SYMMETRY REALIZATION UNDER (5+4) SCHEME IN THE CONTEXT OF MINIMAL EXTENDED SEESAW MECHANISM

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Abstract: This work includes the Abelian group symmetry realization of two-zero textures of neutrino mass matrices ($M_\nu^{4\times4}$) under Minimal Extended Seesaw (MES) Mechanism. MES is an extension of type-I seesaw mechanism which incorporates an additional scalar singlet field 'S' apart from the three right-handed neutrinos. MES mechanism deals with 3×3 form of Dirac neutrino mass matrix M_D and right-handed Majorana neutrino mass matrix M_R, along with 1×3 form of M_S which couples the right-handed neutrinos and the singlet scalar 'S'. In this work, we present the Z_9 cyclic group symmetry realization of those textures of M_D, M_R and M_S which are viable in realizing the two-zero textures of $M_\nu^{4\times4}$ under MES mechanism. In doing so, the Standard Model is extended to include few $SU(2)_L$ scalar doublets to realize the zero textures of M_l, M_D, scalar singlet $'\chi'$ to realize M_R and few scalar singlets $\lambda's$ to realize M_S.

Keywords: keyword1; keyword2; keyword3; keyword4

1. Introduction:

Apart from the three flavours of neutrino, experimental observations from various neutrino oscillation experiments around the globe have hinted towards the presence of an additional flavour of neutrino. For instance, the Liquid Scintillation Neutrino Detector (LSND) [1,2] at Los Alamos was designed to search the $\underline{\nu}_\mu \rightarrow \underline{\nu}_e$ oscillation using high intensity proton beam to produce pions which decays to μ^+ and then finally to $\underline{\nu}_\mu$. With the protons present in the detector, LSND experimental was set up to search for $\underline{\nu}_e$, which if produced will undergo the reaction $\underline{\nu}_e + p \rightarrow e^+ + n$. The e^+ produced in the detector produces Cherenkov radiation with energy $20 < E_e < 60$ MeV and a 2.2 MeV γ produced from neutron capture on a free proton. The final LSND oscillation results shows a clear excess of events with oscillation probability between $\underline{\nu}_\mu$ and $\underline{\nu}_e$ which corresponds to mass squared difference of $\Delta m^2 \sim eV^2$. This was in contradiction with the solar ($\Delta m^2_{sol} \sim 10^{-5} eV^2$) and atmospheric ($\Delta m^2_{atm} \sim 10^{-3} eV^2$) squared mass differences in case of three active neutrino scenario. This is known as LSND anomaly which hints towards the presence of an additional neutrino state with mass of the order of eV scale. Similar anomalous results were also reported by a number of oscillation experiments like MiniBooNE [3], Gallium solar neutrino experiment [4,5] and reactor experiments [6]. However recent experimental results have put strict constraints on the mixing parameters of sterile neutrino and the worlds's most stringent limit [7] has been obtained in the region of $2\times10^{-4}eV^2 \lesssim |\Delta m^2_{41}| \lesssim 0.1eV^2$. Incorporating such intriguing experimental results within the theoretical framework stands to be one of the major challenge. From the theoretical front such incorporation of an eV scale sterile neutrino and its admixture with the three active neutrinos has been studied in the Minimal Extended Seesaw (MES) mechanism [8]. MES is an extension of the canonical-type I seesaw mechanism wherein the Standard Model is extended with an additional gauge singlet field 'S' apart from the three right-handed neutrinos. The neutrino mass matrix in its 4×4 form under MES, involves Dirac neutrino mass matrix (M_D) and right-handed Majorana neutrino mass matrix (M_R) in its 3×3 form while 1×3 form of M_S which couples the right-handed neutrinos and the additional singlet 'S'. In this work we shall be considering those forms of zero textures of M_D, M_R and M_S which can lead to the desired two-zero textures of $M_\nu^{4\times4}$ in the context of MES mechanism. Texture zero models are one of the successful theoretical models which can explain the mixings in both the sectors-quarks and leptons. Zeros in the mass matrix elements is the simplest and transparent way of inducing relations among the physical quantities (masses, mixing angles and CP phases) of the mass matrix and thereby reducing the number of free parameters [9]. Zeros in the neutrino mass matrix can also be imposed considering type-I seesaw mechanism wherein the zeros of $M_D\ and\ M_R$ propagates as zeros of m_ν[10,11,12]. In the context of three neutrino scenario, zero textures of m_ν have also been investigated in the framework of inverse seesaw mechanism [13-17]. In the context of MES mechanism, zeros of $M_\nu^{4\times4}$ can be obtained by considering zero textures of $M_D^{3\times3}$, $M_R^{3\times3}$ and $M_S^{1\times3}$. There are 15 possible two-zero textures in case of three neutrino paradigm, out of which only 7 are experimentally viable [18]. However, in (3+1) scheme, that is, three active and one light sterile neutrino, all the 15 two-zero textures are experimentally allowed [19]. In (3+1) picture, texture zero models have been investigated in a number of papers [20-23]. In this work we shall be relooking into the (5+4) scheme, that is 5 zeros in M_D and 4 zeros in M_R and one-zero texture of M_S which finally propagates as zeros in $M_\nu^{4\times4}$ under MES. Under (5+4) scheme, out of 15 only 9 two-zero textures can be realized [24].

For a mass matrix with zeros in any arbitrary entries, it is always possible to find suitable Abelian symmetry group with an extended scalar sector such that the texture zero originates from these symmetries [25,26]. In this present work, the prospective textures of M_D, M_R and M_S which generates the desired zero texture of $M_\nu^{4\times4}$ are realized using Z_9 Abelian cyclic group symmetry. For this realization the particle content of the Standard Model is extended with few scalar singlets and doublets.

2. Minimal Extended Seesaw (MES) Mechanism

In MES model an additional scalar singlet 'S' is introduced apart from the SM particle content and three right-handed neutrinos [8]. The Lagrangian in this case takes the form

$$L = \underline{\nu_L} M_D \nu_R + \underline{S}^C M_S \nu_R + (1/2)\, \underline{\nu}^C M_R \nu_R + h.c \qquad (1)$$

Here M_S is a (1×3) row matrix as only one extra singlet 'S' is considered. For $M_R > M_S > M_D$ the heavy RH ν decouple and in the basis (ν_L, S^c) and the neutrino mass matrix takes the form

$$M_\nu^{4\times4} = (M_D M_R^{-1} M_D^T \; M_D M_R^{-1} M_S^T \; M_S (M_R^{-1})^T M_D^T \; M_S M_R^{-1} M_S^T\,) \qquad (2)$$

3. Two-zero textures of $M_\nu^{4\times4}$

The 9 two-zero textures [24] which can be realized under (5+4) scheme are listed below:

$A_1 = (0\,0\,X\,X\,0\,X\,X\,X\,X\,X\,X\,X\,X\,X\,X\,X\,)$, $A_2 = (0\,X\,0\,X\,X\,X\,X\,X\,0\,X\,X\,X\,X\,X\,X\,X\,)$,

$B_3 = (X\,0\,X\,X\,0\,0\,X\,X\,X\,X\,X\,X\,X\,X\,X\,X\,)$

$B_4 = (X\,X\,0\,X\,X\,X\,X\,X\,0\,X\,0\,X\,X\,X\,X\,X\,)$, $C = (X\,X\,X\,X\,X\,0\,X\,X\,X\,X\,0\,X\,X\,X\,X\,X\,)$,

$D_1 = (X\,X\,X\,X\,X\,0\,0\,X\,X\,0\,X\,X\,X\,X\,X\,X\,)$

$D_2 = (X\,X\,X\,X\,X\,X\,0\,X\,X\,0\,0\,X\,X\,X\,X\,X\,)$, $E_1 = (0\,X\,X\,X\,X\,0\,X\,X\,X\,X\,X\,X\,X\,X\,X\,X\,)$,

$E_2 = (0\,X\,X\,X\,X\,X\,X\,X\,X\,X\,0\,X\,X\,X\,X\,X\,)$

Here 'X' represents the non-zero entries.

4. (5+4) scheme:

The general (3×3) form of a non-symmetric Dirac neutrino mass matrix M_D and symmetric right-handed Majorana neutrino mass matrix M_R and 1×3 form of M_S are:

$$M_D = (a\,b\,c\,d\,e\,f\;g\,h\,i\,)\,, \qquad M_R = (A\;B\;C\;B\;D\;E\;C\;E\;F\,)\,, \qquad M_S = (s_1\,s_2\,s_3\,) \qquad (3)$$

Under (5+4) scheme, that is 5 zeros in M_D and 4 zeros in M_R, there are 126 possible textures of M_D and 15 possible textures of M_R. Out of 15, only 3 textures of M_R are non-singular which is a requirement of MES mechanism. Out of 126, only 54 textures of M_D along with the 3 four-zero textures and one/two-zero texture of M_S are useful [24] in realizing the two-zero textures of $M_\nu^{4\times4}$ mentioned in section 3. In this work, we shall consider only one-zero texture of M_S for the study.

4.1 Textures under study for symmetry realization:

A number of combinations of M_D , M_R , M_S which lead to the desired two-zero texture of $M_\nu^{4\times4}$ has already been mentioned in Ref. [24]. Out of which, in this work, we shall consider the following textures for symmetry realization under Z_9 cyclic symmetry group.

Table 1: Table showing zero-textures of M_D , M_R , M_S required for MES realization of two-zero textures of $M_\nu^{4\times4}$

Texture	M_D	M_R	M_S
A_1	$(0\ b\ 0\ d\ 0\ 0\ g\ 0\ i\ )$	$(A\ 0\ 0\ 0\ 0\ E\ 0\ E\ 0\ )$	$(s_1\ 0\ s_3\ )$
A_2	$(0\ b\ 0\ d\ 0\ f\ g\ 0\ 0\ )$	$(A\ 0\ 0\ 0\ 0\ E\ 0\ E\ 0\ )$	$(s_1\ 0\ s_3\ )$
B_3	$(a\ 0\ 0\ 0\ e\ 0\ g\ 0\ i\ )$	$(A\ 0\ 0\ 0\ 0\ E\ 0\ E\ 0\ )$	$(s_1\ 0\ s_3\ )$
B_4	$(a\ 0\ 0\ d\ e\ 0\ 0\ 0\ i\ )$	$(A\ 0\ 0\ 0\ 0\ E\ 0\ E\ 0\ )$	$(s_1\ s_2\ 0\ )$
C	$(0\ b\ c\ 0\ e\ 0\ 0\ 0\ i\ )$	$(A\ 0\ 0\ 0\ 0\ E\ 0\ E\ 0\ )$	$(0\ s_2\ s_3\ )$
D_1	$(a\ b\ 0\ 0\ 0\ f\ g\ 0\ 0\ )$	$(A\ 0\ 0\ 0\ 0\ E\ 0\ E\ 0\ )$	$(s_1\ s_2\ 0\ )$
D_2	$(a\ b\ 0\ d\ 0\ 0\ 0\ 0\ i\ )$	$(A\ 0\ 0\ 0\ 0\ E\ 0\ E\ 0\ )$	$(s_1\ s_2\ 0\ )$
E_1	$(0\ b\ 0\ 0\ 0\ f\ 0\ h\ i\ )$	$(A\ 0\ 0\ 0\ 0\ E\ 0\ E\ 0\ )$	$(0\ s_2\ s_3\ )$
E_2	$(0\ b\ 0\ 0\ 0\ f\ 0\ h\ i\ )$	$(A\ 0\ 0\ 0\ 0\ E\ 0\ E\ 0\ )$	$0\quad (s_2\ s_3\ )$

5. Symmetry Realization

For symmetry realization of the textures under MES mechanism we consider the Z_9 symmetry group which consists of the elements: $(1, \omega, \omega^2, \omega^3, \omega^4, \omega^5, \omega^6, \omega^7, \omega^8)$ with $\omega = e^{i2\pi/9}$ being the generator of the group. Throughout the symmetry realization, we shall consider the charge lepton mass matrix M_l to be diagonal. The symmetry realization of texture A_1 is shown below:

The transformations of the leptonic fields under Z_9 are considered as:

$$\begin{aligned} &\underline{D}_{eL} \to \omega^5 \underline{D}_{eL}, && e_R \to \omega e_R, && \nu_{eR} \to \omega^2 \nu_{eR} \\ &\underline{D}_{\mu L} \to \omega^2 \underline{D}_{\mu L}, && \mu_R \to \omega^3 \mu_R, && \nu_{\mu R} \to \omega^4 \nu_{\mu R} \\ &\underline{D}_{\tau L} \to \omega^7 \underline{D}_{\tau L}, && \tau_R \to \omega^2 \tau_R, && \nu_{\tau R} \to \omega^5 \nu_{\tau R} \end{aligned} \quad (4)$$

where $\underline{D}_{jL}, l_R$ and ν_{kR} represents the $SU(2)_L$ doublets, right-handed $SU(2)_L$ singlets and the right-handed neutrinos respectively. Under the above transformation, the bilinears $\underline{D}_{jL} l_R$, $\underline{D}_{jL} \nu_{kR}$, $\nu_{kR}^T C^{-1} \nu_{jR}$ which represents M_l, M_D and M_R takes the following form:

$$M_l = (\omega^6\ \omega^8\ \omega^7\ \omega^3\ \omega^5\ \omega^4\ \omega^8\ \omega\ 1\ ), \quad M_D = (\omega^7\ 1\ \omega\ \omega^4\ \omega^6\ \omega^7\ 1\ \omega^2\ \omega^3\ ),$$
$$M_R = (\omega^4\ \omega^6\ \omega^7\ \omega^6\ \omega^8\ 1\ \omega^7\ 1\ \omega^2\ ), \quad (5)$$

To obtain diagonal form of M_l along with the desired form of M_D, we consider three $SU(2)_L$ doublet Higgs (Φ_1, Φ_2, Φ_3) which transform under Z_9 as:

$$\Phi_1 \to \Phi_1, \qquad \Phi_2 \to \omega^6 \Phi_2, \qquad \Phi_3 \to \omega^5 \Phi_3 \quad (6)$$

The Z_9 invariant Yukawa Lagrangian becomes:

$$-L = Y_{11}^l \underline{D}_{eL}\, \tilde{\Phi}_2 e_R + Y_{22}^l \underline{D}_{eL} \tilde{\Phi}_3 \mu_R + Y_{33}^l \underline{D}_{\tau L} \tilde{\Phi}_1 \tau_R + Y_{12}^D \underline{D}_{eL} \Phi_1 \nu_{\mu R} + Y_{21}^D \underline{D}_{\mu L} \Phi_3 \nu_{eR} + Y_{31}^D \underline{D}_{\tau L} \Phi_1 \nu_{eR} + Y_{33}^D \underline{D}_{\tau L} \Phi_2 \nu_{\tau R} + h.c. \quad (7)$$

The Higg's field (Φ_1, Φ_2, Φ_3) after acquiring vacuum expectation value leads to the following form of M_l and M_D:

$$M_l = (m_e\ 0\ 0\ 0\ m_\mu\ 0\ 0\ 0\ m_\tau\ ), \qquad M_D = (0\ b\ 0\ d\ 0\ 0\ g\ 0\ i\ ) \quad (8)$$

To realize M_R an additional scalar singlet χ is istroduced which transforms as:

$$\chi \to \omega^5 \chi \quad (9)$$

Which leads to the following form of M_R

$$M_R = (A\ 0\ 0\ 0\ 0\ E\ 0\ E\ 0\ ) \qquad (10)$$

The singlet field 'S' must also be given a transformation under Z_9 , otherwise bare mass term of the form $\underline{S}^C S$ will appear. This must be prevented according to MES mechanism. Thus, the following transformation of 'S' is considered-

$$S \to \omega S \qquad (11)$$

To arrive at the desired form of M_S, two additional scalar singlets are considered which transform under Z_9 as

$$\lambda_1 \to \omega^6 \lambda_1, \qquad \lambda_2 \to \omega^3 \lambda_2 \qquad (12)$$

This leads to the following form of M_S

$$M_S = (s_1\ 0\ s_3\ ) \qquad (13)$$

From Eq. (8), (10), (13) it can be seen that the required zero texture of M_D, M_R, M_S which yields the two-zero texture A_1(table 1), can be realized using Z_9 group symmetry realization. The symmetry realization of all the other textures are listed below in Table 2. As for all the textures we have considered the right-handed Majorana mass matrix M_R to be the same (Ref. Table 1). Therefore, we have taken the symmetry realization of the right-handed neutrino singlets (ν_{kR}) to be the same as in Eq. (4). As such the transformation of the scalar singlet χ also remains the same as in Eq. (9). Also, for all the textures, we consider the transformation of the singlet 'S' to be the same as in Eq. (11).

Table 2: Symmetry realization of the two-zero textures.

Texture	$\underline{D}_{eL}, \underline{D}_{\mu L}, \underline{D}_{\tau L}$	e_R, μ_R, τ_R	Φ_1, Φ_2, Φ_3	λ_1, λ_2
A_2	$\omega^5, \omega^3, \omega^7$	$\omega^4, \omega, \omega^3$	$1, \omega^4, \omega$	ω^6, ω^3
B_3	$\omega^3, \omega^5, \omega^4$	$\omega, \omega^7, \omega^5$	$1, \omega^3, \omega^4$	ω^6, ω^3
B_4	$\omega^7, \omega^3, \omega^4$	$\omega^4, \omega, \omega^5$	$1, \omega^4, \omega^2$	ω^6, ω^3
C	$\omega^4, \omega^5, \omega^3$	$\omega^5, \omega^4, 1$	$1, \omega, \omega^3$	ω^4, ω^3
D_1	$\omega^5, \omega, \omega^7$	$\omega^2, \omega^4, \omega^5$	$1, \omega, \omega^3$	ω^6, ω^3
D_2	$\omega^5, \omega, \omega^4$	$\omega^6, \omega^8, \omega^2$	$1, \omega^3, \omega^7$	ω^6, ω^3
E_1	$\omega, \omega^4, \omega^5$	$\omega^7, \omega^5, \omega^3$	$1, \omega^8, \omega^4$	ω^4, ω^3
E_2	$\omega^5, \omega^8, \omega^4$	$1, \omega^7, \omega^5$	$1, \omega^6, \omega^5$	ω^4, ω^3

Conclusion:

In this work, symmetry realization of the two-zero textures of $M_\nu^{4\times4}$ under MES has been performed. For this realization, (5+4) scheme has been considered, where the digits in the bracket represents the number of zeros of M_D and M_R respectively along with one-zero texture of M_S. The required textures of M_D, M_R, M_S have been mentioned in Table 1. I have considered Z_9 Abelian symmetry group for realization of the desired textures. Realization of the zero textures of M_D, M_R, M_S for texture A_1 has been shown elaborately in section 5. It has been observed that on applying field transformation of the $SU(2)_L$ doublets ($\underline{D}_{jL}$), right-handed $SU(2)_L$ singlets (l_R) and the right-handed neutrinos (ν_{kR}) gives rise to the texture of M_l, M_D, M_S as shown in Equation (5). To arrive at the required 5-zero texture of M_D and diagonal form of M_l, we have considered two additional Higgs doublet (Φ_2, Φ_3) with proper transformation along with the Standard model Higgs Φ_1 which remains invariant under Z_9. To arrive at the 4-zero texture of M_R additional scalar singlet χ is introduced which with proper transformation under Z_9 and bare mass terms lead to the desired texture. Also, to prevent bare mass term of the scalar singlet 'S' a transformation has been applied which with two additional scalar singlets λ_1, λ_2 leads to the one-zero texture of M_S. Z_9 symmetry realization of all the other textures are shown in Table 2. For all the textures, field transformation of the right-handed neutrinos (ν_{kR}), scalar singlet χ and singlet 'S' has been considered to the same.

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