

# FINDING CORRELATION COEFFICIENT OF INTUITIONISTIC FUZZY SETS

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**Abstract:** A method for finding correlation coefficient (CC) of fuzzy sets (FSs) is proposed in this article. An FS is characterized here with the help of its membership function with respect to a reference function. The value of the CC is found to be in  $[-1, 1]$  demonstrating the efficacy of formula to give non only the strength of the relationship between FSs but also whether they are positively or negatively correlated. Thus, the propose method looks better than the couple of existing methods available in the literature.

**Keywords:** Intuitionistic fuzzy set; Degree of positive membership; Degree of neutral membership; Degree of negative membership; Reference function; Covariance of fuzzy sets; Discrete set of universe of discourse.

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## 1. Introduction:

The FS was first presented in the mathematical world by Zadeh [1]. After that a lot of researchers worked on the mathematics fuzziness and extended many statistical parameters such as mean, median, mode, standard deviation, regression, and CC etc. [2, 3, 4]. Atanassov [5] has presented intuitionistic fuzzy sets (IFSs) by generalizing Zadeh's fuzzy set [1] and non-membership parameter. In [6], the authors used the conventional statistics to explain an IFS for calculating the CC of IFS. The CC calculated by [6] indicates that the IFSs may be positively or negatively related. In [7, 8], the authors proposed to define the CC between two IFSs as a sum total of the products of the both membership and non-membership values. By presenting the notion of CC and studying the interval-valued intuitionistic fuzzy sets (IVIFSS) in [9], the authors expanded the work done in [7, 8].

Xuan Thao [10] presented a novel definition of CC between IFSs that belongs to  $[-1,1]$  whereas many others' value lies in  $[0,1]$ . A central interval method was used in [11] to determine CC of fuzzy data. The authors of [12] used a straightforward method based on the widely known idea of Pearson correlation coefficient to assess a management plan or decision using fuzzy interval measures. The authors of [13] modified the traditional method to determine whether a set of multivariate fuzzy data can be explained by a multivariate normal distribution. In [14] the researchers presented CC of picture fuzzy sets (PFSs), an extension IFSs [5]. In [15], Sharma & Singh initially suggested CC for FSs and then extended idea to fuzzy soft sets.

In [16], the authors proposed CC of fuzzy interval data by providing five sets of rules to calculate the CC between variables. In [17], Man-Ki & Hyun-Ah have proposed "some properties of fuzzy testing of independence in a bivariate normal distribution by agreement index obtained from the fuzzy random samples". They have also defined the restriction of fuzzy data and fuzzy statistics for observed data with alteration error terms. Then, they introduced some properties of fuzzy correlation coefficient with fuzzy data from the bivariate normal distribution.

In [18], the authors generated fuzzy interval data from crisp data with the help of CC defined in [12] and the generating process was based on the  $\delta$ -cut. In [19, 20], the authors proposed a unique definition of fuzzy numbers in terms a membership function and a reference function and then the membership values are

calculated by taking the difference between the two aforesaid functions. Similar works related to CC were reported in [21-24]

In this article, a formula for evaluating the CC of FSs is defined where FS is characterized a membership function with respect to a reference function [19, 20]. The formula is expressed with the help of mean and covariance defined using membership values where a membership value is computed by taking difference between membership function and reference function. It is to be mentioned here that the FSs are defined over discrete universe of discourse. It has been found that the value CC lies in between -1 and 1.

The paper is ordered as follows. In Section-2, the problem definition is discussed. In Section-3, the proposed method is discussed. Section-4, concludes the article with a brief conclusion.

## 2. Problem Definition:

Let  $X$  be the universe of discourse, a FS [1],  $A$  is described by

$$A = \{(x, \mu_A(x)); x \in X\} \quad (1)$$

where  $\mu_A: X \rightarrow [0, 1]$  is the membership function that produces membership grade of each element  $x \in X$  in  $A$ .

An IFS [5, 6],  $A$  in  $X$  is given by

$$A = \{(x; \mu_A(x), \nu_A(x)); x \in X\} \quad (2)$$

where  $\mu_A: X \rightarrow [0, 1]$  and  $\nu_A: X \rightarrow [0, 1]$  denote the membership and non-membership function of  $A$  respectively with the condition  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$  for every  $x \in X$ . Using (2), the Zadeh's [1] fuzzy set can be expressed as

$$A = \{(x; \mu_A(x), 1 - \mu_A(x)); x \in X\} \quad (3)$$

Let  $\mu_A: X \rightarrow [0, 1]$  and  $\nu_A: X \rightarrow [0, 1]$  be two functions on  $X$  (discrete/continuous) such that  $0 \leq \nu_A(x) \leq \mu_A(x) \leq 1$  and  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$  for every  $x \in X$ , then [19] proposed an alternative definition of fuzzy number as

$$A = \{(x; \mu_A(x), \nu_A(x)); x \in X\} \quad (4)$$

$\mu_A(x)$  is named as membership function and  $\nu_A(x)$  is named reference function of  $A$  in  $X$  such that  $\mu_A(x) - \nu_A(x)$  is the membership value for any  $x \in X$ . The Zadeh's [1] FS can be reproduced using (4) as

$$A = \{(x; \mu_A(x), 0), x \in X\} \quad (5)$$

Accordingly, the authors [19, 20] have redefined most the algebraic operations on fuzzy numbers using both membership and reference functions.

In this article, we propose to work on the new FS of [19] over discrete set of universe. Let  $X = \{x_1, x_2, \dots, x_n\}$  be the universe of discourse, then the FS  $A$  is characterized as follows [19]:

$$A = \{(x_1; \mu_A(x_1), \nu_A(x_1)), (x_2; (\mu_A(x_2), \nu_A(x_2))), \dots (x_n; (\mu_A(x_n), \nu_A(x_n)), x_i \in X\} \quad (6)$$

In other words,

$$A = \{(x_i; \mu_A(x_i), \nu_A(x_i)), x_i \in X; i=1, 2, \dots, n\} \quad (7)$$

where  $\mu_A(x_i) - \nu_A(x_i)$  = the membership value of  $x_i$  in  $A$ .

## 3. Proposed Method:

The proposed technique of computing correlation coefficient of fuzzy sets defined over discrete set of universe of discourse is given as follows.

For the two fuzzy  $A = \{(x_i; (\mu_A(x_i), \nu_A(x_i)), x_i \in X; i=1, 2, \dots, n\}$ , and  $B = \{(x_i; (\mu_B(x_i), \nu_B(x_i)), x_i \in X; i=1, 2, \dots, n\}$  over  $X = \{x_1, x_2, \dots, x_n\}$ , then  $\lambda_A(x_i) = \mu_A(x_i) - \nu_A(x_i)$  and  $\lambda_B(x_i) = \mu_B(x_i) - \nu_B(x_i); x_i \in X; i=1, 2, \dots, n$  are the membership values of  $A$  and  $B$  respectively. Then, the CC between  $A$  and  $B$  is expressed by the formula

$$\rho(A, B) = \frac{cov(A, B)}{\sqrt{cov(A, A).cov(B, B)}} \quad (8)$$

$$\text{where } cov(A, B) = \frac{\sum_{i=1}^n \left( \frac{(\lambda_A(x_i) - \bar{\lambda}_A(x))}{2} \right) \cdot \left( \frac{(\lambda_B(x_i) - \bar{\lambda}_B(x))}{2} \right)}{n-1} = \text{covariance of } A \text{ and } B, \quad (9)$$

$$cov(A, A) = \frac{\sum_{i=0}^n \left( \frac{(\lambda_A(x_i) - \bar{\lambda}_A(x_i))}{2} \right)^2}{n-1} = \text{covariance of the same fuzzy set } A, \quad (10)$$

$$\text{and } cov(B, B) = \frac{\sum_{i=0}^n \left( \frac{(\lambda_B(x_i) - \bar{\lambda}_B(x_i))}{2} \right)^2}{n-1} = \text{covariance of the same fuzzy set } B. \quad (11)$$

$$\text{Mean } (A) = \bar{\lambda}_A(x) = \frac{\sum_{i=1}^n \lambda_A(x_i)}{n} = \frac{\sum_{i=1}^n (\mu_A(x_i) - \nu_A(x_i))}{n} \quad (12)$$

$$\text{Mean } (B) = \bar{\lambda}_B(x) = \frac{\sum_{i=1}^n \lambda_B(x_i)}{n} = \frac{\sum_{i=1}^n (\mu_B(x_i) - \nu_B(x_i))}{n} \quad (13)$$

Using (9), (10) and (11), (8) can be expressed as

$$\begin{aligned} \rho(A, B) &= \frac{\frac{\sum_{i=1}^n \left( \frac{(\lambda_A(x_i) - \bar{\lambda}_A(x))}{2} \right) \cdot \left( \frac{(\lambda_B(x_i) - \bar{\lambda}_B(x))}{2} \right)}{n-1}}{\sqrt{\frac{\sum_{i=0}^n \left( \frac{(\lambda_A(x_i) - \bar{\lambda}_A(x_i))}{2} \right)^2}{n-1} \cdot \frac{\sum_{i=0}^n \left( \frac{(\lambda_B(x_i) - \bar{\lambda}_B(x_i))}{2} \right)^2}{n-1}}} \\ &= \frac{\frac{\sum_{i=0}^n (\lambda_A(x_i) - \bar{\lambda}_A(x_i)) (\lambda_B(x_i) - \bar{\lambda}_B(x_i))}{n}}{\sqrt{\sum_{i=0}^n (\lambda_A(x_i) - \bar{\lambda}_A(x_i))^2 \cdot \sum_{i=0}^n (\lambda_B(x_i) - \bar{\lambda}_B(x_i))^2}} \quad (14) \end{aligned}$$

**Theorem.** If  $A = \{(x_i, \mu_A(x_i), \nu_A(x_i)); x_i \in X, i=1, 2, \dots, n\}$ , and  $B = \{(x_i, \mu_B(x_i), \nu_B(x_i)); x_i \in X, i=1, 2, \dots, n\}$  are two fuzzy sets over a discrete set of universe of discourse  $X$ , then the correlation coefficient  $\rho(A, B)$  lies between -1 and 1.

**Proof.** Case1. Let us suppose that  $\mu_B(x_i) = a \cdot \mu_A(x_i) + b$  and  $\nu_B(x_i) = a \cdot \nu_A(x_i) + b$ , for any two real numbers  $a$  and  $b$ ,  $i=1, 2, \dots, n$ . We choose  $a$  and  $b$  in such a way that the conditions  $0 \leq \nu_A(x) \leq \mu_A(x) \leq 1$ , and  $0 \leq \nu_B(x) \leq \mu_B(x) \leq 1$  are satisfied. Since  $\lambda_B(x_i) = \mu_B(x_i) - \nu_B(x_i) = a \cdot \lambda_A(x_i)$ , we have

$$\bar{\lambda}_B(x_i) = \frac{\sum_{i=1}^n \lambda_B(x_i)}{n} = \frac{\sum_{i=1}^n a \cdot \lambda_A(x_i)}{n} = a \cdot \bar{\lambda}_A(x_i) \quad (8)$$

From (14), we get

$$\begin{aligned} \rho(A, B) &= \frac{\frac{\sum_{i=0}^n (\lambda_A(x_i) - \bar{\lambda}_A(x_i)) (\lambda_B(x_i) - \bar{\lambda}_B(x_i))}{n}}{\sqrt{\sum_{i=0}^n (\lambda_A(x_i) - \bar{\lambda}_A(x_i))^2 \cdot \sum_{i=0}^n (\lambda_B(x_i) - \bar{\lambda}_B(x_i))^2}} \\ &= \frac{\frac{\sum_{i=0}^n (\lambda_A(x_i) - \bar{\lambda}_A(x_i)) (a \cdot \lambda_A(x_i) - a \cdot \bar{\lambda}_A(x_i))}{n}}{\sqrt{\sum_{i=0}^n (\lambda_A(x_i) - \bar{\lambda}_A(x_i))^2 \cdot \sum_{i=0}^n (a \cdot \lambda_A(x_i) - a \cdot \bar{\lambda}_A(x_i))^2}} \\ &= \frac{a \cdot \sum_{i=0}^n (\lambda_A(x_i) - \bar{\lambda}_A(x_i))^2}{|a| \sqrt{\sum_{i=0}^n (\lambda_A(x_i) - \bar{\lambda}_A(x_i))^2}} \end{aligned}$$

$$\frac{a}{|a|}$$

$$= \begin{cases} 1, & \text{if } a > 0 \\ -1, & \text{if } a < 0 \end{cases}$$

Case2. For the cases other than case1, we proceed as follows. By Cauchy-Schwartz's inequality, we have

$$\begin{aligned} & \left( \sum_{i=0}^n (\lambda_A(x_i) - \overline{\lambda_A(x_i)}) (\lambda_B(x_i) - \overline{\lambda_B(x)}) \right)^2 \leq \sum_{i=0}^n (\lambda_A(x_i) - \overline{\lambda_A(x_i)})^2 \cdot \sum_{i=0}^n (a \cdot \lambda_A(x_i) - a \cdot \overline{\lambda_A(x_i)})^2 \\ & \Rightarrow \frac{\left( \sum_{i=0}^n (\lambda_A(x_i) - \overline{\lambda_A(x_i)}) (\lambda_B(x_i) - \overline{\lambda_B(x)}) \right)^2}{\sum_{i=0}^n (\lambda_A(x_i) - \overline{\lambda_A(x_i)})^2 \cdot \sum_{i=0}^n (a \cdot \lambda_A(x_i) - a \cdot \overline{\lambda_A(x_i)})^2} \leq 1 \\ & \Rightarrow (\rho(A, B))^2 \leq 1 \quad (\text{using (7)}) \\ & \Rightarrow -1 \leq \rho(A, B) \leq 1 \end{aligned}$$

#### 4. Conclusion:

In this article, a method of correlation coefficient (CC) of FSs over discrete set of universe of discourse is proposed. The fuzzy sets are defined in terms of membership functions and reference functions and their membership values are computed using difference of membership and reference functions [19, 20]. It has been observed that the value of the CC lies in the closed interval [-1, 1] indicating that the proposed method not only gives the strength of the relationship between FSs but also provide whether the FSs are positively or negatively correlated. So, our method outperforms some of the existing methods in the sense that the existing methods only find the strength of the relationship of the FSs. In future, we will try to study the behavior of other statistical parameters using similar fuzzy setting.

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