

INFLUENCE OF SELF GRAVITY ON THE MOTION OF IONIZING SHOCK FRONT IN A NON-IDEAL GAS

S. Talukdar^{1*} and B. Nath²

¹*Dept. of Mathematics, School of Fundamental and Applied Sciences, Assam Don Bosco University,
Guwahati-782402*

²*Dept. of Mathematics, Sanskriti The Gurukul, Guwahati, Assam, India*

*For correspondence. (talukdarsujit15@gmail.com)

Abstract: The influence of self gravity on ionizing shock front motion in a non-ideal gas in presence of a spatially decreasing magnetic field is examined. The medium ahead of the shock is considered to possess a negligible electrical conductivity and it is assumed that the initial density of the gas obeys a power law. The parametric variations of the variables are observed in the area in the wake of the shock front along with the similarity solutions are also derived and the influences of governing parameters are examined systematically.

Keywords: non-ideal gas, self-gravitational effects, self-similar flow, shock front.

1. Introduction:

Shock waves often arise in the interstellar medium due to the presence of diverse supersonic flows and high energy astrophysical events, such as intense mass loss from massive stars during their late evolutionary stages, super nova explosions, etc. With the leading equations of motion and equilibrium in view of gravitational forces the qualitative behavior of the gaseous mass may be investigated. Numerous studies have focused on homo-thermal flow fields generated behind a spherical shock wave in self-gravitating gases. Shock waves moving through variable density have likewise attracted considerable attention from many authors [1-7] where the gas was modeled as perfect gas. However, at high temperature, the assumption for ideal gas is no longer valid. Therefore, van der Waals model in simplified form is considered an alternative to the ideal gas. Considering the electrical conductivity medium to be infinite several notable researchers [8-10] have investigated self similar flow behind a strong shock determined by a cylindrical piston moving in a non ideal gas medium obeying simplified van der Waals equation of state. But due to the passage of a shock, in many practical cases the medium may becomes highly conductive. Such a shock is called an ionizing shock. Several works has been done in the propagation of an ionizing shock in perfect or non-ideal gas[11-17]. In recent times Nath and Singh [18] have made the analytical study on cylindrical shock wave in rotational axisymmetric non-ideal gas. Chauhan [19] and Singh also have studied magnetogasdynamic shock wave propagation in a rotational axisymmetric self-gravitating non-ideal gas.

In this present study, self similar solution for the flow produced in the rear of an ionizing shock wave travelling through a self gravitating non ideal gas influenced by a magnetic field have been investigated.

2. Basic formulation and boundary constraints:

Under the influence of magnetic field with heat conduction and neglecting viscous stress the following equations govern the motion of the unsteady, adiabatic, spherically symmetric flow of perfectly conducting and self-gravitating gas

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{2\rho u}{r} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \left(\frac{\partial p}{\partial r} + \mu h \frac{\partial h}{\partial r} + \frac{\mu h^2}{r} \right) + \frac{Gm}{r^2} = 0 \quad (2)$$

$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial r} + h \frac{\partial u}{\partial r} + \frac{hu}{r} = 0 \quad (3)$$

$$\frac{\partial m}{\partial r} - 4\pi \rho r^2 = 0 \quad (4)$$

$$\frac{\partial e}{\partial t} + u \frac{\partial e}{\partial r} - \frac{p}{\rho^2} \left[\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right] = 0 \quad (5)$$

A simplified van der Waals equation of state is assumed as follows

$$p = \frac{\Gamma \rho T}{1 - b\rho}, \quad e = C_v T = \frac{p(1 - b\rho)}{\rho(\gamma - 1)} \quad (6)$$

The following assumptions are relevant to the current investigation

- (i) Low conducting medium is considered which due to passage of a strong shock becomes highly conducting.
- (ii) The initial gas density and the initial magnetic field are considered to vary as some power-law variations with respect to distance.
- (iii) It is assumed that, as a result of the pressure applied by a piston, the entire energy of the flow field behind the shock accelerated with time.
- (iv) The gas is unspoken to be stationary ahead of the shock.
- (v) A spherical shock is supposed to be moving through an undisturbed non-ideal gas with non-uniform density, $\rho = Ar^{-\omega}$, $0 \leq \omega < 3$, where A and ω are constants.
- (vi) The initial magnetic field in the medium is considered to vary as $h = Br^{-k}$, $0 < k < 1.5$, where B and k are constants.
- (vii) Due to the passage of the shock, the gas is highly ionized and the electrical conductivity is infinitely large.

The flow parameters in the region directly ahead of the shock front are:

$$u_1 = 0, \quad \rho_1 = AR^{-\omega}, \quad h_1 = BR^{-k}$$

$$p_1 = \left[\frac{2\pi G A^2}{(3-\omega)(\omega-1)} + \left(\frac{2-\omega}{\omega-1} \right) \frac{\mu B^2}{2} \right] R^{-2k}, \quad (k = \omega-1) \quad (7)$$

$$m_1 = \frac{4\pi A}{3-\omega} R^{3-\omega}$$

At gas-ionizing shock wave, the jump conditions are given by:

$$\left. \begin{aligned} \rho_1 V &= \rho_2 (V - u_2) = m s_1 \\ p_2 - p_1 &= m_s u_2 \\ e_1 + \frac{p_1}{\rho_1} + \frac{1}{2} V^2 &= e_2 + \frac{p_2}{\rho_2} + \frac{1}{2} (V - u_2)^2 \\ h_1 &= h_2, \quad m_1 = m_2 \end{aligned} \right\} \quad (8)$$

(The conditions immediately ahead and behind the shock front are implied by the subscripts '1' and '2' respectively)

From equation (8), we get

$$u_2 = (1-\beta)V, \quad \rho_2 = \frac{\rho_1}{\beta}, \quad p_2 = \left[\frac{1}{\gamma M^2} + (1-\beta) \right] \rho_1 V^2, \quad m_2 = \frac{4\pi A}{3-\omega} R^{3-\omega}, \quad h_2 = h_1, \quad (9)$$

The quantity β ($0 < \beta < 1$) is obtained by the relation

$$\beta = \frac{(\gamma-1) + 2\bar{b} + 2M^{-2}}{\gamma+1}. \quad (10)$$

In the flow field behind the shock the total energy is specified by

$$E = E_0 t^s, \quad s \geq 0 \quad (11)$$

where E_0 and s are constants.

2.1. self-similar transformations:

We define the following characteristic parameters as

$$[a] = [A] \quad (12)$$

$$[d] = \left[\frac{E_0}{A} \right] \quad (13)$$

The non-dimensional independent variable η in this is given by

$$\eta = \left[\frac{v E_0}{A} \right]^{1/(s-\omega)} r t^{-\delta} \quad (14)$$

$$\text{where } \delta = \frac{2+s}{5-\omega}$$

(15)

In the explicit form the shock propagation law is as follows:

$$R = \left[\frac{vE_0}{A} \right]^{\frac{1}{(5-\omega)}} t^\delta \quad (16)$$

$$\text{Hence, we have } \frac{V}{V_0} = \left[\frac{R}{R_0} \right]^{\frac{(\delta-1)}{\delta}} \quad (17)$$

The fluid variables are expressed as:

$$\left. \begin{aligned} u &= VU(\eta), \\ \rho &= \rho_1 D(\eta), \\ p &= V^2 \rho_1 P(\eta), \\ m &= \rho_1 R^3 N(\eta), \\ \sqrt{\mu} h_1 &= \sqrt{\rho_1} V H(\eta) \end{aligned} \right\} \quad (18)$$

The shock-Mach number and the Alfven Mach number ought to be constants for the existence of similarity solutions, hence

$$\delta = \frac{2}{\omega} \quad \text{and} \quad \omega = \frac{10}{4+s} \quad (19)$$

where, $1 < \omega < 2$ ($1 < s < 6$) or $2 < \omega < 2.5$ ($0 < s < 1$).

The equations of motion from (1) to (5) can be transformed into a system of ordinary differential equations.

$$(U-\eta) \frac{dD}{d\eta} + D \frac{dU}{d\eta} + \left(\frac{2U}{\eta} - \omega \right) D = 0 \quad (20)$$

$$(U-\eta) \frac{dU}{d\eta} + \left(\frac{\delta-1}{\delta} \right) U + \frac{1}{D} \frac{dP}{d\eta} + \frac{H}{D} \left(\frac{dH}{d\eta} + \frac{H}{\eta} \right) + \frac{G_0 N}{\eta^2} = 0 \quad (21)$$

$$(U-\eta) \frac{dH}{d\eta} + \left(\frac{\delta-1}{\delta} - \frac{\omega}{2} \right) H + H \frac{dU}{d\eta} + \frac{HU}{\eta} = 0 \quad (22)$$

$$\frac{dN}{d\eta} = 4\pi D \eta^2 \quad (23)$$

$$(1-\bar{b}D)(U-\eta) \frac{dP}{d\eta} - (U-\eta) \frac{\gamma P}{D} \frac{dD}{d\eta} + 2 \left(\frac{\delta-1}{\delta} \right) (1-\bar{b}D)P + (\gamma-1+\bar{b}D)\omega P = 0 \quad (24)$$

where,

$$G_0 = \frac{(3-\omega)(\omega-1)}{2\pi} \left[\frac{1}{\gamma M^2} + \frac{\omega-2}{2(\omega-1)} M_A^{-2} \right] \quad (\text{The gravitational parameter}) \quad (25)$$

Equations (20)-(24), yield

$$\frac{dD}{d\eta} = -\left(\frac{D}{U-\eta}\right)\left[\frac{dU}{d\eta} + \frac{2U}{\eta} - \omega\right] \quad (26)$$

$$\frac{dH}{d\eta} = -\frac{H}{U-\eta}\left[\frac{dU}{d\eta} + \frac{U}{\eta} + \frac{\delta-1}{\delta} - \frac{\omega}{2}\right] \quad (27)$$

$$\frac{dP}{d\eta} = -D\left[\left\{(U-\eta) - \frac{H^2}{D(U-\eta)}\right\}\frac{dU}{d\eta} + \left(\frac{\delta-1}{\delta}\right)U - \frac{H^2}{D(U-\eta)}\left(\frac{U}{\eta} + \frac{\delta-1}{\delta} - \frac{\omega}{2}\right) + \frac{H^2}{D\eta} + \frac{G_0 N}{\eta^2}\right] \quad (28)$$

$$\frac{dU}{d\eta} = \left[\frac{(1-\bar{b}D)D(U-\eta)}{H^2(1-\bar{b}D) + \gamma P - D(1-\bar{b}D)(U-\eta)^2}\right] \times \left[\left(\frac{\delta-1}{\delta}\right)U - \frac{H^2}{D(U-\eta)}\left(\frac{U}{\eta} + \frac{\delta-1}{\delta} - \frac{\omega}{2}\right) + \frac{H^2}{D\eta} + \frac{G_0 N}{\eta^2}\right] - \left[\frac{2\gamma PU}{\eta D(1-\bar{b}D)(U-\eta)} - \left\{2\left(\frac{\delta-1}{\delta}\right) - \omega\right\}\frac{P}{D(U-\eta)}\right] \quad (29)$$

$$\frac{dN}{d\eta} = 4\pi D\eta^2 \quad (30)$$

Using the transformations (18), the shock conditions (9) are reduces to

$$U(1) = (1-\beta), \quad D(1) = \frac{1}{\beta}, \quad H(1) = M_A^{-1}, \quad P(1) = \left[\frac{1}{\gamma M^2} + (1-\beta)\right], \quad N(1) = \frac{4\pi}{(3-\omega)} \quad (31)$$

Further the inner expanding surface obligatory to satisfy the following kinematic condition:

$$U(\eta_p) = \eta_p \quad (32)$$

2.2 Results and discussion:

In order to get physical insights into the problem the distributions of the variables in the flow-field behind the shock front are yield by integrating the equations (26)-(30) numerically, employing the fourth order Runge-Kutta method. The values of the parameters involved are chosen for implementing the numerical integration as such:

$\gamma = \frac{5}{3}$; $M = 5, 10, 15$; $M_A^{-2} = 0.005, 0.01$; $\bar{b} = 0, 0.05, 0.1$; $\omega = 1.1, 1.2, 1.3$, $\bar{b} = 0$ (corresponds to the ideal

gas). The exponent in the shock propagation law is given by $\delta = \frac{\omega}{2}$ and the exponent in the law of variation of

the total energy behind the shock is specified by $s = \frac{10-4\omega}{\omega}$, where ω tends for the exponent of law of variation

of the initial density

The density ratio and the position of the inner expanding surface in different cases are tabulated.

Due to the rise in the non-idealness parameter, the impact of can be summarized as:

- To increase the value of β , i.e. to decrease the shock strength.
- To increase the gap between the inner expanding surfaces from the shock front, this indicates that the gas behind the shock is less condensed.

Because of the rise in the value of exponent of the density variation the impact is

- (i) To increase the gap the inner expanding surface from the shock front.

Table 1: Tabulated values of the density ratio β and position of the inner expanding surface η_p for diverse

values of ω and $\bar{b}$ and $\gamma = \frac{5}{3}$, $M^2 = 5$, $M_A^{-2} = 0.01$.

ω	$\bar{b}$	β	η_p
1.1	0	0.4000	0.857427
	0.05	0.4375	0.841868
	0.1	0.4750	0.825170
1.2	0	0.4000	0.849694
	0.05	0.4375	0.835003
	0.1	0.4750	0.819260
1.3	0	0.4000	0.841013
	0.05	0.4375	0.827580
	0.1	0.4750	0.813081

Fig 1: variant of non dimensional flow variables for different values of (a) velocity, (b) density, (c) pressure, (d)

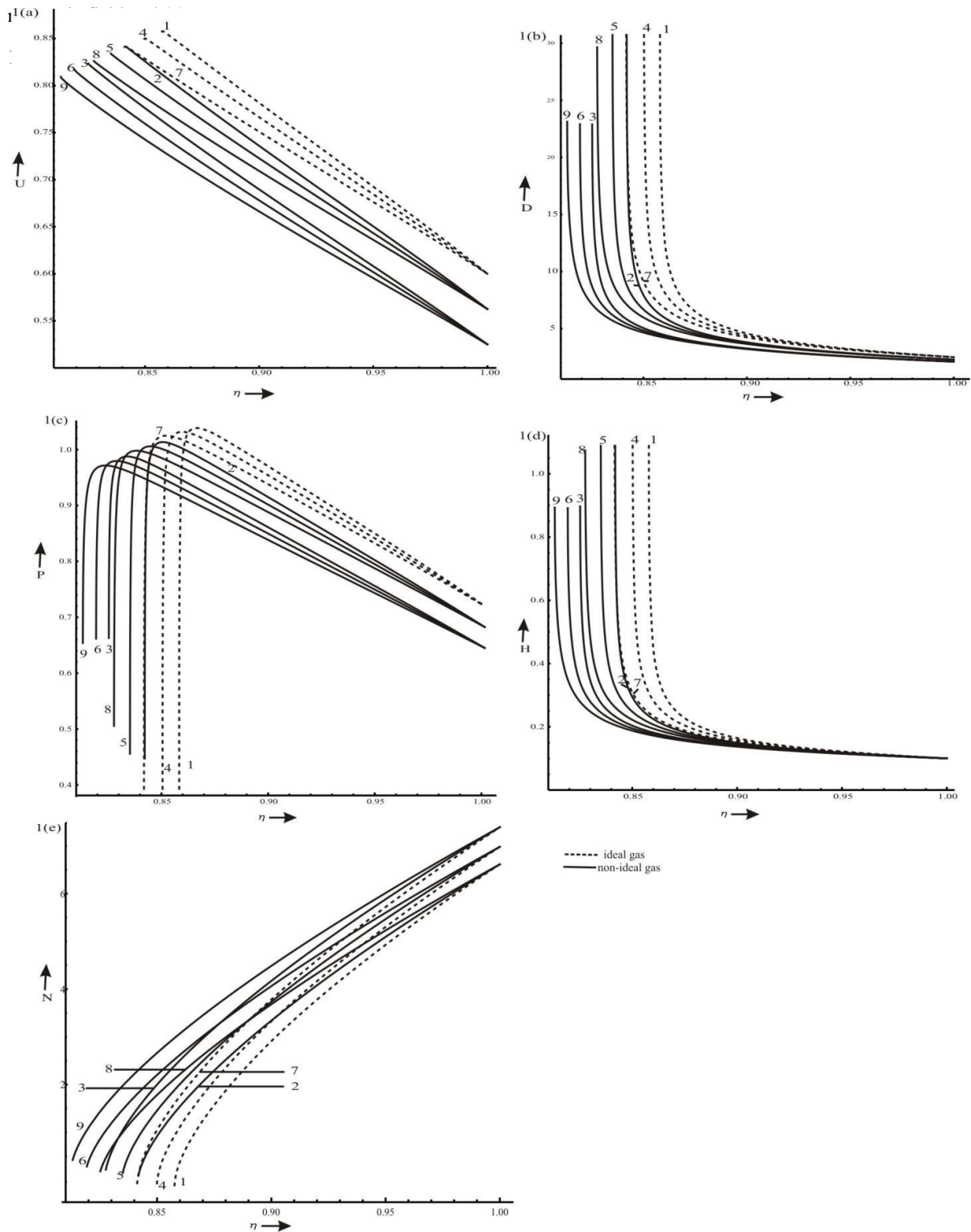


Fig 1(a), 1(b), 1(d) indicates that the non-dimensional velocity U , density D and mass N increases beside the path of the shock front towards the inner expanding surface. Also fig 1(c) shows that the non-dimensional pressure P

increases attains a maximum and then decreases rapidly near the inner surface. Fig 1(e) demonstrates that the non-dimensional mass decreases as we proceed towards the inner expanding surface.

Due to the rise in the value of the non-idealness parameter that impacts:

- (i) To retard the non-dimensional velocity U , density D , magnetic field H and pressure P .
- (ii) To increase the non-dimensional mass N .

The impact of rise in the value of exponent of the density variation ω is

- (i) To retard the velocity, density, magnetic field, pressure and to boost the mass at any point in the flow-field behind the shock.

Nomenclature:

r	independent spatial coordinate,
t	independent time variable,
u	velocity of the fluid,
ρ	density,
P	pressure,
H	magnetic field in the azimuthal direction,
e	inner energy per unit mass,
μ	magnetic permeability,
M	mass inside the sphere,
G	gravitational constant.
Γ	gas constant,
$C_v T = \frac{\Gamma}{\gamma - 1}$	specific heat at constant volume,
γ	proportion of specific heats,
b	van der Waals excluded volume,
R	shock radius,
V_0	velocity at the instant it is produced,
R_0	radius of the shock at the instant it is produced,
$M = \left(\frac{\rho_1 V^2}{\gamma P_1} \right)^{1/2}$	shock Mach number referred to frozen speed of sound
$M_A = \left(\frac{\rho_1 V^2}{\mu h_1^2} \right)^{1/2}$	Alfven Mach number,
η_p	value of η at the inner expanding surface.

Acknowledgement: The authors acknowledge Assam Don Bosco University for providing the necessary requirements.

Conflict of Interests

The authors declare no conflict of interest.

References:

- [1] S. C. Purohit, Self-similar homothermal flow of self-gravitating gas behind shock wave. J. Phys. Society, 36, 288-292, 1974.
- [2] J. B. Singh, Self-similar ionizing shock waves with radiation heat flux, Indian Journal of Technology, 21, 315-318, 1983.
- [3] A. Sakurai Propagation of spherical waves in stars. J. Fluid Mech., 14, 436-453, 1956.
- [4] M. H. Rogers, Analytic solutions for the blast-waves problem with an atmosphere of varying density, Astrophys. J., 125, 478-493, 1957.
- [5] P. Rosenau and S. Frankenthal, Equatorial propagation of axisymmetric magnetohydrodynamic shocks, Phys. Fluids, 19, 1889-1899, 1976.
- [6] P. H. Roberts and C. C. Wu, Structure and stability of a spherical implosion, Phys. Lett. A, 213, 50-64, 1996.
- [7] P. H. Roberts and C. C. Wu, The shock wave theory of sonoluminescence. In Shock focussing effects in medical science and sonoluminescence, Edited by R.C. Srivastava, D. Leutloff, K. Takayama and H. Groning, Springer-Verlag, Heidelberg, 2003.
- [8] J. P. Vishwakarma, A. K. Maurya and K. K. Singh, Self-similar adiabatic flow headed by a magnetogasdynamics cylindrical shock wave in a rotating non-ideal gas, Geo. Astro. Fluid Dyn., 101(2), 155-168, 2007.
- [9] H. P. Greenspan, Similarity solution for a cylindrical shock magnetic field interaction, Phys. Fluids, 5, 255-259, 1962.
- [10] C. Greinfinger and J. D. Cole, Similarity solution for cylindrical magnetohydrodynamic blast waves, Phys. Fluids 5, 1597-1607, 1962.
- [11] A. H. Christer, Self-similar cylindrical ionizing shock and detonation waves, Z Angew Math. Mech. 52, 11-22, 1972.
- [12] M. P. Rangarao and B. V. Ramana, Self-similar cylindrical magnetogasdynamics and ionizing shock waves. Indian Int. J Engng. Sci, 11, 337-351, 1973.
- [13] J. B. Singh and P. R. Vishwakarma, Self-similar solutions in the theory of flare-ups in novae I, Astrophys. Space Sci., 95, 94-104, 1983.
- [14] J. P. Vishwakarma and M. Singh, Ionizing cylindrical shock waves in a rotating homogeneous non-ideal gas. Int. J. Mech. Appl. 2(2), 14-23, 2012.
- [15] K. K. Singh and B. Nath, Self-similar cylindrical ionizing shock wave in a rotating axisymmetric gas flow. Indian J. Sci. Tech., 2013.
- [16] G. Nath and S. Singh, Approximate analytical solution for ionizing cylindrical shock wave in rotational axisymmetric non-ideal gas: Isothermal flow, Can. J. Phys., 98(11), 1077-1089, 2020.

- [17] S. Chauhan and D. Singh, An analysis of magnetogasdynamic shock wave propagation in a rotational axisymmetric self-gravitating nonideal gas, Eur. Phys. J. Plus, 139(10), 941, 2024.