DECISION MAKING WITH SIMPLEX MODEL: A CASE STUDY

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Abstract: Decision making is a big deal for a manufacturer. Simplex model is a vibrant tool for the decision makers. This paper focuses on of the most used simplex technique to take the decision of producing optimum number of items for earning maximum profit. Here, a drinking water manufacturing company has been undertaken and the information regarding different items produced, number of units produced, labour hour required, profit earned for each of the items has been collected. Linearity of the information is checked with help of coefficient of determination. Profit function is generated along with the underlying set of constraints and the convexity is also verified and eventually a linear programming problem of the company is formulated. Solving the problem by the powerful simplex technique in Excel-Solver, the maximum profit as well as the optimum number of items to be produced are obtained.

Keywords: linear programming; optimization; simplex technique; convexity; coefficient of determination

1. Introduction:

The linear programming (LP) is a mathematical method to provide an optimal solution for the problems where objective and requirements are both linear. The Linear Programming technique came to limelight during Second World. Scientists were involved to find out a scientific way to win over the other countries of the world. The same technique is now used in many industries for how many to produce in order to earn maximum profit. Most common methods in this regard are graphical and simplex methods.

Linear Programming was first introduced by Leonid Kantorovich in 1939. He developed the earliest linear programming problems that were used by the army during WWII in order to reduce the costs of the army and increase the efficiency in the battlefield. The method was a secret because of its use in war-time strategies, until 1947 when George B. Dantzig published the simplex method and John von Neuman developed the theory of duality. After WWII, many industries began adopting linear programming for its usefulness in planning optimization.

Dantzig's original linear programming example was to find the best assignment of 70 people to 70 jobs. In order to select the best assignment requires a lot of computing power; the number of possible configurations exceeds the number of particles in the observable universe. However, by posing the problem as a linear program and applying the simplex algorithm, it takes only a moment to find the optimum solution. The theory behind linear programming drastically reduces the number of possible optimal solutions that must be checked.

Linear programming (LP) may be defined as the problem of maximizing or minimizing a linear function that is subjected to linear constraints. The constraints may be equalities or inequalities. The Optimization problems involve the calculation of profit and loss. Linear programming problems are an important class of optimization problems that helps to find the feasible region and optimize the solution in order to have the highest or lowest value of the function.

In other words, linear programming is considered as an optimization method to maximize or minimize the objective function of the given mathematical model with the set of some requirements which are represented in the linear relationship. The main aim of the linear programming problem is to find the optimal solution.

Linear programming is the method of considering different inequalities relevant to a situation and calculating the best value that is required to be obtained in those conditions. Some of the assumptions taken while working with linear programming are:

- The number of constraints should be expressed in the quantitative terms.
- The relationship between the constraints and the objective function should be linear.
- The linear function (i.e., objective function) is to be optimized.

The basic components of the LP are as follows:

- Decision Variables, Constraints, Data, Objective Functions.

Our project is based on the following companies:

- Ozonised Purell Packaged Drinking Water Company

1.1 literature review:

To discuss the application of Linear Programming in various practical situations, a Literature review has been done as follows:
Render et. al. in the year 2012 [1], puts an emphasis on model building and computer applications to show how the techniques presented in the text are used in business. This text's use of software also allows instructors to focus on the managerial problem, while spending less time on the mathematical details of the algorithms. In the eleventh edition, Excel 2010 has been incorporated throughout the text and an even greater emphasis on modelling is provided.

Hasib and Hasan 2013 [2], in his research paper they represented formulating linear programming in real life problem using computer techniques AMPL and LINGO.

Taha in the year 2011 [3] ,the first formal activities of OR were initiated in England during World War II, when a team of British scientists set out to make scientifically based decisions regarding the best utilization of war materiel. After the war, the ideas advanced in military, they have taken note of this important statement during the preparation of the ninth edition, making every effort to introduce the art of modelling in OR.

Taylor in the year 2013 [4], a simple straightforward approach to modelling and solution techniques. Introduction to Management Science shows how to approach decision-making problems in a straightforward, logical way. Through the use of clear explanations and learn how to solve problems and make decisions based on the results. The eleventh edition reflects latest version of Excel and provides many new problems for instructors to assign.

Murty in the year 1983 [5], The goal of this journal is to provide a central forum for the distribution of timely information about network problems, their design and mathematical analysis, as well as efficient algorithms for carrying out optimization on networks. The nonstandard modelling of diverse processes using networks and network concepts is also of interest. Consequently, the disciplines that are useful in studying networks are varied, including applied mathematics, operations research, computer science, discrete mathematics and economics.

Dantzing and Thapa in the year 2003 [6], Linear programming represents one of the major applications of mathematics to business, industry, and economics. It provides a methodology for optimizing an output given that is a linear function of a number of inputs. George dantazing is widely regarded as the founder of the subject with his invention of the simplex algorithm in the 1940's. This second volume is intended to add to the theory of the items discussed in the first volume. It also includes additional advanced topics such as variants of the simplex methods, interior point methods.

Vazirani in the year 2001 [7], This book covers the dominant theoretical approaches to the approximate solution of hard combinatorial optimization and enumeration problems. It contains elegant combinatorial theory, useful and interesting algorithms, and deep results about the intrinsic complexity of combinatorial problems. Its clarity of exposition and excellent selection of exercises will make it accessible and appealing to all those with a taste for mathematics and algorithms.

By George and B. Dantzig in the year 1997 [8], the Story about How It Began: Some legends, a little about its historical sign-cancel, and comments about where its many mathematical programming extensions may be headed. Industrial production, resources in the economy, the exertion of military or in a war—all require the coordination of interrelated activities. What these complex undertakings share in common is the task of constructing a statement of actions to be.

Neter and Nachtchaim in the year 1996 [9], This article presents necessary and sufficient conditions to be satisfied by the best linear unbiased predictor of future observations in the general linear model in order to have a simple form. Under these conditions, the predictors have an expression similar to that in the uncorrelated case and some parameters related to the co variances between some observations need not to be known.

Kudragavtsen in the year 1981 [10], now a days mathematical method is widely applied in planning of natural economy, organization of industry control, business decision, transportation, engineering, telecommunications, elaboration of military operations etc. From the general point of view, the problems of control and planning are usually reduced to a choice of a certain system of numerical parameters or a function ensuring the most effective achievement of the pre planed aim (optimum plan) with the limited possible resources taken into account. To estimate the effectiveness of a plan, introduce the plan quantity index expressed in term of the plan characteristics and attaining the extremism value for an optimal plan. For the large number of practically interesting problems the objective function is expressed linearly in term of plan characteristics, the permissible values of the parameters also obeying linear equalities or inequalities.

1.2 Objectives:

On the basis of the literature studied, the following objectives are carried out in this project--

- Optimizing the number of units per item manufactured by the companies.
- Optimizing the profit

2. Materials and Methods:

2.1 Materials:

This case study is based on the Ozonised Purell Packaged Drinking Water Company. This company was established on 2016 and it is situated in Nalbari, Assam, India. Products prepared by this company are: 20-liter, 2-liter, 1-liter and Half-liter water bottles. Information gathered from this company are displayed in Table 2.1.

Table 2.1: Materials of Ozonised Purell Packaged Drinking Water Company:

PRODUCT	20 LITER BOTTLE				2 LITER BOTTLE			
YEAR	Size	Profit	Labour hour	Metal	Size	Profit	Labour Hour	Metal
2016	15000	100200	5000	307500	25000	10000	834	52500
2017	16500	124000	5500	338250	28000	136000	930	58800
2018	18000	150000	6000	369000	35000	180000	1166	73500
2019	21000	170000	7000	430500	45000	220000	1500	94500
2020	25000	220000	8334	512500	50000	250000	1666	105000
PRODUCT	1 LITER BOTTLE				HALF LITER BOTTLE			
2016	Size	Profit	Labour Hour	Metal	Size	Profit	Labour Hour	Metal
2017	30000	80000	500	36000	45000	70000	750	27000
2018	40000	120000	668	48000	54000	120000	900	32400
2019	60000	155000	1000	72000	60000	154000	1000	36000
2020	75000	230000	1250	90000	74000	190000	1240	44400
	80000	250000	1340	96000	90000	230000	1500	54000

2.2 Methodology:

- Define Decision variables.
- Formulation of objective function.
- Formulation of the constraints.
- Non-negative constraints.

The linear relationship should be examined and this is done with the help of R^2 value (coefficient of determination). This value should be reported for a relationship between quantity of production of each kind of products and profits, and quantity of production and labour hours, quantity of production and metal, and time progressing and size of production. Calculation and examination of R^2 is done in MS-EXCEL which is followed by F-ratio. Practically speaking, a higher R^2 indicates that a linear model fits the data well; therefore, if the R^2 is high, a large F statistic should follow, indicating strong evidence that at least some of the coefficients are non-zero. The p-value of the F-statistic is also implemented for double verification of the existence of linear relationship. The examination is done at 5% level of significance. If the p-value is less than the level of significance, then the model fits the linearity.

For obtaining the objective function, coefficients of the profit per unit for each of the products is taken from regression analysis table. For the constraints, the coefficients of each of the products are considered. t-statistic and p-value conform the normality of the data.

To check the convexity of a function, we need to examine the second-order derivative conditions. Specifically, for a function $f: R^n \rightarrow R$ to be convex, its Hessian matrix must be positive semidefinite.
The Hessian matrix H for a function f(x) is given by the matrix of second partial derivatives:

$$H_{ij} = \frac{\partial^2 f}{\partial x_i \partial x_j}$$

3. Results and discussion:

3.1 Results: mathematical formulation:

3.1.1 Status of Relationship:

Status of relationships between product and profit, product and labour hour, product and metal required and impact of time of progressing and unit of the products are displayed in table 3.1.1.

Table 3.1.1: Status of Relationship

RELATIONSHIP	R^2	F-value	p-value	Status
The impact of unit of produced 20 litter bottle and profit	0.98221	165.6332	0.001012	Linear
The impact of unit of produced 2 litter bottle and profit	0.809797	12.7726	0.037449	Linear
The impact of unit of produced 1 litter bottle and profit	0.969734	96.12185	0.002255	Linear
The impact of unit of produced Half litter bottle and profit	0.95639	65.7912	0.003912	Linear
The impact of unit of produced 20 litter bottle and labour Hour	1	1.9E+08	8.4E-13	Linear
The impact of unit of produced 2 litter bottle and labour Hour	0.999983	173530.1	3.05E-08	Linear
The impact of unit of produced 1 litter bottle and labour Hour	0.999954	64902.75	1.33E-07	Linear
The impact of unit of produced Half litter bottle and labour Hour	0.999907	32357.92	3.79E-07	Linear
The impact of unit of produced 20 litter bottle and metal	1	6.96E+32	1.2E-49	Linear
The impact of unit of produced 2 litter bottle and metal	1	3.94E+32	2.82E-49	Linear
The impact of unit of produced 1 litter bottle and metal	1	2.13E+32	7.09E-49	Linear
The impact of unit of produced Half litter bottle and metal	1	7.26E+34	1.13E-52	Linear
The impact of time of progressing and unit of 20 litter bottle	0.949763	56.71654	0.004853	Linear
The impact of time of progressing and unit of 2 litter bottle	0.97333	109.4878	0.001863	Linear
The impact of time of progressing and unit of 1 litter bottle	0.969415	95.08696	0.002291	Linear
The impact of time of progressing and unit of Half litter bottle	0.967072	88.1068	0.002561	Linear

3.1.2. Status of Objective Function:

Status of objective function is shown in table 3.1.2.

Table 3.1.2: Status of objective function

Products	Coefficient (Profit per unit)	t –value	p-value
20 Litter bottle	11.436	12.87	0.001
2 Litter bottle	7.859	3.574	0.037
1 Litter bottle	3.274	9.804	0.002
Half Litter bottle	3.417	8.111	0.004

Table 3.1.2 shows the profit per unit (coefficients) of 20 litter-bottle, 2 litter-bottle, 1 litter-bottle and half-litter-bottle along with their significance at 5% level of significance. All the p-values are less than the level of significance and hence they significant for consideration. Thus, the objective function thus obtained is:

$$Max.profit = 11.436x_1 + 7.859x_2 + 3.274x_3 + 3.417x_4 ----(3.1)$$

3.1.3 Status of Constraints:

Status of Constraints is presented in table 3.1.3.

Table 3.1.3: Status of constraints

Products	Coefficient		t-value	p-value
20 Litter bottle	0.333	Labour	13793.95	0.000
2 Litter bottle	0.033	Labour	416.5694	0.000
1 Litter bottle	0.017	Labour	2 54.7602	0.000
Half Litter bottle	0.017	Labour	179.8831	0.000
20 Litter bottle	20.5	Metal	2.64E+16	0.000
2 Litter bottle	2.1	Metal	1.98E+16	0.000
1 Litter bottle	1.2	Metal	1.46E+16	0.000
Half Litter bottle	0.6	Metal	2.69E+17	0.000

The constraints thus obtained here are:

$$\left.\begin{array}{c} x_1 \leq 26450 \\ x_2 \leq 56700 \\ x_3 \leq 97500 \\ x_4 \leq 97600 \\ 0.333x_1 + 0.033x_2 + 0.017x_3 + 0.017x_4 \leq 13966.8 \\ 20.5x_1 + 2.1x_2 + 1.2x_3 + 0.6x_4 \leq 836855 \\ x_1, x_2, x_3, x_4 \geq 0 \end{array}\right\} - - - - (3.2)$$

3.1.4 Convexity:

In our case, the objective function is $f(x) = 11.436x_1 + 7.859x_2 + 3.274x_3 + 3.417x_4$. Since this is a linear function, all the second partial derivatives will be zero, and the Hessian matrix will be a matrix of zeros. A matrix of zeros is always positive semidefinite, so the function is convex.

In summary, the function $f(x) = 11.436x_1 + 7.859x_2 + 3.274x_3 + 3.417x_4$ is convex.

3.1.5 Solution:

To know the result of objective function (3.1) with respect to the set of constraints (3.2), MS-Excel Solver has been incorporated. Table 3.1.5 portrays below the complete result of the linear programming problem consists of the function (3.1) and set of restrictions (3.2).

Table 3.1.5: Ozonised Purell Packaged Drinking Water Purification Results:

Variables	Solution		
x_1	26363.36		
x_2	56700		
x_3	97500		
x_4	97600		
Objective			
Maximize	1399811		
Constraints			
		Inequality	RHS
1	26363.36	≤	26450
2	56700	≤	56700
3	97500	≤	97500
4	97600	≤	97600
5	13966.8	≤	13966.8
6	835078.9	≤	836855

7	26363.36	≥	0
8	56700	≥	0
9	97500	≥	0
10	97600	≥	0

3.2 Discussion:

In section 3.1.1, the status of relationship and linearity is examined and is found linear. These are verified with the application of R^2 and p-value. In section 3.1.2, coefficients of the objective function are obtained and finally the objective function is generated which is shown in function (3.1). in section 3.1.3, the coefficients of the constraints of the problem are obtained and eventually the set of constraints is formulated which is shown in function (3.2). Clubbing function (3.1) and (3.2), the formulation of the LP problem is framed. It is a simple LP situation. It can be solved by various software. Excel Solver is one of them. Adopting simplex method in Excel Solver in section 3.1.5, the problem is solved. The objective function gives an amount of Rs. 1399811 and $x_1 = 26363, x_2 = 56700, x_3 = 97500$ and $x_4 = 97600$. These results help us to infer that maximum profit for the year 2021 has been found to be Rs. 1399811, units of 20 litre bottle to be produced is 26363, unit of 2 litre bottle to be produced is 56700, unit of 1 litre bottle to be produced is 97500 and unit of Half litre to be produced is 97600.

4. Conclusion:

In this case study, a company has been taken for studying the application of linear programming problem to maximize the profit and determine the number of units of each product to be produced. In section 1.2, the objectives are framed. To meet the objectives, the information is processed in section 3.1.1, 3.1.2 and 3.1.3. in section 3.1.4, convexity of the problem is justified. Section 3.1.5 brings out the solution of the entire problem. This has been discussed in section 3.2. From the discussion, the following conclusions are inferred out:

- Information from the company are collected.
- Simplex method is adopted in Excel solver.
- Maximum profit for the year 2021 found to be: Rs. 1399811.
- Number of 20 litre bottle to be produced is 26363.
- Number of 2 litre bottle to be produced is 56700.
- Number of 1 litre bottle to be produced is 97500.
- Number of Half litre to be produced is 97600.

In this study, the company undertaken falls within a certain jurisdiction only. The same can be extended to work for all small-scale as well as large scale industries and students of mathematics can take it as a source of income by rendering the service to the industries.

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