

AN INVESTIGATION OF BIPOLAR FUZZY COMPLEX IN SEMIRING

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Abstract: In this study investigates the fundamental notion of bipolar fuzzy complex ideals and examines their key properties. It introduces the concept of bipolar fuzzy complex ideals within semirings, broadening the traditional understanding of ideals by incorporating a more adaptable and expressive approach to membership values. Furthermore, the paper analyzes several crucial properties and notable outcomes related to semiring ideals under the framework of bipolar fuzzy logic.

Keywords: Bipolar fuzzy sets; Bipolar complex fuzzy idelas; Ideals; fully idempotens ideals; Semiring.

1. Introduction:

Several algebraic notions are used to extend the description of an associative ring as both a near-ring and a semiring. These structures are especially helpful in a number of practical mathematics fields, such as formal language theory, automata theory, and optimization. A semiring is an algebraic structure that contains binary associative pairs of addition and multiplication operations, where addition is distributive over addition and multiplication is commutative. The formal definition of a semiring was first presented by Vandiver in 1935. Since then, several scholars have studied the semiring theory in great detail in order to describe its different types. Weakly regular semiring and totally idempotent semiring are two important extensions of ring theory to the theory of semiring.

A member of a set is binary in classical set theory; it either belongs to the set or it does not. But classical sets cannot capture the uncertainty or partial truth that characterizes many real-world scenarios. Lotfi A. Zadeh developed the idea of fuzzy sets in 1965 in order to solve this [8]. A more flexible representation of imprecise or ambiguous information is made possible by the degree of membership that each element in a fuzzy set has, which ranges from 0 to 1. On top of this, human decision-making is based on bipolar judgmental thinking, which has both positive and negative aspects. Effects and side effects, teamwork, and competition are a few instances of two-sided choices. This meant that a fuzzy set that deals with bipolar judgment was needed. In 1998, Zhang introduced the bipolar fuzzy set. It addresses the level of membership of an element that fulfills the property as well as the level of membership of that element that satisfies the counter property. Furthermore, the membership degree range of a bipolar fuzzy set is extended from $[0,1]$ to $[-1,1]$. Zhou and Li later presented the idea of bipolar fuzzy subsemirings and ideals of semirings.

Naturally, real numbers were not the end of the number system's evolution. In the past, complex numbers were derived from real ones. Likewise, one could reasonably think that the same idea could be used to extend fuzzy set theory. This results in a so-called complex fuzzy set, which is characterized by a membership function that accepts values over the complex domain. The innovative complex fuzzy notion was introduced by Ramot, D. et al. in 2002. In contrast to ordinary fuzzy sets, which have membership values that fall between $[0,1]$, complex fuzzy sets are defined by membership functions that fall on the complex plane's unit circle.

2. Definition and Results:

Definition 2.1. A bipolar complex fuzzy subset $A = (\xi_{rA} + i\xi_{iA}, \zeta_{rA} + i\zeta_{iA})$ of a semiring R is said to be a bipolar complex fuzzy subsemiring of R if

- $\xi_{rA}(g+h) \geq \xi_{rA}(g) \wedge \xi_{rA}(h)$ and $\xi_{iA}(g+h) \geq \xi_{iA}(g) \wedge \xi_{iA}(h)$
- $\xi_{rA}(gh) \geq \xi_{rA}(g) \wedge \xi_{rA}(h)$ and $\xi_{iA}(gh) \geq \xi_{iA}(g) \wedge \xi_{iA}(h)$
- $\zeta_{rA}(g+h) \leq \zeta_{rA}(g) \vee \zeta_{rA}(h)$ and $\zeta_{iA}(g+h) \leq \zeta_{iA}(g) \vee \zeta_{iA}(h)$
- $\zeta_{rA}(gh) \leq \zeta_{rA}(g) \vee \zeta_{rA}(h)$ and $\zeta_{iA}(gh) \leq \zeta_{iA}(g) \vee \zeta_{iA}(h)$ for all $g, y \in R$.

Definition 2.2. A bipolar complex fuzzy subset $A = (\xi_{rA} + i\xi_{iA}, \zeta_{rA} + i\zeta_{iA})$ of a semiring R is said to be a bipolar fuzzy left(right) ideal of R if

- $\xi_{rA}(g+h) \geq \xi_{rA}(g) \wedge \xi_{rA}(h)$ and $\xi_{iA}(g+h) \geq \xi_{iA}(g) \wedge \xi_{iA}(h)$
- $\xi_{rA}(gh) \geq \xi_{rA}(h)(\xi_{rA}(gh) \geq \xi_{rA}(g)$ and $\xi_{iA}(gh) \geq \xi_{iA}(h)(\xi_{iA}(gh) \geq \xi_{iA}(g)$
- $\zeta_{rA}(g+h) \leq \zeta_{rA}(g) \vee \zeta_{rA}(h)$ and $\zeta_{iA}(g+h) \leq \zeta_{iA}(g) \vee \zeta_{iA}(h)$
- $\zeta_{rA}(gh) \leq \xi_{rA}(h)(\zeta_{rA}(gh) \leq \zeta_{rA}(g)$ and $\zeta_{iA}(gh) \leq \xi_{iA}(h)(\zeta_{iA}(gh) \leq \zeta_{iA}(g)$ for all $g, y \in R$.

Definition 2.3. Let $A = (\xi_{rA} + i\xi_{iA}, \zeta_{rA} + i\zeta_{iA})$ and $B = (\xi_{rB} + i\xi_{iB}, \zeta_{rB} + i\zeta_{iB})$ be two bipolar complex fuzzy subsets of a semiring R , then the sum of bipolar fuzzy subsets A and B is defined as $A+B = ((\xi_{rA} + \xi_{rB}) + i(\xi_{iA} + \xi_{iB}), (\zeta_{rA} + \zeta_{rB}) + i(\zeta_{iA} + \zeta_{iB}))$ for all $g \in R$, where

- $(\xi_{rA} + \xi_{rB})(g) = \begin{cases} \bigvee_{g=a+b} \{\xi_{rA}(a) \wedge \xi_{rB}(b)\} \\ 0, \text{if } g \text{ is not expressible as } g = a+b \end{cases}$,
- $(\xi_{iA} + \xi_{iB})(g) = \begin{cases} \bigvee_{g=a+b} \{\xi_{iA}(a) \wedge \xi_{iB}(b)\} \\ 0, \text{if } g \text{ is not expressible as } g = a+b \end{cases}$,
- $(\zeta_{rA} + \zeta_{rB})(g) = \begin{cases} \bigwedge_{g=a+b} \{\zeta_{rA}(a) \vee \zeta_{rB}(b)\} \\ 0, \text{if } g \text{ is not expressible as } g = a+b. \end{cases}$ and
- $(\zeta_{iA} + \zeta_{iB})(g) = \begin{cases} \bigwedge_{g=a+b} \{\zeta_{iA}(a) \vee \zeta_{iB}(b)\} \\ 0, \text{if } g \text{ is not expressible as } g = a+b. \end{cases}$

Definition 2.4. Let $A = (\xi_{rA} + i\xi_{iA}, \zeta_{rA} + i\zeta_{iA})$ and $B = (\xi_{rB} + i\xi_{iB}, \zeta_{rB} + i\zeta_{iB})$ be two bipolar complex fuzzy subsets of a semiring R , then the product of bipolar fuzzy subsets A and B is defined as $A \circ B = (\xi_{rA} \circ \xi_{rB} + i(\xi_{iA} \circ \xi_{iB}), \zeta_{rA} \circ \zeta_{rB} + i(\zeta_{iA} \circ \zeta_{iB}))$ for all $g \in R$, where

- $(\xi_{rA} \circ \xi_{rB})(g) = \begin{cases} \bigvee_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigwedge_{i=1}^n (\xi_{rA}(a_i) \wedge \xi_{rB}(b_i)) \right\}, & \text{if } g = \sum_{i=1}^n a_i b_i, \\ 0, & \text{otherwise.} \end{cases}$
- $(\xi_{iA} \circ \xi_{iB})(g) = \begin{cases} \bigvee_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigwedge_{i=1}^n (\xi_{iA}(a_i) \wedge \xi_{iB}(b_i)) \right\}, & \text{if } g = \sum_{i=1}^n a_i b_i, \\ 0, & \text{otherwise.} \end{cases}$
- $(\zeta_{rA} \circ \zeta_{rB})(g) = \begin{cases} \bigwedge_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigvee_{i=1}^n (\zeta_{rA}(a_i) \vee \zeta_{rB}(b_i)) \right\}, & \text{if } g = \sum_{i=1}^n a_i b_i, \\ 0, & \text{otherwise.} \end{cases}$

$$\bullet \quad (\zeta_{iA} \circ \zeta_{iB})(g) = \begin{cases} \bigwedge_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigvee_{i=1}^n (\zeta_{iA}(a_i) \vee \zeta_{iB}(b_i)) \right\}, & \text{if } g = \sum_{i=1}^n a_i b_i, \\ 0, & \text{otherwise.} \end{cases}$$

Example 1. Let $R = \mathbb{N}_0$ be the semiring of non-negative integers under usual addition and multiplication. Define two bipolar complex fuzzy subsets A and B of R as follows:

$$\begin{aligned} \xi_{rA}(n) &= \begin{cases} 1 & \text{if } 2 \mid n, \\ 0.5 & \text{if } 2 \nmid n, \end{cases} & \xi_{iA}(n) &= \begin{cases} 0.8 & \text{if } 2 \mid n, \\ 0.3 & \text{if } 2 \nmid n, \end{cases} \\ \zeta_{rA}(n) &= \begin{cases} -0.5 & \text{if } 2 \mid n, \\ -0.2 & \text{if } 2 \nmid n, \end{cases} & \zeta_{iA}(n) &= \begin{cases} -0.6 & \text{if } 2 \mid n, \\ -0.3 & \text{if } 2 \nmid n, \end{cases} \\ \xi_{rB}(n) &= \begin{cases} 1 & \text{if } 3 \mid n, \\ 0.4 & \text{if } 3 \nmid n, \end{cases} & \xi_{iB}(n) &= \begin{cases} 0.7 & \text{if } 3 \mid n, \\ 0.25 & \text{if } 3 \nmid n, \end{cases} \\ \zeta_{rB}(n) &= \begin{cases} -0.4 & \text{if } 3 \mid n, \\ -0.1 & \text{if } 3 \nmid n, \end{cases} & \zeta_{iB}(n) &= \begin{cases} -0.5 & \text{if } 3 \mid n, \\ -0.15 & \text{if } 3 \nmid n. \end{cases} \end{aligned}$$

Thus $A = (\xi_{rA} + i\xi_{iA}, \zeta_{rA} + i\zeta_{iA})$ and $B = (\xi_{rB} + i\xi_{iB}, \zeta_{rB} + i\zeta_{iB})$. It follows upon easy inspection that A , B and $A + B$ all satisfy the criteria of bipolar fuzzy left and right ideals of R . This motivates the formulation of the next theorem.

Theorem 1. $A+B$ is a bipolar fuzzy complex left(right) ideal of a semiring R , if $A = (\xi_{rA} + i\xi_{iA}, \zeta_{rA} + i\zeta_{iA})$ and $B = (\xi_{rB} + i\xi_{iB}, \zeta_{rB} + i\zeta_{iB})$ are bipolar fuzzy complex left(right) ideals of R .

Proof. Let $A=(\xi_{rA}, \zeta_{rA})$, $B=(\xi_{rB}, \zeta_{rB})$ are two bipolar fuzzy left ideals of a semiring R and $g, h \in R$. Now if $(\xi_{rA} + \xi_{rB})(g) = 0$ or $(\xi_{rA} + \xi_{rB})(h) = 0$, $(\xi_{iA} + \xi_{iB})(g) = 0$ or $(\xi_{iA} + \xi_{iB})(h) = 0$, $(\zeta_{rA} + \zeta_{rB})(g) = 0$ or $(\zeta_{rA} + \zeta_{rB})(h) = 0$, and $(\zeta_{iA} + \zeta_{iB})(g) = 0$ or $(\zeta_{iA} + \zeta_{iB})(h) = 0$ then

$$(\xi_{rA} + \xi_{rB})(g) \wedge (\xi_{rA} + \xi_{rB})(h) = 0 \leq (\xi_{rA} + \xi_{rB})(g+h), (\xi_{iA} + \xi_{iB})(g) \wedge (\xi_{iA} + \xi_{iB})(h) = 0 \leq (\xi_{iA} + \xi_{iB})(g+h)$$

$$\text{and } (\zeta_{rA} + \zeta_{rB})(g) \vee (\zeta_{rA} + \zeta_{rB})(h) = 0 \geq (\zeta_{rA} + \zeta_{rB})(g+h). \text{ and } (\zeta_{iA} + \zeta_{iB})(g) \vee (\zeta_{iA} + \zeta_{iB})(h) = 0 \geq (\zeta_{iA} + \zeta_{iB})(g+h).$$

If $(\xi_{rA} + \xi_{rB})(g) \neq 0$, $(\xi_{rA} + \xi_{rB})(h) \neq 0$, $(\zeta_{rA} + \zeta_{rB})(g) \neq 0$, $(\zeta_{rA} + \zeta_{rB})(h) \neq 0$, $(\xi_{iA} + \xi_{iB})(g) \neq 0$, $(\xi_{iA} + \xi_{iB})(h) \neq 0$, $(\zeta_{iA} + \zeta_{iB})(g) \neq 0$ and $(\zeta_{iA} + \zeta_{iB})(h) \neq 0$, then by definition

$$(\xi_{rA} + \xi_{rB})(g) = \bigvee_{g=a+b} \{\xi_{rA}(a) \wedge \xi_{rB}(b)\}, (\xi_{iA} + \xi_{iB})(g) = \bigvee_{g=a+b} \{\xi_{iA}(a) \wedge \xi_{iB}(b)\}$$

$$(\xi_{rA} + \xi_{rB})(h) = \bigvee_{h=c+d} \{\xi_{rA}(c) \wedge \xi_{rB}(d)\}, (\xi_{iA} + \xi_{iB})(h) = \bigvee_{h=c+d} \{\xi_{iA}(c) \wedge \xi_{iB}(d)\}$$

$$(\zeta_{rA} + \zeta_{rB})(g) = \bigwedge_{g=a+b} \{\zeta_{rA}(a) \vee \zeta_{rB}(b)\}, (\zeta_{iA} + \zeta_{iB})(g) = \bigwedge_{g=a+b} \{\zeta_{iA}(a) \vee \zeta_{iB}(b)\}$$

$$(\zeta_{rA} + \zeta_{rB})(h) = \bigwedge_{h=c+d} \{\zeta_{rA}(c) \vee \zeta_{rB}(d)\}, (\zeta_{iA} + \zeta_{iB})(h) = \bigwedge_{h=c+d} \{\zeta_{iA}(c) \vee \zeta_{iB}(d)\}.$$

Thus,

$$\begin{aligned} (\xi_{rA} + \xi_{rB})(g) \wedge (\xi_{rA} + \xi_{rB})(h) &= \left(\bigvee_{g=a+b} \{\xi_{rA}(a) \wedge \xi_{rB}(b)\} \right) \wedge \left(\bigvee_{h=c+d} \{\xi_{rA}(c) \wedge \xi_{rB}(d)\} \right) \\ &= \bigvee_{g=a+b} \bigvee_{h=c+d} \{(\xi_{rA}(a) \wedge \xi_{rB}(b)) \wedge (\xi_{rA}(c) \wedge \xi_{rB}(d))\} \\ &= \bigvee_{g=a+b} \bigvee_{h=c+d} \{(\xi_{rA}(a) \wedge \xi_{rA}(c)) \wedge (\xi_{rB}(b) \wedge \xi_{rB}(d))\} \leq \bigvee_{g=a+b} \bigvee_{h=c+d} \{\xi_{rA}(a+c) \wedge \xi_{rB}(b+d)\} \end{aligned}$$

$= (\xi_{rA} + \xi_{rB})(g+h)$ and

$$\begin{aligned} (\xi_{iA} + \xi_{iB})(g) \wedge (\xi_{iA} + \xi_{iB})(h) &= \left(\bigvee_{g=a+b} \{\xi_{iA}(a) \wedge \xi_{iB}(b)\} \right) \wedge \left(\bigvee_{h=c+d} \{\xi_{iA}(c) \wedge \xi_{iB}(d)\} \right) \\ &= \bigvee_{g=a+b} \bigvee_{h=c+d} \{(\xi_{iA}(a) \wedge \xi_{iB}(b)) \wedge (\xi_{iA}(c) \wedge \xi_{iB}(d))\} = \bigvee_{g=a+b} \bigvee_{h=c+d} \{(\xi_{iA}(a) \wedge \xi_{iA}(c)) \wedge (\xi_{iB}(b) \wedge \xi_{iB}(d))\} \leq \\ &= \bigvee_{g=a+b} \bigvee_{h=c+d} \{\xi_{iA}(a+c) \wedge \xi_{iB}(b+d)\} = (\xi_{iA} + \xi_{iB})(g+h) \end{aligned}$$

Also,

$$\begin{aligned} (\zeta_{rA} + \zeta_{rB})(g) \vee (\zeta_{rA} + \zeta_{rB})(h) &= \left(\bigwedge_{g=a+b} \{\zeta_{rA}(a) \vee \zeta_{rB}(b)\} \right) \vee \left(\bigwedge_{h=c+d} \{\zeta_{rA}(c) \vee \zeta_{rB}(d)\} \right) \\ &= \bigwedge_{g=a+b} \bigwedge_{h=c+d} \{(\zeta_{rA}(a) \vee \zeta_{rB}(b)) \vee (\zeta_{rA}(c) \vee \zeta_{rB}(d))\} \\ &= \bigwedge_{g=a+b} \bigwedge_{h=c+d} \{(\zeta_{rA}(a) \vee \zeta_{rA}(c)) \vee (\zeta_{rB}(b) \vee \zeta_{rB}(d))\} \geq \bigwedge_{g=a+b} \bigwedge_{h=c+d} \{\zeta_{rA}(a+c) \vee \zeta_{rB}(b+d)\} = (\zeta_{rA} + \zeta_{rB})(g+h), \text{ and} \end{aligned}$$

$$\begin{aligned} (\zeta_{iA} + \zeta_{iB})(g) \vee (\zeta_{iA} + \zeta_{iB})(h) &= \left(\bigwedge_{g=a+b} \{\zeta_{iA}(a) \vee \zeta_{iB}(b)\} \right) \vee \left(\bigwedge_{h=c+d} \{\zeta_{iA}(c) \vee \zeta_{iB}(d)\} \right) \\ &= \bigwedge_{g=a+b} \bigwedge_{h=c+d} \{(\zeta_{iA}(a) \vee \zeta_{iB}(b)) \vee (\zeta_{iA}(c) \vee \zeta_{iB}(d))\} \\ &= \bigwedge_{g=a+b} \bigwedge_{h=c+d} \{(\zeta_{iA}(a) \vee \zeta_{iA}(c)) \vee (\zeta_{iB}(b) \vee \zeta_{iB}(d))\} \geq \bigwedge_{g=a+b} \bigwedge_{h=c+d} \{\zeta_{iA}(a+c) \vee \zeta_{iB}(b+d)\} \\ &= (\zeta_{iA} + \zeta_{iB})(g+h). \end{aligned}$$

Again if $(\xi_{rA} + \xi_{rB})(g) = 0$, $(\zeta_{rA} + \zeta_{rB})(g) = 0$, $(\xi_{iA} + \xi_{iB})(g) = 0$ and $(\zeta_{iA} + \zeta_{iB})(g) = 0$ then, $(\xi_{rA} + \xi_{rB})(g) \leq (\xi_{rA} + \xi_{rB})(yx)$, $(\xi_{iA} + \xi_{iB})(g) \leq (\xi_{iA} + \xi_{iB})(yx)$, $(\zeta_{iA} + \zeta_{iB})(g) \geq (\zeta_{iA} + \zeta_{iB})(yx)$ and $(\zeta_{rA} + \zeta_{rB})(g) \geq (\zeta_{rA} + \zeta_{rB})(yx)$.

If $(\xi_{rA} + \xi_{rB})(g) \neq 0$, $(\xi_{iA} + \xi_{iB})(g) \neq 0$, $(\zeta_{iA} + \zeta_{iB})(g) \neq 0$ and $(\zeta_{rA} + \zeta_{rB})(g) \neq 0$ then,

$$\begin{aligned} (\xi_{rA} + \xi_{rB})(g) &= \bigvee_{g=a+b} \{\xi_{rA}(a) \wedge \xi_{rB}(b)\} \leq \bigvee_{g=a+b} \{\xi_{rA}(ya) \wedge \xi_{rB}(yb)\} \\ &\leq \bigvee_{yg=c+} \{\xi_{rA}(c) \wedge \xi_{rB}(d)\} = (\xi_{rA} + \xi_{rB})(yx) \end{aligned}$$

$$\begin{aligned} \text{and } (\xi_{iA} + \xi_{iB})(g) &= \bigvee_{g=a+b} \{\xi_{iA}(a) \wedge \xi_{iB}(b)\} \leq \bigvee_{g=a+b} \{\xi_{iA}(ya) \wedge \xi_{iB}(yb)\} \\ &\leq \bigvee_{yg=c+} \{\xi_{iA}(c) \wedge \xi_{iB}(d)\} = (\xi_{iA} + \xi_{iB})(yx) \end{aligned}$$

Also,

$$\begin{aligned} (\zeta_{rA} + \zeta_{rB})(g) &= \bigwedge_{g=a+b} \{\zeta_{rA}(a) \vee \zeta_{rB}(b)\} \geq \bigwedge_{g=a+b} \{\zeta_{rA}(ya) \vee \zeta_{rB}(yb)\} \\ &\geq \bigwedge_{yg=c+} \{\zeta_{rA}(c) \vee \zeta_{rB}(d)\} = (\zeta_{rA} + \zeta_{rB})(yx) \text{ and} \end{aligned}$$

$$\begin{aligned} (\zeta_{iA} + \zeta_{iB})(g) &= \bigwedge_{g=a+b} \{\zeta_{iA}(a) \vee \zeta_{iB}(b)\} \geq \bigwedge_{g=a+b} \{\zeta_{iA}(ya) \vee \zeta_{iB}(yb)\} \\ &\geq \bigwedge_{yg=c+} \{\zeta_{iA}(c) \vee \zeta_{iB}(d)\} = (\zeta_{iA} + \zeta_{iB})(yx). \end{aligned}$$

Therefore, $A+B$ is a bipolar fuzzy complex left ideal of R .

Example 2. Consider the semiring $R = \{0, \alpha, 1\}$ defined by the followings tables.

$\cdot$	0	α	1
0	0	0	0
α	0	α	1
1	0	α	1

$+$	0	α	1
0	0	α	1
α	α	α	α
1	1	α	1

Consider $S = \{0, \alpha\}$, and $A = (\xi_{rA} + i\xi_{iA}, \zeta_{rA} + i\zeta_{iA})$, $B = (\xi_{rB} + i\xi_{iB}, \zeta_{rB} + i\zeta_{iB})$ are two bipolar complex fuzzy sets defined as:

$$\begin{aligned}\xi_{rA}(x) &= \begin{cases} 0.35, & \text{if } x \in S \\ 0.15, & \text{if } x \notin S \end{cases}, \xi_{iA}(x) = \begin{cases} 0.42, & \text{if } x \in S \\ 0.12, & \text{if } x \notin S \end{cases} \\ \zeta_{rA}(x) &= \begin{cases} 0.15, & \text{if } x \in S \\ 0.27, & \text{if } x \notin S \end{cases}, \zeta_{iA}(x) = \begin{cases} 0.12, & \text{if } x \in S \\ 0.17, & \text{if } x \notin S \end{cases} \\ \xi_{rB} &= \begin{cases} 0.33, & \text{if } x \in S \\ 0.12, & \text{if } x \notin S \end{cases}, \xi_{iB} = \begin{cases} 0.25, & \text{if } x \in S \\ 0.19, & \text{if } x \notin S \end{cases} \\ \zeta_{rB}(x) &= \begin{cases} 0.25, & \text{if } x \in S \\ 0.65, & \text{if } x \notin S \end{cases} \text{ and } \zeta_{iB}(x) = \begin{cases} 0.24, & \text{if } x \in S \\ 0.72, & \text{if } x \notin S \end{cases}\end{aligned}$$

Then it can be easily verified that A, B and $A \circ B$ are bipolar fuzzy left as well as right ideal of R . This observation motivates us to investigate the following theorem.

Theorem 2. $A \circ B$ is a bipolar fuzzy complex left (right) ideal of a semiring R , if $A = (\xi_{rA} + i\xi_{iA}, \zeta_{rA} + i\zeta_{iA})$ and $B = (\xi_{rB} + i\xi_{iB}, \zeta_{rB} + i\zeta_{iB})$ are two bipolar fuzzy complex left(right) ideals of R .

Proof. Let $A = (\xi_{rA} + i\xi_{iA}, \zeta_{rA} + i\zeta_{iA})$ and $B = (\xi_{rB} + i\xi_{iB}, \zeta_{rB} + i\zeta_{iB})$ are two bipolar fuzzy complex left(right) ideals of a semiring R and let $g, y \in R$.

Now if $(\xi_{rA} \circ \xi_{rB})(g) = 0$ or $(\xi_{rA} \circ \xi_{rB})(h) = 0$, $(\xi_{iA} \circ \xi_{iB})(g) = 0$ or $(\xi_{iA} \circ \xi_{iB})(h) = 0$, $(\zeta_{rA} \circ \zeta_{rB})(g) = 0$ or $(\zeta_{rA} \circ \zeta_{rB})(h) = 0$, and $(\zeta_{iA} \circ \zeta_{iB})(g) = 0$ or $(\zeta_{iA} \circ \zeta_{iB})(h) = 0$ then

$$\begin{aligned}(\xi_{rA} \circ \xi_{rB})(g) \wedge (\xi_{rA} \circ \xi_{rB})(h) = 0 &\leq (\xi_{rA} \circ \xi_{rB})(g+h), (\xi_{iA} \circ \xi_{iB})(g) \wedge (\xi_{iA} \circ \xi_{iB})(h) = 0 \leq (\xi_{iA} \circ \xi_{iB})(g+h), \\ (\zeta_{rA} \circ \zeta_{rB})(g) \vee (\zeta_{rA} \circ \zeta_{rB})(h) = 0 &\geq (\zeta_{rA} \circ \zeta_{rB})(g+h) \quad \text{and} \quad (\zeta_{iA} \circ \zeta_{iB})(g) \vee (\zeta_{iA} \circ \zeta_{iB})(h) = 0 \geq (\zeta_{iA} \circ \zeta_{iB})(g+h).\end{aligned}$$

If $(\xi_{rA} \circ \xi_{rB})(g) \neq 0$, $(\xi_{rA} \circ \xi_{rB})(h) \neq 0$, $(\zeta_{rA} \circ \zeta_{rB})(g) \neq 0$, $(\zeta_{rA} \circ \zeta_{rB})(h) \neq 0$, $(\xi_{iA} \circ \xi_{iB})(g) \neq 0$, $(\xi_{iA} \circ \xi_{iB})(h) \neq 0$ and $(\zeta_{iA} \circ \zeta_{iB})(g) \neq 0$, $(\zeta_{iA} \circ \zeta_{iB})(h) \neq 0$,

then by definition

$$\begin{aligned}(\xi_{rA} \circ \xi_{rB})(g) &= \bigvee_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigwedge_{i=1}^n (\xi_{rA}(a_i) \wedge \xi_{rB}(b_i)) \right\}, (\xi_{iA} \circ \xi_{iB})(g) = \bigvee_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigwedge_{i=1}^n (\xi_{iA}(a_i) \wedge \xi_{iB}(b_i)) \right\} \\ (\xi_{rA} \circ \xi_{rB})(h) &= \bigvee_{h=\sum_{j=1}^m c_j d_j} \left\{ \bigwedge_{j=1}^m (\xi_{rA}(c_j) \wedge \xi_{rB}(d_j)) \right\}, \\ (\xi_{iA} \circ \xi_{iB})(h) &= \bigvee_{h=\sum_{j=1}^m c_j d_j} \left\{ \bigwedge_{j=1}^m (\xi_{iA}(c_j) \wedge \xi_{iB}(d_j)) \right\}\end{aligned}$$

$$(\zeta_{rA} \circ \zeta_{rB})(g) = \bigwedge_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigvee_{i=1}^n (\zeta_{rA}(a_i) \vee \zeta_{rB}(b_i)) \right\}, \quad (\zeta_{iA} \circ \zeta_{iB})(g) = \bigwedge_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigvee_{i=1}^n (\zeta_{iA}(a_i) \vee \zeta_{iB}(b_i)) \right\}$$

$$(\zeta_{rA} \circ \zeta_{rB})(h) = \bigwedge_{h=\sum_{j=1}^m c_j d_j} \left\{ \bigvee_{j=1}^m (\zeta_{rA}(c_j) \vee \zeta_{rB}(d_j)) \right\},$$

$$(\zeta_{iA} \circ \zeta_{iB})(h) = \bigwedge_{h=\sum_{j=1}^m c_j d_j} \left\{ \bigvee_{j=1}^m (\zeta_{iA}(c_j) \vee \zeta_{iB}(d_j)) \right\}.$$

Thus,

$$\begin{aligned} & (\xi_{rA} \circ \xi_{rB})(g) \wedge (\xi_{rA} \circ \xi_{rB})(h) \\ &= \left[\bigvee_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigwedge_{i=1}^n (\xi_{rA}(a_i) \wedge \xi_{rB}(b_i)) \right\} \right] \wedge \left[\bigvee_{h=\sum_{j=1}^m c_j d_j} \left\{ \bigwedge_{j=1}^m (\xi_{rA}(c_j) \wedge \xi_{rB}(d_j)) \right\} \right] \\ &= \bigvee_{g=\sum_{i=1}^n a_i b_i} \bigvee_{h=\sum_{j=1}^m c_j d_j} \left[\bigwedge_{i=1}^n \{ \xi_{rA}(a_i) \wedge \xi_{rB}(b_i) \} \right] \wedge \left[\bigwedge_{j=1}^m \{ \xi_{rA}(c_j) \wedge \xi_{rB}(d_j) \} \right] \\ &\leq \bigvee_{g+h=\sum_{k=1}^q e_k f_k} \left[\bigwedge_{k=1}^q (\xi_{rA}(e_k) \wedge \xi_{rB}(f_k)) \right] = (\xi_{rA} \circ \xi_{rB})(g+h), \\ & (\xi_{iA} \circ \xi_{iB})(g) \wedge (\xi_{iA} \circ \xi_{iB})(h) \\ &= \left[\bigvee_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigwedge_{i=1}^n (\xi_{iA}(a_i) \wedge \xi_{iB}(b_i)) \right\} \right] \wedge \left[\bigvee_{h=\sum_{j=1}^m c_j d_j} \left\{ \bigwedge_{j=1}^m (\xi_{iA}(c_j) \wedge \xi_{iB}(d_j)) \right\} \right] \\ &= \bigvee_{g=\sum_{i=1}^n a_i b_i} \bigvee_{h=\sum_{j=1}^m c_j d_j} \left[\bigwedge_{i=1}^n \{ \xi_{iA}(a_i) \wedge \xi_{iB}(b_i) \} \right] \wedge \left[\bigwedge_{j=1}^m \{ \xi_{iA}(c_j) \wedge \xi_{iB}(d_j) \} \right] \\ &\leq \bigvee_{g+h=\sum_{k=1}^q e_k f_k} \left[\bigwedge_{k=1}^q (\xi_{iA}(e_k) \wedge \xi_{iB}(f_k)) \right] = (\xi_{iA} \circ \xi_{iB})(g+h) \end{aligned}$$

and $(\zeta_{rA} \circ \zeta_{rB})(g) \vee (\zeta_{rA} \circ \zeta_{rB})(h)$

$$\begin{aligned} &= \left[\bigwedge_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigvee_{i=1}^n (\zeta_{rA}(a_i) \vee \zeta_{rB}(b_i)) \right\} \right] \vee \left[\bigwedge_{h=\sum_{j=1}^m c_j d_j} \left\{ \bigvee_{j=1}^m (\zeta_{rA}(c_j) \vee \zeta_{rB}(d_j)) \right\} \right] \\ &= \bigwedge_{g=\sum_{i=1}^n a_i b_i} \bigwedge_{h=\sum_{j=1}^m c_j d_j} \left[\bigvee_{i=1}^n (\zeta_{rA}(a_i) \vee \zeta_{rB}(b_i)) \right] \vee \left[\bigvee_{j=1}^m (\zeta_{rA}(c_j) \vee \zeta_{rB}(d_j)) \right] \\ &\geq \bigwedge_{g+h=\sum_{k=1}^q e_k f_k} \left[\bigvee_{k=1}^q (\zeta_{rA}(e_k) \vee \zeta_{rB}(f_k)) \right] \\ &= (\zeta_{rA} \circ \zeta_{rB})(g+h), \text{ and } (\zeta_{iA} \circ \zeta_{iB})(g) \vee (\zeta_{iA} \circ \zeta_{iB})(h) \\ &= \left[\bigwedge_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigvee_{i=1}^n (\zeta_{iA}(a_i) \vee \zeta_{iB}(b_i)) \right\} \right] \vee \left[\bigwedge_{h=\sum_{j=1}^m c_j d_j} \left\{ \bigvee_{j=1}^m (\zeta_{iA}(c_j) \vee \zeta_{iB}(d_j)) \right\} \right] \\ &= \bigwedge_{g=\sum_{i=1}^n a_i b_i} \bigwedge_{h=\sum_{j=1}^m c_j d_j} \left[\bigvee_{i=1}^n (\zeta_{iA}(a_i) \vee \zeta_{iB}(b_i)) \right] \vee \left[\bigvee_{j=1}^m (\zeta_{iA}(c_j) \vee \zeta_{iB}(d_j)) \right] \end{aligned}$$

$$\geq \bigwedge_{g+h=\sum_{k=1}^q e_k f_k} \left[\bigvee_{k=1}^q (\zeta_{iA}(e_k) \vee \zeta_{iB}(f_k)) \right] = (\zeta_{iA} \circ \zeta_{iB})(g+h).$$

Again if $(\xi_{rA} \circ \xi_{rB})(g) = 0$, $(\xi_{iA} \circ \xi_{iB})(g) = 0$ then $(\xi_{rA} \circ \xi_{rB})(g) \leq (\xi_{rA} \circ \xi_{rB})(yx)$, $(\xi_{iA} \circ \xi_{iB})(g) \leq (\xi_{iA} \circ \xi_{iB})(yx)$.

And if $(\zeta_{rA} \circ \zeta_{rB})(g) = 0$, $(\zeta_{iA} \circ \zeta_{iB})(g) = 0$, then $(\zeta_{rA} \circ \zeta_{rB})(g) \geq (\zeta_{rA} \circ \zeta_{rB})(yx)$, $(\zeta_{iA} \circ \zeta_{iB})(g) \geq (\zeta_{iA} \circ \zeta_{iB})(yx)$.

Now if $(\xi_{rA} \circ \xi_{rB})(g) \neq 0$, $(\xi_{iA} \circ \xi_{iB})(g) \neq 0$ then

$$\begin{aligned} (\xi_{rA} \circ \xi_{rB})(g) &= \bigvee_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigwedge_{i=1}^n (\xi_{rA}(a_i) \wedge \xi_{rB}(b_i)) \right\} \\ &\leq \bigvee_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigwedge_{i=1}^n (\xi_{rA}(y a_i) \wedge \xi_{rB}(y b_i)) \right\} \\ &\leq \bigvee_{yg=\sum_{j=1}^m c_j d_j} \left\{ \bigwedge_{j=1}^m (\xi_{rA}(c_j) \wedge \xi_{rB}(d_j)) \right\} = (\xi_{rA} \circ \xi_{rB})(yx) \\ \text{and } (\xi_{iA} \circ \xi_{iB})(g) &= \bigvee_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigwedge_{i=1}^n (\xi_{iA}(a_i) \wedge \xi_{iB}(b_i)) \right\} \\ &\leq \bigvee_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigwedge_{i=1}^n (\xi_{iA}(y a_i) \wedge \xi_{iB}(y b_i)) \right\} \\ &\leq \bigvee_{yg=\sum_{j=1}^m c_j d_j} \left\{ \bigwedge_{j=1}^m (\xi_{iA}(c_j) \wedge \xi_{iB}(d_j)) \right\} = (\xi_{iA} \circ \xi_{iB})(yx). \end{aligned}$$

And if $(\zeta_{rA} \circ \zeta_{rB})(g) \neq 0$, $(\zeta_{iA} \circ \zeta_{iB})(g) \neq 0$, then

$$\begin{aligned} (\zeta_{rA} \circ \zeta_{rB})(g) &= \bigwedge_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigvee_{i=1}^n (\zeta_{rA}(a_i) \vee \zeta_{rB}(b_i)) \right\} \\ &\geq \bigwedge_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigvee_{i=1}^n (\zeta_{rA}(y a_i) \vee \zeta_{rB}(y b_i)) \right\} \\ &\geq \bigwedge_{yg=\sum_{j=1}^m c_j d_j} \left\{ \bigvee_{j=1}^m (\zeta_{rA}(c_j) \vee \zeta_{rB}(d_j)) \right\} = (\zeta_{rA} \circ \zeta_{rB})(yx) \\ \text{and } (\zeta_{iA} \circ \zeta_{iB})(g) &= \bigwedge_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigvee_{i=1}^n (\zeta_{iA}(a_i) \vee \zeta_{iB}(b_i)) \right\} \geq \bigwedge_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigvee_{i=1}^n (\zeta_{iA}(y a_i) \vee \zeta_{iB}(y b_i)) \right\} \\ &\geq \bigwedge_{yg=\sum_{j=1}^m c_j d_j} \left\{ \bigvee_{j=1}^m (\zeta_{iA}(c_j) \vee \zeta_{iB}(d_j)) \right\} = (\zeta_{iA} \circ \zeta_{iB})(yx). \end{aligned}$$

Thus, $A \circ B$ is a bipolar fuzzy complex left ideal of R . $\square$

Theorem 3. For a bipolar fuzzy complex right ideal $A = (\xi_{rA} + i\xi_{iA}, \zeta_{rA} + i\zeta_{iA})$ and a bipolar fuzzy left ideal $B = (\xi_{rB} + i\xi_{iB}, \zeta_{rB} + i\zeta_{iB})$ of a semiring R , the conditions $\xi_{rA} \circ \xi_{rB} \leq \xi_{rA} \wedge \xi_{rB}$, $\xi_{iA} \circ \xi_{iB} \leq \xi_{iA} \wedge \xi_{iB}$ and $\zeta_{rA} \circ \zeta_{rB} \geq \zeta_{rA} \vee \zeta_{rB}$, $\zeta_{iA} \circ \zeta_{iB} \geq \zeta_{iA} \vee \zeta_{iB}$ are satisfied.

Proof. Since, $A = (\xi_{rA} + i\xi_{iA}, \zeta_{rA} + i\zeta_{iA})$ is a bipolar fuzzy complex right ideal, therefore, we have $\xi_{rA} \circ \chi_R^P \leq \xi_{rA}$, $\xi_{iA} \circ \chi_R^P \leq \xi_{iA}$ and $\zeta_{rA} \circ \chi_R^N \geq \zeta_{rA}$, $\zeta_{iA} \circ \chi_R^N \geq \zeta_{iA}$.

Now, since $\xi_{rB} \leq \chi_R^P$, $\xi_{iB} \leq \chi_R^P$, $\zeta_{rA} \geq \chi_R^P$ and $\zeta_{iA} \geq \chi_R^P$, we have $\xi_{rA} \circ \xi_{rB} \leq \xi_{rA} \circ \chi_R^P \leq \xi_{rA}$, $\xi_{iA} \circ \xi_{iB} \leq \xi_{iA} \circ \chi_R^P \leq \xi_{iA}$, $\zeta_{rA} \circ \zeta_{rB} \geq \zeta_{rA} \circ \chi_R^N \geq \zeta_{rA}$, $\zeta_{iA} \circ \zeta_{iB} \geq \zeta_{iA} \circ \chi_R^N \geq \zeta_{iA}$.

Similarly, $\xi_{rA} \circ \xi_{rB} \leq \chi_R^P \circ \xi_{rB} \leq \xi_{rB}$, $\xi_{iA} \circ \xi_{iB} \leq \chi_R^P \circ \xi_{iB} \leq \xi_{iB}$, $\zeta_{rA} \circ \zeta_{rB} \geq \chi_R^N \circ \zeta_{rB} \geq \zeta_{rB}$ and $\zeta_{iA} \circ \zeta_{iB} \geq \chi_R^N \circ \zeta_{iB} \geq \zeta_{iB}$.

Hence, $\xi_{rA} \circ \xi_{rB} \leq \xi_{rA} \wedge \xi_{rB}$, $\xi_{iA} \circ \xi_{iB} \leq \xi_{iA} \wedge \xi_{iB}$ and $\zeta_{rA} \circ \zeta_{rB} \geq \zeta_{rA} \vee \zeta_{rB}$, $\zeta_{iA} \circ \zeta_{iB} \geq \zeta_{iA} \vee \zeta_{iB}$. $\square$

Theorem 4. *The bipolar fuzzy left ideal $A = (\xi_{rA} + i\xi_{iA}, \zeta_{rA} + i\zeta_{iA})$ and the bipolar fuzzy right ideal $B = (\xi_{rB} + i\xi_{iB}, \zeta_{rB} + i\zeta_{iB})$ of a semiring R is regular if $\xi_{rA} \wedge \xi_{rB} = \xi_{rA} \circ \xi_{rB}$, $\xi_{iA} \wedge \xi_{iB} = \xi_{iA} \circ \xi_{iB}$, $\zeta_{rA} \vee \zeta_{rB} = \zeta_{rA} \circ \zeta_{rB}$ and $\zeta_{iA} \vee \zeta_{iB} = \zeta_{iA} \circ \zeta_{iB}$.*

Proof. Since, $A = (\xi_{rA} + i\xi_{iA}, \zeta_{rA} + i\zeta_{iA})$ and $B = (\xi_{rB} + i\xi_{iB}, \zeta_{rB} + i\zeta_{iB})$ are two bipolar fuzzy left and right ideals of R , respectively. Let R be a regular semiring, then there exists $a \in R$ if $g \in R$ so that $g = gaxg$. Therefore, we have

$$\begin{aligned} \xi_{rA} \circ \xi_{rB}(g) &= \bigvee_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigwedge_{i=1}^n (\xi_{rA}(a_i) \wedge \xi_{rB}(b_i)) \right\} \\ &\geq (\xi_{rA}(ga) \wedge \xi_{rB}(g)) \geq (\xi_{rA}(g) \wedge \xi_{rB}(g)) = (\xi_{rA} \wedge \xi_{rB})(g). \end{aligned}$$

$$\begin{aligned} \xi_{iA} \circ \xi_{iB}(g) &= \bigvee_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigwedge_{i=1}^n (\xi_{iA}(a_i) \wedge \xi_{iB}(b_i)) \right\} \geq (\xi_{iA}(ga) \wedge \xi_{iB}(g)) \\ &\geq (\xi_{iA}(g) \wedge \xi_{iB}(g)) = (\xi_{iA} \wedge \xi_{iB})(g). \end{aligned}$$

$$\begin{aligned} \zeta_{rA} \circ \zeta_{rB}(g) &= \bigwedge_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigvee_{i=1}^n (\zeta_{rA}(a_i) \vee \zeta_{rB}(b_i)) \right\} \leq (\zeta_{rA}(ga) \vee \zeta_{rB}(g)) \\ &\leq (\zeta_{rA}(g) \vee \zeta_{rB}(g)) = (\zeta_{rA} \vee \zeta_{rB})(g) \text{ and} \\ \zeta_{iA} \circ \zeta_{iB}(g) &= \bigwedge_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigvee_{i=1}^n (\zeta_{iA}(a_i) \vee \zeta_{iB}(b_i)) \right\} \leq (\zeta_{iA}(ga) \vee \zeta_{iB}(g)) \\ &\leq (\zeta_{iA}(g) \vee \zeta_{iB}(g)) = (\zeta_{iA} \vee \zeta_{iB})(g) \end{aligned}$$

Now, from the Theorem 3, we have $\xi_{rA} \circ \xi_{rB} \leq \xi_{rA} \wedge \xi_{rB}$, $\xi_{iA} \circ \xi_{iB} \leq \xi_{iA} \wedge \xi_{iB}$ and $\zeta_{rA} \circ \zeta_{rB} \geq \zeta_{rA} \vee \zeta_{rB}$, $\zeta_{iA} \circ \zeta_{iB} \geq \zeta_{iA} \vee \zeta_{iB}$.

Hence, $\xi_{rA} \wedge \xi_{rB} = \xi_{rA} \circ \xi_{rB}$, $\xi_{iA} \wedge \xi_{iB} = \xi_{iA} \circ \xi_{iB}$, $\zeta_{rA} \vee \zeta_{rB} = \zeta_{rA} \circ \zeta_{rB}$ and $\zeta_{iA} \vee \zeta_{iB} = \zeta_{iA} \circ \zeta_{iB}$. $\square$

Theorem 5. *For a weakly regular semiring R , all the bipolar fuzzy complex right ideals of R are idempotent.*

Proof. Suppose $A = (\xi_{rA} + i\xi_{iA}, \zeta_{rA} + i\zeta_{iA})$ is a bipolar fuzzy right ideal of R and let $g \in R$. Then,

$$\begin{aligned} (\xi_{rA} \circ \xi_{rA})(g) &= \bigvee_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigwedge_{i=1}^n (\xi_{rA}(a_i) \wedge \xi_{rA}(b_i)) \right\} \leq \bigvee_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigwedge_{i=1}^n (\xi_{rA}(a_i)) \right\} \\ &\leq \bigvee_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigwedge_{i=1}^n (\xi_{rA}(a_i b_i)) \right\} \leq \bigvee_{g=\sum_{i=1}^n a_i b_i} \{ \xi_{rA}(\sum_{i=1}^n a_i b_i) \} = \xi_{rA}(g). \end{aligned}$$

$$(\xi_{iA} \circ \xi_{iA})(g) = \bigvee_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigwedge_{i=1}^n (\xi_{iA}(a_i) \wedge \xi_{iA}(b_i)) \right\} \leq \bigvee_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigwedge_{i=1}^n (\xi_{iA}(a_i)) \right\}$$

$$\leq \bigvee_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigwedge_{i=1}^n (\xi_{iA}(a_i b_i)) \right\} \leq \bigvee_{g=\sum_{i=1}^n a_i b_i} \{ \xi_{iA}(\sum_{i=1}^n a_i b_i) \} = \xi_{iA}(g).$$

$$\begin{aligned} \text{Also, } (\zeta_{rA} \circ \zeta_{rA})(g) &= \bigwedge_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigvee_{i=1}^n (\zeta_{rA}(a_i) \vee \zeta_{rA}(b_i)) \right\} \geq \bigwedge_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigvee_{i=1}^n (\zeta_{rA}(a_i)) \right\} \\ &\geq \bigwedge_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigvee_{i=1}^n (\zeta_{rA}(a_i b_i)) \right\} \geq \bigwedge_{g=\sum_{i=1}^n a_i b_i} \left\{ \zeta_{rA} \left(\sum_{i=1}^n a_i b_i \right) \right\} = \zeta_{rA}(g) \end{aligned}$$

and

$$\begin{aligned} \text{Also, } (\zeta_{iA} \circ \zeta_{iA})(g) &= \bigwedge_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigvee_{i=1}^n (\zeta_{iA}(a_i) \vee \zeta_{iA}(b_i)) \right\} \geq \bigwedge_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigvee_{i=1}^n (\zeta_{iA}(a_i)) \right\} \\ &\geq \bigwedge_{g=\sum_{i=1}^n a_i b_i} \left\{ \bigvee_{i=1}^n (\zeta_{iA}(a_i b_i)) \right\} \geq \bigwedge_{g=\sum_{i=1}^n a_i b_i} \{ \zeta_{iA}(\sum_{i=1}^n a_i b_i) \} = \zeta_{iA}(g). \end{aligned}$$

Now, since R is right weakly regular, we have $x \in (xR)^2$. This indicates that there exist $r_i, s_i \in R$ such that $g = \sum_{i=1}^n g r_i g s_i$. Thus, $\xi_{rA}(g) = \xi_{rA}(g) \wedge \xi_{rA}(g) \leq \xi_{rA}(g r_i) \wedge \xi_{rA}(g s_i)$, $\xi_{iA}(g) = \xi_{iA}(g) \wedge \xi_{iA}(g) \leq \xi_{iA}(g r_i) \wedge \xi_{iA}(g s_i)$ for $1 \leq i \leq n$. This indicates

$$\xi_{rA}(g) \leq \bigwedge_{i=1}^n (\xi_{rA}(g r_i) \wedge \xi_{rA}(g s_i)) \leq \bigvee_{g=\sum_{j=1}^m a_j b_j} \left\{ \bigwedge_{j=1}^m (\xi_{rA}(a_j) \wedge \xi_{rA}(b_j)) \right\} = (\xi_{rA} \circ \xi_{rA})(g).$$

$$\xi_{iA}(g) \leq \bigwedge_{i=1}^n (\xi_{iA}(g r_i) \wedge \xi_{iA}(g s_i)) \leq \bigvee_{g=\sum_{j=1}^m a_j b_j} \left\{ \bigwedge_{j=1}^m (\xi_{iA}(a_j) \wedge \xi_{iA}(b_j)) \right\} = (\xi_{iA} \circ \xi_{iA})(g)$$

and $\zeta_{rA}(g) = \zeta_{rA}(g) \vee \zeta_{rA}(g) \geq \zeta_{rA}(g r_i) \vee \zeta_{rA}(g s_i)$, $\zeta_{iA}(g) = \zeta_{iA}(g) \vee \zeta_{iA}(g) \geq \zeta_{iA}(g r_i) \vee \zeta_{iA}(g s_i)$, for $1 \leq i \leq n$. This indicates

$$\zeta_{rA}(g) \geq \bigvee_{i=1}^n (\zeta_{rA}(g r_i) \vee \zeta_{rA}(g s_i)) \geq \bigwedge_{g=\sum_{j=1}^m a_j b_j} \bigvee_{j=1}^m (\zeta_{rA}(a_j) \vee \zeta_{rA}(b_j)) = (\zeta_{rA} \circ \zeta_{rA})(g),$$

$$\zeta_{iA}(g) \geq \bigvee_{i=1}^n (\zeta_{iA}(g r_i) \vee \zeta_{iA}(g s_i)) \geq \bigwedge_{g=\sum_{j=1}^m a_j b_j} \bigvee_{j=1}^m (\zeta_{iA}(a_j) \vee \zeta_{iA}(b_j)) = (\zeta_{iA} \circ \zeta_{iA})(g).$$

Hence, $(\xi_{rA} \circ \xi_{rA})(g) = \xi_{rA}(g)$, $(\xi_{iA} \circ \xi_{iA})(g) = \xi_{iA}(g)$, $(\zeta_{rA} \circ \zeta_{rA})(g) = \zeta_{rA}(g)$ and $(\zeta_{iA} \circ \zeta_{iA})(g) = \zeta_{iA}(g)$. Thus, $A \circ A = A$. Therefore, A is idempotent. $\square$

3. Conclusion:

In this paper, we explore the concept of bipolar fuzzy complex set and how they relate to semiring structure. A thorough review and analysis of bipolar fuzzy complex ideals in semirings are presented in this study. We examine several outcomes of bipolar fuzzy complex ideals and subsemirings of a semiring and construct a number of significant outcomes pertaining to addition and multiplication algebraic operations in this extended fuzzy framework.

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