HAFELE-KEATING ATOMIC CLOCK EXPERIMENT AND THE UNIVERSAL TIME-DILATION LAW

Robert J. Buenker*
Fachbereich C-Mathematik und Naturwissenschaften, Bergische Universität Wuppertal, Gaussstr. 20, D-42097 Wuppertal, Germany
*For correspondence. (rjbuenker@gmail.com)

Abstract: The results of the timing experiments using atomic clocks carried onboard circumnavigating airplanes [J. C. Hafele and R. E. Keating, Science 177, 168-172 (1971)] are reviewed. It is pointed out that the finding that the eastward-flying clock arrived back at the airport of origin with less elapsed time than its westward-flying counterpart was not expected based on the conventional interpretation of the Lorentz transformation (LT). The latter predicts that a moving clock always runs slower than one that is stationary in the rest frame of the observer. In order to satisfactorily represent the observed timing results it was necessary to assume that clocks slow in inverse proportion to $\gamma=(1-v_{ECM}^2c^{-2})^{-0.5}\approx0.5v_{ECM}^2c^{-2}$**,** where v_{ECM} is the speed of the clock relative to the earth's center of mass (ECM) and c is the speed of light in free space. A similar result was obtained earlier in experiments in which an x-ray source and absorber were mounted on a high-speed rotor and the reference frame for computing clock speeds is the rotor axis. The common proportionality relationship for the two fundamental experiments is therefore referred to as the Universal Time-dilation Law (UTDL). Calculations of elapsed times based on the HK interpretation are presented which speak against the traditional LT interpretation of symmetric time dilation when clocks are not subject to unbalanced forces. A different space-time transformation is presented which is consistent with the UTDL, unlike the LT, but also satisfies both of Einstein's postulates of relativity and is compatible with the relativistic velocity transformation (RVT).

Keywords: time dilation; Lorentz transformation (LT); Universal Time-Dilation Law (UTDL); alternative Global Positioning System-Lorentz transformation (GPS-LT); Newton-Voigt transformation (NVT); relativistic velocity transformation (RVT)

1. Introduction:

One of the most striking predictions of Einstein's Special Theory of Relativity (STR) [1] was time dilation (TD), the slowing down of clocks in motion relative to the observer. The observations of wavelengths emitted from accelerated light sources in 1938 by Ives and Stilwell [2] provided the first experimental verification of this phenomenon. More direct evidence came in the form of frequency shifts [3-5] measured using the Mössbauer effect for an x-ray source and absorber mounted on a high-speed rotor. Sherwin [6] pointed out, however, that the latter experiments did not conform to the traditional STR interpretation based on the Lorentz transformation (LT), according to which it is completely *ambiguous* which of two moving clocks runs at a slower rate. This conclusion was then underscored in 1971 when Hafele and Keating reported their experimental results for cesium atomic clocks carried onboard circumnavigating airplanes [7,8]. It was found that clocks on the eastward-bound airplane came back to the airport of origin with significantly less elapsed time than one that had stayed at rest there throughout the entire time of the flight, while their westward-flying counterparts returned with greater elapsed times than any of the others. The asymmetry of the clock rates observed in both the airplane and rotor experiments has been attributed to the fact that in both cases there is a unique inertial reference frame from which the speeds of the clocks should be measured, the rotor axis in the one case [6] and the rest frame (ECM) of the earth's polar axis in the other [7,8]. It has been argued that the symmetry for any pair of clocks implied by direct application of the LT *would* be observed in the hypothetical case when no unbalanced forces are present. However, there has never been an experimental verification of the latter prediction.

The Hafele-Keating model on which they based their predictions [7] of the elapsed times of the atomic clocks will be considered in detail in the following discussion. Particular attention will be placed on the changeover from asymmetric to symmetric time dilation expected on this basis for the case when all unbalanced forces acting on the clocks are suddenly eliminated.

2. Test calculations of the Hafele-Keating model:

For the sake of making the following calculations in a meaningful but computationally simple manner, it will be assumed that both flights took place at the Equator, and thus covered a distance of 40000 km before arriving back at the airport. If the average speed v in both directions was 800 km/h, the LT predicts on the basis of time dilation that *each* airplane clock ran slower than the stationary airport clock by a factor of 2.75×10^{-13}. This value is obtained by first computing the v/c ratio of 7.41×10^{-7} (c is the speed of light in free space, 3×10^{5} km/s) and then evaluating the Einstein time-dilation factor $\gamma - 1 = (1 - v^2 c^{-2})^{-0.5} - 1 \approx 0.5 v^2 c^{-2} = 2.75 \times 10^{-13}$. The elapsed time of each round-the world flight is calculated to be 40000 km/800 kmh= 1.8×10^{5} s. Therefore, according to the LT, both airplane clocks should have arrived back at the airport with 5×10^{-8} s = 50 ns less time than the airport clock (assuming that the respective altitudes of the two planes were approximately the same during the flights).

What HK found instead is that the westward-flying airplane had 273±7 ns more time on its clock than that left behind at the airport, whereas the eastward-flying clock had lost 59±10 ns relative to the airport clock [8]. They were only able to reconcile these results with the LT predictions by assuming that the speeds used to compute the values of the time-dilation γ factors have to be determined relative to a unique rest frame, namely that of the earth's "non-rotating polar axis." The justification for this restriction was that this rest frame, unlike those of the airplane and airport clocks, is inertial since it alone is not affected by the earth's rotation. The speeds (all in the easterly direction) of the various clocks relative to this reference are 1600 km/h for the airport clock (since the earth's rotational speed at the Equator has this value), and 800 (1600-800) and 2400 (1600 + 800) km/h for the westward- and eastward-flying clocks, respectively. On this basis, it is found that the three clocks should all have lost time with respect to a hypothetical clock located at the earth's center of mass (ECM) because it is stationary with respect to the above reference frame: 49 ns for the westward-flying, 198 ns for the airport, and 444 ns for the eastward-flying clock. Taking time differences, one therefore expects that the clock flying west should return with 149 ns more, while the one flying east with 246 ns less than the airport clock. Taking into account the details of the actual flight paths, HK found the corresponding results stated above of +96 ns (instead of 149 ns) and -184 ns (instead of -246 ns), respectively. A correction for the gravitational speeding up of clocks (ghc^{-2}) for the two on the airplane also needed to be made [7]. On this basis HK found that the expected time differences for the airplane clocks relative the airport clock agreed quite well with the measured values (-59 ± 10 ns vs -40 ns predicted for the eastward flight and 273± 7 ns vs 275 ns predicted for the westward flight).

It should be realized that HK's success in reconciling their experimental results with theory was only possible because they eschewed the traditional application of the LT that is found in textbooks dealing with relativity. In essence, they concluded that the LT prediction that a moving clock always runs slower than a stationary one is only applicable for perfectly inertial systems. This amounts to a two-tiered procedure for applying the LT to predict the amount of time dilation [6]: the effect is *asymmetric* when the objects of the timing measurements are under the influence of unbalanced forces, but *symmetric* when this is not the case.

The above arrangement suggest some interesting hypothetical situations when the boundary between asymmetric and symmetric time dilation is suddenly crossed. For example, consider the case of a slowly rotating planet in the context of the HK experiment. As long as the speed v of the (easterly-directed) orbital rotation is close to but still greater than zero, one must expect that the westward-flying clock will arrive back at the airport with (slightly) more elapsed time than either the airport clock or its counterpart that flew in the opposite direction (asymmetric time dilation). Yet, if the rotation stops completely, the LT prediction would revert back to the standard symmetric interpretation. Thus, from the standpoint of the eastward-flying airplane, the clock flying westward is moving at a relative speed of 1600 km/h in the original example. Accordingly, the westward-flying clock should arrive back at the airport with 200 ns *less* elapsed time than that carried onboard the eastward-flying plane. At the same time, one would have to conclude from the viewpoint of the observer on the westward-flying airplane that his onboard clock would return with 200 ns *more* elapsed time since the eastward-flying plane flies at a speed of 1600 km/h relative to him as well. That raises the obvious question as to how the two clocks could arrive back at the airport with each showing less (more) time than the other. In short, the LT prediction of symmetric time dilation is impossible to realize in practice.

On the other hand, a much more feasible outcome results in this case if we continue to use the procedure that assumes asymmetric time dilation. Lowering the earth's orbital speed to exactly zero would simply mean that each airplane clock moves with the same (800 kmh) speed relative to the ECM, in which case the prediction is that both clocks return to the airport with 50 ns less elapsed time than the clock left behind there. That would be the limiting value for both airplane clocks as the orbital speed approaches zero from both directions, exactly as one would expect based on the elapsed-time results obtained as a function of v. One therefore expects perfectly continuous behavior for both airplane clocks based on the asymmetric time-dilation interpretation, up to and including the null orbital-velocity limit.

In summary, the results of the Hafele-Keating (HK) experiments with circumnavigating atomic clocks do not mesh with the predictions of the Lorentz transformation (LT). This is the case despite the HK's *ex post facto* attempt to reformulate Einstein's relativity theory to allow for a departure from its original interpretation of exclusively symmetric time dilation.

One can escape from this seemingly hopeless conundrum by demanding that the results of a revised theory be *directly* consistent with all experimental findings obtained to date. At the same time, it only makes sense to also require that Einstein's two postulates of relativity still be satisfied in the new theory. To this end it is important to see that asymmetric dilation has also been observed in all other time-dilation experiments, particularly the tests made with x-ray radiation employing the Mössbauer effect [3-5]. As in the HK experiment, it was found that the periods of the associated clocks (absorber and source) mounted on a high-speed rotor are inversely proportional to γ (v_i), where v_i is the clock's speed relative to the rotor axis. It is seen that this inverse proportionality is identical with that found in the HK study, whereby the rotor axis now takes the place of the ECM as the rest frame from which clock speeds are to be measured.

One can generalize these results (Universal Time-dilation Law or UTDL [9]) and also bring them into a form suitable for use in a space-time transformation by simply requiring that the elapsed times measured in the two rest frames satisfy the equation: $\Delta t'=\Delta t/Q$, where $Q = \gamma(v_i')/\gamma(v_i)$. The same formula is used to adjust atomic clocks carried onboard satellites of the Global Positioning System [10,11]. One can take advantage of a degree of freedom in the formulation of the LT to obtain the new transformation, which has previously been referred to as the GPS-LT [12-14], or more aptly as the Newton-Voigt transformation (NVT) [15]. The NVT contains the above proportionality between measured times explicitly and also satisfies both of Einstein's postulates in addition to being compatible with the relativistic velocity transformation (RVT) [1,16]. It also avoids an incompatibility between the LT predictions of remote non-simultaneity and proportional time dilation (Clock Puzzle) [17].

3. Popular theory of time dilation and the second postulate:

One of the main lessons to be learned from the HK experiment is that the rates of clocks vary with their speeds relative to a particular rest frame. No light pulses are involved in any way. This fact is significant in itself because of the way in which time dilation is often explained in standard textbooks on relativity. In that view one needs to consider a particular case when a light pulse travels in a perpendicular direction relative to a given observer (O'), say along the y' axis of an airplane. Some other observer (O) sees the airplane moving along the x axis with speed v relative to O' and therefore he will see the light pulse traveling in a *diagonal direction.* If the light pulse travels a distance L to arrive at a screen that is stationary in the rest frame of the airplane, it is assumed that O located at the airport will see the same light pulse travel a distance of $D = (L^2 + v^2\Delta T^2)^{0.5}$ before reaching the screen, whereby ΔT is the elapsed time on his stationary proper clock for this to occur. The time difference ΔT is longer than is required for the *same* light pulse to reach the screen according to the proper clock at rest with O', namely $\Delta T' = L/c$. Using the RVT, one comes to the conclusion that $D = c\Delta T= \gamma L = \gamma c\Delta T'$ and $\Delta T=\gamma\Delta T'$ (using the same definition of γ as in Sect. II). On this basis it is concluded that the proper clock in the rest frame of O' on the airplane must have run γ times *slower* than the corresponding proper clock used by O at the airport. According to the standard interpretation, this difference is said to be the result of time dilation in the rest frame of the airplane.

The above argument can be refuted on the following grounds. To begin with, why do we need to restrict our consideration to the case of a light pulse moving in the *transverse* direction? If the light pulse travels in the same (x) direction as O' relative to O, for example, it no longer follows that $\Delta T=\gamma\Delta T'$ according to the above argument

The real reason why the textbook justification of time dilation is specious, however, is the incorrect way in which it uses Einstein's light-speed postulate (LSP) to come to its conclusion. The latter states that the speed of light in free space is equal to c independent of the state of motion of the observer *when measured relative to the source from which it is emitted*. The above argument assumes that this source is stationary in *both* the rest frames of O and O', a clear impossibility. Accordingly, the speed of the light pulse is claimed to be equal to c for both observers. Since D is greater than L, it therefore follows that $\Delta T > \Delta T'$ since both supposedly agree that the light pulse travels with speed c from the same origin to the same point on the airplane's screen during the corresponding time.

The way to resolve the issue and still remain consistent with the LSP is to assume that *two* light pulses are involved, each emitted from the same origin at the same time. One (P') is emitted from a source that is stationary with respect to O' traveling on an airplane, while the other (P) is emitted from a different source that is stationary with respect to O located at the airport . P' reaches the screen on the airplane in time $T'=L/c$, i.e. with x',y' coordinates: (0, L). The speed of P' is assumed thereby to be c along the y' axis. This value of the speed is in accord with the LSP, since P' is emitted from the stationary source on the airplane. The corresponding x,y coordinates for O are (vT', L) since the airplane has moved along the x axis with speed v during the time that P' travels between the source and the screen on the airplane.

The light pulse P must move along the diagonal mentioned in the original example. The corresponding velocity will be computed using the relativistic velocity transformation (RVT). The components are thereby found to be $(v,c/\gamma)$. The total speed is $(v^2+\gamma^{-2}c^2)^{0.5} = c$. This value is consistent with the LSP because it is computed relative to the light source at the airport from which P has been emitted. This result is in accord with the relativistic treatment of the aberration of starlight at the zenith [18], which indicates an aberration angle of $\tan^{-1}(\gamma v/c)$ from the horizontal (x) axis. At time T', P is therefore located at $(vT', cT'/\gamma)=(vT',L/\gamma)$. In the airplane's coordinate system, this is $(0,L/\gamma)$. Thus, P intersects the line between the source and the screen on the airplane at this time, but at a point which falls short of the screen by a factor of $1/\gamma$.

In order to reach the screen on the airplane, P must travel for a longer time, arriving there at time $T=\gamma T'$. At that point, its coordinates are (vT,L) relative to the airport, and (0,L) from the vantage point of O' on the airplane. Meanwhile, P' is located farther along the above line on the airplane, i.e. beyond the screen, namely at $(0,\gamma L)$. This is not "time dilation." Rather, the fact that the time $T=\gamma T'$ needed for light pulse P to reach the screen on the airplane is greater than for P' to do so merely reflects the fact that it had to travel a longer distance from its source at the airport than did P' from its respective /different source on the airplane. It also should be noted that in the classical theory, the two light pulses would be expected to arrive at exactly the same time (T=T') at the screen on the airplane. This is because the speed of light was effectively taken to be infinite in the classical view, in which case γ has a value of unity.

4. Einstein's Equivalence Principle and the UTDL:

The experiment with circumnavigating atomic clocks does not allow for a definite answer to be given to the question of what causes time dilation. What it does instead is to provide a simple prescription as to how to accurately predict the amount by which clock rates slow as a result of their motion. As discussed in Sect. II, the following proportionality relation suffices quantitatively to obtain this information [9]:

$$\tau_1\gamma(v_{10}) = \tau_2\gamma(v_{20}). \quad (1)$$

In order to apply it, it is necessary to determine the speed v_{i0} of a given clock relative to the ECM [7,8]. The corresponding elapsed time measured on this clock for a definite segment of the flight's trajectory is then given by τ_i.

A decade earlier, measurements of the Doppler frequency shift using an x-ray source and absorber mounted on a high-speed rotor [3-5] had given the following result:

$$\Delta\nu/\nu = (R_a^2 - R_s^2)\, \omega^2/2c^2, \quad (2)$$

In this case the respective speeds of the absorber and source clocks are given by $R_a\, \omega$ and $R_s\, \omega$, and thus eq. (2) fits in perfectly with the HK eq. (1) since a frequency ν is equal to the reciprocal of a time period τ and thus $\Delta\nu/\nu= (\nu_a-\nu_s)/ \nu_s= \nu_a /\nu_s - 1= \tau_s/\tau_a-1$, which according to eq. (1) is therefore equal to $[\gamma(R_a\, \omega)/ \gamma(R_s\, \omega)]-1$ to a suitable degree of approximation. The rotor axis therefore has the same function in these experiments as the ECM in the HK counterparts. Einstein used a version of eq. (1) in his 1905 paper [1] to predict that a clock at the Equator would run slower than one at a Pole and also that a clock attached to an electron moving in a closed path would return to the origin with less time than one left behind at the origin of the flight. Eq. (1) is therefore deserving of the title "Universal Time-dilation Law" or UTDL [9,19,20].

Despite the above similarity in the theoretical descriptions of the rotor and circumnavigating airplane experiments, there is at least one area where they were markedly different. The three different groups of authors of the rotor studies [3-5] all invoked Einstein's equivalence principle (EP) [21] in arriving at eq. (2), whereas there is no mention of it in the HK report. They did this by associating the rest frame of the clock on the outer rim of the rotor with a lower gravitational potential than that of the x-ray source. They apparently recognized [6] that the experiment could not be plausibly explained in terms of the standard prediction of STR of symmetric time dilation in view of the anti-symmetric character of eq. (2).

By contrast, HK make no mention of the EP in their theoretical discussion. It is easy to see why because in their experiment both a kinetic and gravitational effect on clock rates could be observed. The whole idea of the EP is that accelerating an object is equivalent to changing its position in a gravitational field, with the implication that the two interactions are just different sides of the same coin. The HK study of atomic clocks shows to the contrary that they are clearly distinguishable from one another and that the effects they have on clock rates are both separate and additive. It also should be noted that the use of the EP in the rotor study was not entirely consistent since it requires that one believe that the clock at the *lower potential* (with the greater speed relative to the rotor axis) should have the *higher energy* as well as the lower frequency. One also should be aware of the fact that the speed of light increases with gravitational potential [21, 22], whereas it is critical in STR [1] that it have the same value in all rest frames located at the same gravitational potential. The equivalence is less than perfect in other words. In short, the EP is not a good fit for either the HK or the rotor studies.

A relatively simple theoretical method for predicting changes in the rates of clocks nonetheless arises from the HK study. With a minimum of information about its location and speed relative to a particular rest frame, it is possible to compute two constants that completely determine its rate relative to some standard clock. This approach is used quantitatively in the GPS navigation technology [10, 11] to adjust clocks on satellites so that they run at the same rate as their counterparts on the earth's surface. The UTDL is used in one case and Einstein's red-shift formula [21] in the other. In short, the laws that govern how the rates of clocks change with kinetic acceleration and gravity are well understood even if the underlying for these relationships is a matter of some dispute. Finally, it has been argued that integral powers of the same two constants can also be used to determine the ratios of other dynamical quantities such as speed, mass, energy and distance [23,24].

5. Conclusion:

Time dilation is the slowing of the rates of all types of clocks that occurs by virtue of their motion relative to a specific rest frame. The HK experiments [7-8] with circumnavigating atomic clocks demonstrate that the rate of a given clock is inversely proportional to $\gamma(v_{ECM})$, where v_{ECM} is its speed relative to the ECM. Contrary to the standard prediction of symmetric time dilation by the LT [1], these results clearly indicate that the slowing of clock rates is a perfectly objective phenomenon, i.e. it is always possible to say which of two clocks has the slower rate. This relationship between elapsed times measured in different rest frames is mirrored exactly [see eq. (1)] in the studies of the transverse Doppler effect using a high-speed rotor [3-5], in which case the reference frame from which speeds are to be determined is the rotor axis.

In none of these cases does the time dilation occur because of the emission of light pulses. The HK and rotor results therefore do not agree with the standard STR interpretation of time dilation as being the consequence of a light pulse having to move in different directions from the viewpoint of two observers in order to arrive at the same time at a given destination. This argument is shown in Sect. III to be inconsistent with the requirements of the light speed postulate (LSP). Instead, the experiments indicate that eq. (1) can be used reliably to predict the relative rates of clocks in motion without revealing a clear theoretical justification for this relationship. As such, it has been referred to as the Universal Time-dilation Law (UTDL), similar in principle to other laws of physics such as those used in kinetics and thermodynamics, for which no clearly consistent justification is possible either.

Finally, although the UTDL is not consistent with the LT, it is possible to have a space-time transformation which does agree with it completely and still satisfy both of Einstein's postulates of relativity [1], as well as be consistent with the relativistic velocity transformation (RVT). It has been referred to as the GPS-LT, in recognition of its direct relevance to the timing procedures used to adjust atomic clocks carried onboard satellites of the Global Positioning System.

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