELL'S EQUATIONS, THE RELATIVITY PRINCIPLE AND THE OBJECTIVITY OF MEASUREMENT

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Abstract: The interpretation of the velocity parameter in Maxwell's equations and the Lorentz Force law is examined. It is shown that the predicted trajectories of electrons in an electromagnetic field depend on the state of motion of the observer. This ambiguity can be removed by assuming that the origin of the field is the unique reference system for determining the electron velocity in these equations. The same convention is recommended for the definition of the momentum of particles in general. Accordingly, it is assumed that observers will only disagree on the results of measurements because they employ different systems of physical units in their respective rest frames. This situation needs to be recognized in applying Galileo's Relativity Principle (RP), particularly with respect to the transformation properties of physical laws in different rest frames.

Keywords: Maxwell's equations; Lorentz force law; four-vectors; asymmetric time dilation; Newton-Voigt Transformation (NVT); amended version of the relativity principle; objectivity of measurement

1. Introduction:

The recognition of the central role of the speed of light in free space in the theory of electromagnetism stimulated the development of relativity theory in the late 19th century. Four-vectors [1,2] are generally considered to be an indispensible feature of the resulting theory. The concept was first introduced by Minkowski [3] in an attempt to formalize new relations between space and time that were a key feature of Einstein's Special Theory of Relativity (STR) [4]. Pais [1] has pointed out that terms such as "spacelike vector" and "timelike vector" that are in regular use today in discussing this general subject originated with this 1908 paper. It is interesting to note that Einstein was originally dismissive of Minkowski's methods, referring to them as "superfluous learnedness" [1], although he later acknowledged that they had been helpful in his development of his general theory of relativity [5] less than a decade later. It was shown that the key concept of Lorentz invariance in STR [4] can be conveniently described in terms of a scalar product between space-time four-vectors. An application to the mathematical description of Galileo's Relativity Principle [RP] was also noted. It was the consensus of 19th-century physicists that satisfying the RP means that the laws of physics must be invariant to a space-time transformation. The concept of "covariance" was applied to four-vectors of various kinds in relativity theory in order to insure the desired invariance properties.

Recently [6,7], however, it has been pointed out that the time-dilation experiments which have been carried out over the years are actually at odds with Einstein's STR [4]. The transverse Doppler effect studies of Hay et al. [8] and the experiments with circumnavigating atomic clocks carried out by Hafele and Keating [9] in 1971, as well as the everyday experience with the Global Positioning System (GPS) [10], indicate that it is always possible to determine which of two clocks runs slower at any given time (asymmetric time dilation). However, the cornerstone of STR, the Lorentz transformation (LT), predicts on the contrary that it is a matter of perspective which of two clocks in relative motion runs slower (symmetric time dilation). As a consequence, the whole concept of four-vectors is called into question because it clearly rests on the LT and its prediction of Lorentz invariance. In the following, it will be shown that even the long-cherished belief in the requirement that the laws of physics need to be invariant to a Lorentz transformation requires modification after careful attention is paid to its consequences in the theory of electromagnetic interactions.

2. Inconsistency in the predicted trajectories of accelerated electrons:

The standard relativistic treatment of electromagnetic interactions is based on the premise that the components of the electric **E** and magnetic **B** field vectors transform according to the following equations [11] (c is the speed of light in free space, 299792458 ms^{-1}):

$$E_x' = E_x \qquad B_x' = B_x$$
$$E_y' = \gamma (E_y - vc^{-1}B_z) \qquad B_y' = \gamma (B_y + vc^{-1}E_z)$$
$$E_z' = \gamma (E_z + vc^{-1}B_y) \qquad B_z' = \gamma (B_z - vc^{-1}E_y). \tag{1}$$

Einstein derived this set of relations [4] by assuming that Maxwell's equations must be invariant to a Lorentz transformation (LT) of spatial and time coordinates between different rest frames. It was further assumed that the components of the electromagnetic force **F** on charged particles e are given in terms of the above field components by the Lorentz Force equation:

$$\mathbf{F} = e\,(\mathbf{E} + c^{-1}\mathbf{v} \times \mathbf{B}). \tag{2}$$

In this equation it has been generally assumed that **v** is the velocity of charged particles relative to the observer, a point which will prove worthy of further discussion subsequently.

There is ample evidence [12] that the Lorentz Force satisfies the equation of motion expected from Newton's Second Law, namely:

$$e\,(\mathbf{E} + c^{-1}\mathbf{v} \times \mathbf{B}) = d/dt\,(\gamma\mu\mathbf{v}), \tag{3}$$

i.e. the force **F** equals the time rate of change for the relativistic momentum $\mathbf{p} = \gamma\mu\mathbf{v}$, with $\gamma = (1-v^2c^{-2})^{-0.5}$ and μ is the rest mass of the particle/electron. Nonetheless, as will be seen from the following concrete example which makes use of this equation, there is still an uncertainty in the definition of **v** therein when the observer is located in a different rest frame than that of the laboratory.

Consider the effects of an electromagnetic field with only the two components, E_x and B_y, acting on an electron. From the point of view of an observer located at the origin of the field, the electron will initially move along the x axis. This is because the force **F** in eq. (2) only depends on E_x at the instant the field is applied since the value of v=0 negates any effect from the corresponding magnetic field component B_y. This situation changes as time goes by and the electron is accelerated to non-zero speeds. The $\mathbf{v} \times \mathbf{B}$ term in eq. (2) gradually produces a force component in the z direction, causing the electron to veer away from its initial path. Depending on the relative strengths of the constant values of E_x and B_y, the amount of deflection can be quite significant over time. This situation is easily reproduced in the laboratory and there is no doubt that it is consistent with the Lorentz Force law.

Next consider the same example from the perspective of an observer co-moving with the electron. Since the speed v of the electron relative to the observer is zero at all times, it follows according to the transformation law of eq. (1) as well as eq. (2) that the *magnetic field has no effect.* As a result one expects that, from the perspective of this observer, the electron continues indefinitely along a straight line parallel to the x axis. This predicted trajectory is therefore clearly distinguishable from that discussed first from the vantage point of the laboratory observer.

This behavior raises the question of whether it is reasonable to expect that the electron would appear to follow a different path for the two observers. No one has ever ridden along with an accelerated electron or other charged particle to verify that the predicted straight-line trajectory would actually be found by such an observer. Since the curved path expected from the laboratory perspective is routinely observed, however, it would therefore seem on the contrary *that the straight-line result is pure fiction,* an artifact of a physically unrealistic theory.
Does this example prove that Galileo's RP does not apply to electromagnetic interactions? Clearly not. The reason is because there is another quite straightforward way to satisfy both Maxwell's equations and the RP at the same time, namely to insist that all observers, regardless of their state of motion, *see exactly the same results of any given interaction*. In particular, the hypothetical observer co-moving with the accelerated electron must record the same curved trajectory as is viewed from the laboratory perspective.

The measured values for the parameters of the electron's path may still differ for the two observers, however. This is *because the units in which they express their respective measured values may not be the same.* We know, for example, from the time-dilation experiments [8,9] mentioned in the Introduction that the clocks they employ to measure elapsed times can run at different rates. This fact does not change the above conclusion about the trajectory of the electron in the above example, however. There is no reason to doubt that all observers should agree that a curved path is followed as a consequence of the interaction of crossed electric and magnetic fields.

3. The velocity definition:

There is a detail that needs to be considered in both Maxwell's equations and the Lorentz Force law which is crucial for deciding how to apply the RP to electromagnetic interactions. It is the interpretation of the velocity that appears in both expressions. At some point in history, physicists came to the consensus that **v** is the velocity of the electrons or other charged particles *relative to the observer* in any given interaction. This decision has quite important consequences vis-a-vis the measurement process in general. It means that the results of any measurement are thought to depend on the perspective of the observer. *Measurement is subjective*, in other words.

This was an astonishing departure from the prevailing attitude of physicists in the preceding centuries. It was previously taken for granted that measurement had an *absolute character* and that all observers could agree on values such as the length and weight of an object. A confusing aspect of measurement was clearly that each observer could express his measured results in a different set of units and therefore obtain different numerical values for the same quantity. However, this eventuality did not change the fact that people could always agree on which of two lengths or weights was larger. More quantitatively, it could safely be assumed that the ratio of two measured values of the same type must be independent of the choice of units. *Measurement was both rational and objective* and thus, if carried out properly, could serve as a fair basis for trading practices. Yet now, physicists were claiming that quantities such as electric and magnetic fields vary with the state of motion of the observer.

There is a clear alternative interpretation of the velocities which appear in Maxwell's equations and the Lorentz Force law, however, one which eliminates the need to assume that observers can disagree on the trajectories of particles affected by these interactions. It is simply necessary to assume that the variable **v** in these equations is the velocity of the electron *relative to the rest frame where the electromagnetic field originates*. This is a quantity which all observers can agree upon at least in principle. Just changing the unit in which velocity measurements are expressed can have no effect on the measured trajectory of the particle. In particular, an observer co-moving with the electron in the example of the previous section can therefore use Maxwell's equations and the Lorentz Force law to conclude that the path being followed is exactly the same as reported by his counterpart located at the origin of the electromagnetic field, except perhaps for a difference in the sets of physical units in which each expresses his results.

The above interpretation allows for a much less restrictive interpretation of the RP. It is not necessary that the form of the physical law describing this or any other interaction be invariant to a particular space-time transformation in an arbitrarily chosen rest frame. In the case of electromagnetic interactions, it is only necessary that the same laws, in this case Maxwell's equations and the Lorentz Force law, apply in any rest frame *where the electron currently exists*. Its velocity **v** relative to the origin of the interaction uniquely determines the magnitudes of the electric and magnetic fields as well as the corresponding force acting on it. By contrast, the velocity of the electron relative to the observer himself plays no direct role in determining such quantities, thereby removing any element of subjectivity from the process. All observers, regardless of their own state of motion, must agree on the results of the interaction, except that they will generally not agree on the numerical values of their measurements because of differences in their respective choice of physical units.

It therefore suffices if the law in question faithfully predicts the results of the interaction in any given rest frame, regardless of the observer's current state of motion. Phipps [13] has pointed out that Ampère's original law of ponderomotive force action exerted by an infinitesimal element of neutral current $I_2 d\vec{s}_2$ upon another element $I_1 d\vec{s}_1$, has the form [14,15]:

$$\vec{F}_{21(Ampere)} = \frac{\mu_0}{4\pi}\frac{I_1 I_2 \vec{r}}{r^3}\left[\frac{3}{r^2}\left(\vec{r}\cdot d\vec{s}_1\right)\left(\vec{r}\cdot d\vec{s}_2\right)\right] - 2\left(d\vec{s}_1\cdot d\vec{s}_2\right), \quad (4)$$

where $\vec{r} = \vec{r}_1 - \vec{r}_2$ is the relative position vector of the elements and ($\mu_0 / 4\pi$) is a units factor yielding force in N for current in amperes, is symmetrical between 1 and 2 subscripts, and proportional to $\vec{r}$. Thus, it rigorously obeys Newton's third law of equality and co-linearity of action-reaction between current elements, which requires $\vec{F}_{21} = -\vec{F}_{12}$ on a detailed element-by-element basis. It has nonetheless generally been rejected by physicists in favor of the Lorentz Force law because of its transformation properties not shared by eq. (4).

The latter, when similarly expressed, takes the form [14]

$$\vec{F}_{21(Lorentz)} = \frac{\mu_0}{4\pi}\frac{I_1 I_2}{r^3}\left[-\left(d\vec{s}_1 \cdot d\vec{s}_2\right)\vec{r} + \left(d\vec{s}_1 \cdot \vec{r}\right)d\vec{s}_2\right] \tag{5}$$

It is asymmetrical in subscripts 1 and 2, and not proportional to $\vec{r}$, so that it disobeys Newton's third law in two ways. More details about this topic may be found in Phipps's original work [13]. The point to be emphasized in the present discussion is that there is no reason to reject eq. (4) once one agrees that the velocities of the electrons in the current elements I_1 and I_2 are to be measured relative to the rest frame in which the electromagnetic field originates.

4. Four-vectors and Lorentz invariance:

There is a similar problem with subjectivity in the Lorentz transformation (LT) given below:

$$\Delta t' = \gamma\,(\Delta t - v\Delta x c^{-2}) = \gamma\,\eta^{-1}\,\Delta t \tag{6a}$$

$$\Delta x' = \gamma\,(\Delta x - v\Delta t) \tag{6b}$$

$$\Delta y' = \Delta y \tag{6c}$$

$$\Delta z' = \Delta z. \tag{6d}$$

These equations are given in terms of intervals of space Δx, Δy and Δz and time Δt and their primed counterparts, i.e. $\Delta x = x_2 - x_1$, $\Delta x' = x_2' - x_1'$ etc. for two events [c is the speed of light, v is the relative speed of the participating inertial systems S and S' moving along a common x,x' coordinate axis and $\gamma = (1 - v^2c^{-2})^{-0.5}$]. In addition, the quantity η is defined as $(1 - vc^{-2}\Delta x/\Delta t)^{-1}$; in Einstein's original derivation [4], Δx/Δt is the velocity component (u_x) of an object in uniform translation as viewed by a stationary observer in S. One of the key predictions of the LT is that a moving clock will always appear to run slower than its identical counterpart at rest. Thus, once again STR [4] claims that the results of measurements are a matter of perspective. Each observer thinks that it is the other's clock that has the slower rate.

There has been much debate over the last century about whether such a situation is physically realizable. As already mentioned in the Introduction, however, experiments which have been carried out to test this prediction have proven decidedly negative in this respect. In the Hafele-Keating study [9], for example, it has been demonstrated that the atomic clocks on the airplane flying eastward run slower than those left behind at the origin of the flight, whereas those moving in the westerly direction run faster than both of the latter. The GPS methodology [10] relies on the assumption that an atomic clock on a satellite runs slower than its identical counterpart on the ground once one takes account of gravitational effects.

It has been shown in earlier work [16-18] that there is an alternative space-time transformation (referred to as the Newton-Voigt transformation NVT) which satisfies both of Einstein's postulates of STR [4] and assumes, in contrast to eq. (6a) of the LT, that the measured elapsed times of the two observers in S and S' are strictly proportional to one another:

$$\Delta t' = \Delta t/Q \tag{7a}$$

$$\Delta x' = \eta\,(\Delta x - v\Delta t) \tag{7b}$$

$$\Delta y' = \eta\Delta y/\gamma Q \tag{7c}$$

$$\Delta z' = \eta\Delta z/\gamma Q. \tag{7d}$$

The parameter Q in the NVT equations is fixed for any pair of inertial systems. In a typical case it is defined in terms of the speeds v_0 and v_0' of S and S' relative to a specific rest frame [19]. In the Hafele-Keating study [9], the earth's center of mass serves as this unique system, for example. The corresponding value of the parameter is then found experimentally to be:

$$Q = \gamma(v_0')/\gamma(v_0). \tag{8}$$

The point to be emphasized in the present discussion is that the symmetric relationship between elapsed times expected from the LT has always been contradicted in actual experiments. The assumption of clock-rate proportionality in the GPS-LT of eqs. (7a-d) has been quantitatively verified in both the Hay et al. [8] and Hafele-Keating [9] studies. *Time dilation is exclusively asymmetric and space and time are unequivocally distinct entities.* The conclusion from the empirical data is unequivocal. The LT is invalid and all of its predictions therefore need to be carefully reconsidered. This includes most especially the idea that space and time are inextricably mixed [20]. Newton was right and Einstein was wrong [21].

Minkowski's four-vector approach depends wholly on the LT. This was the point that Einstein was making when he dismissed it [1] as "superfluous learnedness." All that is done is to put STR in the framework of linear/affine spaces. One defines the spatial variables in the LT of eqs. (6a-d) as follows: $x_1=\Delta x$, $x_2=\Delta y$, $x_3=\Delta z$. Then, instead of using elapsed time directly, a fourth vector is defined as $ic\Delta t$. The Lorentz invariance condition is obtained by summing the squares of the four LT relations,

$$\Delta x'^2 + \Delta y'^2 + \Delta z'^2 - c^2\Delta t'^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2\Delta t^2. \tag{9}$$

In terms of the Minkowski four-vector, $\mathbf{x}=(x_1,x_2,x_3,x_4)$, this equation becomes a relation between scalar products:

$$x\bullet x = \mathbf{x'}\bullet\mathbf{x'}. \tag{10}$$

The beautiful simplicity of eq. (10) doesn't change the fact that the LT on which it is based is invalid [19,22]. There is a corresponding invariance relationship for the NVT. Expressed in its most symmetric form, it is:

$$\eta' Q'^{-1}(\Delta x'^2 + \Delta y'^2 + \Delta z'^2 - c^2\Delta t'^2) = \eta Q^{-1} (\Delta x^2 + \Delta y^2 + \Delta z^2 - c^2\Delta t^2). \tag{11}$$

In this expression η' is obtained from η by interchanging the primed and unprimed variables and changing the sign of v, i.e. $\eta'= (1+vc^{-2}\,\Delta x'/\Delta t')^{-1}$. It should be noted that the following identity [20] holds for these two quantities:

$$\eta\eta' = \gamma^2 . \tag{12}$$

In order to satisfy the RP it is also necessary that $QQ'=1$, i.e. that eq. (7a) is consistent with its inverse, $\Delta t = Q\Delta t' = \Delta t'/Q'$. As a result, eq. (11) has the equivalent form:

$$(\Delta x'^2 + \Delta y'^2 + \Delta z'^2 - c^2\Delta t'^2) = \varepsilon^2 (\Delta x^2 + \Delta y^2 + \Delta z^2 - c^2\Delta t^2), \tag{13}$$

with $\varepsilon=\eta(\gamma Q)^{-1}$. The LT has a corresponding value of $\varepsilon=1$ in eq. (9). Inverting eq. (13) shows that $\varepsilon\varepsilon'=1$ is the condition for satisfying the RP, i.e. where ε' is obtained in the usual way (Galilean inversion) from ε by interchanging primed and unprimed variables and reversing the sign of v.

The NVT is thus seen to satisfy the RP condition but not $\varepsilon^2=1$. This is a significant distinction for the four-vector formalism. It means that the set of 4x4 matrices a_{ij} describing the LT form a group. It should be noted, however, that this relationship only holds if the velocity vectors in the pairs of a_{ij} matrices lie in the same direction. The corresponding set of matrices for the NVT do not form a group, however. That is a physically irrelevant point, however, since there is no *a priori* reason that the true space-time transformation should exhibit group properties.

An attractive feature of Minkowski's formlism is that the LT transformation matrix $A= a_{ij}$ is orthogonal, i.e. its transpose A'satisfies the relationship:

$$A'= A^{-1}. \tag{14}$$

It also can be used in a similarity transformation for the four-tensor F of the electromagnetic field [23]:

$$F' = A\,FA', \tag{15}$$

i.e., where F' has the same form in the other rest frame.

It needs to be emphasized, however, that a comparable set of relationships is obtained when one uses the NVT of eqs. (7a-d) to form the Minkowski matrix, $B=\varepsilon A$, with $\varepsilon=\eta(\gamma Q)^{-1}$. The same relationship between the electric and magnetic fields is obtained with the NVT as with the LT because of the homogeneity of Maxwell's equations (for the same reason that both transformations satisfy Einstein's light-speed postulate [1,24,25]). In this case, $B'= \varepsilon' A'$ and $\varepsilon'=\eta' (\gamma Q')^{-1}$. One therefore obtains the equivalent matrix relationships as in eqs. (14,15) since $\varepsilon\varepsilon'=1$:

$$B'= B^{-1}. \tag{16}$$

$$F'= BFB'. \tag{17}$$

There is a distinction between the four-vector approach and ordinary linear spaces that needs to be taken into account, however. There is an axiom in the strictly mathematical definition which states that in order for a set to qualify as a linear space it must have a *unique zero element.* This condition is not satisfied in the Minkowski definition because all light-vectors have zero magnitude by definition in eq. (9), i.e. each side has a null value. This state of affairs has a more serious consequence when it comes to defining the four-tensors that are used to represent the NVT and other space-time transformations in general. In the case of the NVT, this situation manifests itself because there are two mathematically equivalent linear combinations of the x_i vectors to define the time vector x_4'. One has already been discussed in connection with the B matrix above. The fourth row has two non-zero elements in the B transformation matrix, similarly as for A, from which it differs by the constant factor $\varepsilon=\eta(\gamma Q)^{-1}$. An alternative matrix C is obtained if eq. (7a) is used instead. In that case the fourth row contains only a single non-zero element, namely $c_{44}=Q^{-1}$. The other three rows are identical to those in B. The fourth column of the transpose matrix C' therefore also has only one non-zero element, $c_{44}'=Q'^{-1}$. As a consequence, one finds that the condition equivalent to eq. (16) for the B matrix does not apply to C, i.e. $C'\neq C^{-1}$.

If one carries out the transformation of Maxwell's equations in the conventional manner employed by Einstein [4], it is clear that the results are exactly the same whether one uses eq. (7a) directly or eq. (6a) with the right-hand side multiplied with $\varepsilon=\eta(\gamma Q)^{-1}$, or for that matter with any other value of ε. As mentioned above, this equivalence is guaranteed by the homogeneity of Maxwell's equations. The difference between the results of the matrix operations in the Minkowski formalism simply points out the necessity of choosing a specific form for the transformation equations in order to obtain the desired result. This situation is already evident from the fact that a particular factor, namely ic, for Δt and Δt' must be used to satisfy eq. (10). On the one hand, this specificity does not prevent one from obtaining the "right answers" using the four-vector formalism, but on the other, it supports Einstein's critique of the method as being unnecessarily complicated ("superficial").

5. Accelerated rest frames and the relativity principle:

The case of an electron being acted upon by an electromagnetic field is an example of a more general situation in which a particle undergoes acceleration as a result of a locally applied force. The energy E and momentum **p** of the particle also combine to form a four-vector which satisfies the following relationship in STR [4] between different rest frames:

$$E^2-p^2c^2 = E'^2-p'^2c^2 = \mu^2c^4, \tag{18}$$

where μ is the rest mass of the particle. This equation can be derived from the experimental results obtained by Bucherer [26] for the variation of the mass m of accelerated electrons with speed v relative to the laboratory:

$$m = \gamma\ (v)\ \mu. \tag{19}$$

When combined with Einstein's mass-energy relation [4], this equation can be converted to

$$E = mc^2 =\gamma\mu c^2= \gamma E_0, \tag{20}$$

where E_0 is referred to as the rest energy of the particle. Squaring eq. (20) leads back to eq. (18) since

$$E^2\gamma^{-2} = E^2\ (1-v^2c^{-2}) = E^2 - E^2v^2c^{-2} = E^2 -(E^2c^{-4})v^2c^2 = E^2 - m^2v^2c^2 =$$
$$E^2 - p^2c^2 = E_0{}^2 = \mu^2c^4 = E'^2-p'^2c^2. \tag{21}$$

Note that E' and **p**' in the last term correspond to a different rest frame than the original and therefore to a different value of the particle's speed (v') relative to the laboratory.

There are two points to be emphasized in the above derivation. First, eqs. (19-20) refer to measurements made from the perspective of the laboratory in Bucherer's experiments [26], i.e. v and γ (v) are determined relative to this rest frame. Second, the situation is the same as for electromagnetic interactions, as discussed in Sects. II-III. The *speed of the observer relative to the particle is irrelevant* in determining the values of E, m and **p**. All observers see the same *absolute* values of these quantities. If the observer has also undergone acceleration relative to the origin/laboratory, his standard unit of mass will differ from that employed in the laboratory.

The situation is the same as for time dilation. One needs to know the speed v' of the observer relative to the origin of the interaction as well as the speed v of the particle relative to the same rest frame. The conversion factor between the observer's unit of mass and that employed in the laboratory rest frame is the same (Q) as for time dilation in eq. (8). In this case eqs. (19,20) must be replaced by the relations:

$$m = Q\ \mu \tag{22}$$
$$E = Q\ E_0, \tag{23}$$

This is a critical distinction for the four-vector formalism, however. The E,**p** four-vector no longer satisfies the scalar product relation in eq. (18) when $Q\neq\gamma$. This only occurs when the *observer is stationary in the rest frame where the force causing the particle acceleration occurs*, which is the case in Bucherer's experiments [26]. On the other hand, Q=1 for the observer co-moving with the electron, so he will obtain the rest values of E and m, i.e. E_0 and μ. His value for the momentum **p** will be μ**v**, however, where **v** is the velocity of the electron relative to the laboratory, not simply p=0. This conclusion is again consistent with the discussion in Sects. II-III for electromagnetic interactions. Momentum is determined by the velocity of the particle relative to the rest frame in which the relevant force was applied, *not by the speed of the observer relative to either the particle or this origin.* The same situation holds for clock rates in the Hafele-Keating experiments [9].

Note that all observers must agree on the value of **v** to be consistent with the light-speed constancy postulate [27]. This also means that the unit of distance must change in the same manner with rest frame as time and mass, i.e. with the same conversion factor Q as in eq. (8). Accordingly, the conversion factor for speed, the ratio of distance to elapsed time, is $Q^0=1$. More details concerning conversion factors for other physical properties may be found elsewhere [27, 28].

As a final topic in this section, consider the four-vector relationship for frequencies ν and wavelengths γ. For this purpose it is convenient to use the definitions of circular frequency $\omega=2\pi\nu$ and wave vector $\mathbf{k}=2\pi/\boldsymbol{\lambda}$. There is again an invariance condition for the associated scalar product, in this case:

$$\omega^2-k^2c^2=0. \quad (24)$$

This relationship only holds for light in free space, however, in which case $\omega/k = \lambda\nu = c$. It has special significance [29] because of the quantum mechanical relationships for photons: $E = h\nu$ and $p = h/\lambda$. There is thus a close connection between the E,**p** and ω,**k** four-vectors for this case.

6. Conclusion:

The relativity principle (RP) was originally intended by Galileo to apply exclusively to inertial systems, i.e. under the influence of no unbalanced forces. Einstein and his contemporaries sought to extend the RP to apply to electrons undergoing acceleration due to an electromagnetic field. His first postulate of relativity, which states in broad terms that the laws of physics are the same in all inertial systems, falls short of this objective and has been a source of confusion for physicists ever since its inception. Examination of his arguments with regard to electromagnetic interactions shows that what he actually did was to assume that Maxwell's equations must hold in the *non-inertial* rest frame of an accelerated electron. He was led under this assumption to conclude that the electric and magnetic fields must undergo continuous mixing as the electron increases its speed. The resulting transformation, as well as the Lorentz Force law itself, contain a parameter **v** which is assumed to be the velocity of the electron. It has generally been concluded that this implies that the forces acting on the electron vary with the perspective of the observer because ***v** is assumed to be the velocity of the electron relative to the observer.* In Sect. II of the present work, it has been shown that this interpretation leads to a physically untenable result with regard to the trajectory of the electron. It implies that an observer co-moving with the electron will find that it moves continuously along a straight line since v=0 from his perspective, whereas his counterpart who remains stationary in the rest frame of the laboratory where the electromagnetic force originates finds instead that the electron follows a curved path.

There is a straightforward way to eliminate this dilemma, namely to remove the observer from *active* participation in the interaction. The measurement process becomes completely objective as a consequence. This is accomplished in the present case by assuming that the velocity of the accelerated particle *is uniquely defined relative to the origin of the electromagnetic field.* All observers, regardless of their own state of motion, agree on this value of the velocity and also on all other results of the interaction *on an absolute basis*. The only source of disagreement must be due to the fact that different systems of physical units are employed by the various observers. This requirement suggests an amended version of the RP: The laws of physics are the same in all inertial systems *but the physical units in which their results are expressed can and do vary from one system to another.*

The same conclusion applies to the definition of momentum $\mathbf{p}=\gamma\mu\mathbf{v}$ for an accelerated particle; the velocity **v** of the particle is always taken relative to the rest frame in which the force causing acceleration has been applied. Again, there is no disagreement among different observers as to the absolute value of the momentum, both in its magnitude and direction, even though differences in the numerical value occur because of their respective choices of the fundamental units of inertial mass, distance and time.

The above discussion also raises questions about the transformation properties of physical laws in general. The essential point is that the laws of physics must be accurate for the stationary observer at the origin of an applied force. His rest frame plays a unique role in the physical description of forces. Observers in other rest frames must simply agree on the results of the interaction (after appropriate changes in units are made) by virtue of another basic physical principle: the objectivity of the measurement process. The latter is distinct from Galileo’s RP but perfectly consistent with it. Einstein's derivation of the electromagnetic field transformation assumes that Maxwell's equations and the Lorentz Force law must be invariant to a Lorentz space-time transformation. However, the consequences of this assumption with regard to the prediction of electron trajectories from different vantage points that has been alluded to above suggests that this interpretation of the RP is overly restrictive. This conclusion is reinforced by the fact that the above restrictions have led to the acceptance of a form of the Lorentz Force law that violates Newton's third law [see eq. (5)] to the exclusion of the original version of Ampère’s law of ponderomotive force that does satisfy it, namely eq, (4) [13].

It is important to note that the LT is not actually required in Einstein's derivation of the field transformation. The NVT of eqs. (7a-d), which unlike the LT is consistent with the experimental observations of asymmetric time dilation, has the same capability. This degree of freedom in the choice of the space-time transformation for this purpose was first pointed out by Lorentz and was also known to Einstein at the time of his landmark paper on STR. The popular four-vector formalism can also be couched in the framework of the NVT, although care must be taken to avoid an inherent redundancy in Minkowski's methodology. The overriding advantage of the NVT is that it is compatible with the objectivity principle of the measurement process as well as with Einstein's two postulates of relativity.

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