# PUBLIC REVOLUTION: A MATHEMATICAL MODELLING

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Abstract: People may become agitated against the incumbents because of many reasons. The offended public meet the common men and motivate them to become dissidents. This way the number of dissidents' hikes and they raise their points against the stakeholders. This is called public revolution. This can be mathematically modelled. In this article, Kermack-Mckendrik's famous SI model is used to express the situation. The common men are considered as susceptible and the dissidents are considered as infectives. The more interaction between common men and dissidents, the more chance of public revolution. The whole situation is organized with a system of ordinary differential equations. The basic reproduction number is called the basic dissidence number. If the basic dissidence number is higher than 1, then the chance of public revolution sustains.

Keywords: Public revolution; mathematical modelling; ordinary differential equations; equilibrium points; basic dissidence number

## 1. Introduction:

Millions of people reportedly took part in the citizenship amendment act (CAA) protests across India, according to media sources. However, because demonstrations took place in various locations and at various times, it is challenging to estimate the precise number. The biggest demonstrations were held in Delhi, Mumbai, Kolkata, and Bangalore. There were counter-protests in favour of the CAA in addition to the marches, however they were not as many as the anti-CAA protests. Opponents contend that the CAA violates India's secular constitution, while the Indian government claims that it is important to safeguard persecuted religious minorities in neighbouring nations. Overall, the CAA demonstrations were an important political development in India and brought to light the country's ongoing discussions about citizenship, identity, and religious freedom.

Between 1979 and 1985, the Assam Movement, sometimes referred to as the Assam Agitation, was a well-liked uprising in the Indian state of Assam. The movement's main concerns were illegal immigration from Bangladesh and how it affected Assam's population, economy, and culture. People in Assam, who were worried about the effects of illegal immigration on their lives and livelihoods, overwhelmingly supported the campaign. The movement had a major influence on Indian politics as well since it sparked the creation of the Asom Gana Parishad (AGP), a local political organisation that promoted Assamese rights. In general, the Assam Movement was an important occasion in the histories of Assam and India.

A revolution is defined as a forcible or peaceful overthrow of an existent government or social order, in favor of a new system. The term "revolution" also can mean an instance of revolving. Independent of the concrete motive of action, revolution, terrorism, and social unrest are discussed as having a negative impact on the affected society, as they harm economic growth and development and result in the destruction of human and physical capital, as well as a decrease in international trade and foreign direct investment.

Since the late 1970s, a truly remarkable revolution has swept public management around the world. Understanding this revolution means sorting through three issues: the basic ideas of reform; the connections between the reforms and governmental processes, like budgeting and personnel; and the links between these processes and governance. These reforms have proven surprisingly productive but, in the process, they have raised a new generation of fundamentally important issues that have been largely unexplored[1].

To manage public tasks, institutions with a distinct personality from central government organizations have been established in the majority of the world's existing legal systems. Although there is no universal approach in this respect, these organizations are valuable instruments for the central government in carrying out specialized executive and general responsibilities that require independence and distance from political concerns. Non-governmental public institutions in our nation are also among these organizations established to deliver public services. Iran's public non-governmental entities and organizations were identified in 1987, following the passage of the country's Public Accounting Law. Article 3 of the Civil Service Management Law, passed in 2007, defines non-governmental organizations and public institutions as distinct organizational units. that have legal independence. Non-governmental public institutions and institutions are those that were founded with the sanction of the Islamic Council and receive more than half of their yearly budget from non-governmental sources while also providing public services. The legislature defined non-governmental institutions and public institutions in Article 5 of the Public Accounting Law passed in 1987, which states, Public Non-Governmental Institutions: "They are specific organizational units formed with the permission of the law to perform tasks and services with a public aspect." Such institutions are also referred to as non-governmental organizations in various international agreements. It has been stated that non-governmental organizations are organizations that are not governments. Although not directly part of the political structure, they serve a significant role as intermediaries between individuals and the ruling authorities, and even the society itself[2].

A public revolution is a stage in the evolution of contradictions in relationships between people, groups of individuals, or a community as a whole, characterised by the existence of opposing interests, goals, and views of the interacting individuals. Revolution can be latent or obvious, and they are caused by an absence of compromise or, in some cases, conversation between two or more groups[3].

With the inclusion of the control function for a conflict, a model of ethno-social conflict based on diffusion equations was suggested. A model built on Langevin's diffusion equation was created. The concept was founded on the notion that people communicate in society via a communicative field. This field, which is generated by everyone in a community, functions as a model of individual information interaction. Furthermore, the management is incorporated into the system via the dissipation function. The set of equations for a divergent diffusion type was solved[4].

A multidisciplinary interference can help the stakeholders or the dissidents in estimating or predicting the strength of revolution. Researchers are looking for the mathematical concept of public revolution in many angles.

**1.1.** Objectives

- To know the meaning of Revolution in a mathematical way.
- To propose a mathematical model on public revolution.
- To see the stability of the proposed model.
- To find the fundamental dissidence number.

## 2. Materials and methods

### **2.1.** Mathematical formulation

In epidemiology, the Susceptible-Infective (SI) model is a mathematical representation of how infectious illnesses propagate. According to this paradigm, people are divided into two groups: susceptible (S) and infective (I). Individuals who are susceptible are those who are not yet infected but have the potential to become so, whereas infective people are those who are already sick and have the potential to spread the illness to others. Although the SI model was initially created for the study of infectious illnesses, it has also been used in other fields, such as the study of popular uprisings. The concept may be applied in this situation to explain how ideas, beliefs, and attitudes propagate among a population.

According to the SI model, in public revolution, susceptible people are those who have not yet been persuaded or pledged to the cause, whereas infective people are those who have been persuaded and are actively promoting the revolution's message. By assuming that infected individuals interact with susceptible individuals and change them into members of the infective class, the spread of the revolution may be modelled. The dynamics of a popular revolution may be studied using the SI model, along with the variables that might influence its success or failure. The model may be used, for instance, to research the effect of government repression on the growth of the revolution or the function of social networks and the media in disseminating the revolution's message.

### **2.2.** Assumptions

- N(t)+1 be the total population at time t
- C(t) be the total vulnerable population at time t
- D(t) be the total dissident population at time t
- β be the rate of contact between vulnerable human and dissident human.
- η be the death rate of dissident population

**2.3.** Mathematical Model

On the basis of the above assumptions and models suggested by Karmack-McKendrik [5], [6] is incorporated with slight modification to study the growth of the public revolution as follows:

$$\left.\begin{aligned}\frac{dC}{dt} &= A - \beta CD \\ \frac{dD}{dt} &= \beta CD - \eta D\end{aligned}\right\} ------(2.3.1)$$

Such that $C + D = N + 1, C > 0, D \geq 0, N > 0, A > 0, \beta > 0, t \geq 0, \eta > 0 --- (2.3.2)$

3. Results and discussion

**3.1.** Positivity and Boundedness of the state variables:

Proposition: Suppose $\big(C(t), D(t)\big)$ be the solution of the system (2.3.1).

If the initial condition $(C_0,\ D_0)$ is in a space, then ∃ a unique positive solution $\big(C(t), DJ(t)\big)\forall\ t \geq 0$ such that the solution will remain in the space with probability one. The solution $(C, D\ )$is defined in the interval $[0, \infty)$, where $N(t) + 1 = C(t) + D(t)$.

Proof: It is assumed that $(C_0,\ D_0) \in \Theta$, the space. Therefore, the coefficients of the equations of (2.3.1) are Lipchitz continuous. Hence, for any given initial condition $(C_0, D_0) \in \Theta, \exists$ a unique local solution $\big(C(t), D(t)\big)\forall\ t \in [0, T)$, where T is final time.

Solution of $\frac{dC}{dt} = A - \beta CD$ gives the result as:

$$=> \frac{dC}{dt} = A - \beta C(N + 1 - C)$$

$$=> \frac{dC}{dt} + \beta C(N + 1 - C) = A$$

$$IF = e^{\int_0^t \beta C(N+1-C)dt} = e^{\beta C(N+1-C)t}$$

$$C(t)e^{\beta C(N+1-C)T} = \int_0^t A.e^{\beta C(N+1-C)t}dt = A\left[\frac{e^{\beta C(N+1-C)t} - 1}{\beta(N + 1 - C)}\right] > 0\ as\ e^x > 1$$

$$=> C(t) > 0$$

Similarly, we can prove that $D(t) > 0$

It can be deduced that $C(t) + D(t) \leq AT\ \forall\ t \in [0, T)$.

Adding all the equations of (2.3.1), we get

$$\frac{d(N + 1)}{dt} \leq \frac{dC}{dt} + \frac{dD}{dt} = A - \eta$$

$$=> \frac{dN}{dt} \leq A$$

$$=> dN \leq Adt$$

$$=> N \leq At + constant$$

$$=> \lim_{t \to \infty} \sup(N) \leq AT \;\; \forall\, t \in [0, T)$$

Hence the state variables are positive and the system is bounded.

**3.2.** Equilibrium point
Equating the equations of system (2.3.1) to zero, we get

Equilibrium point is $E(C,\; D) = \; E\left(\frac{\eta}{\beta},\; \frac{A}{\eta}\right)$

**3.3.** Fundamental Dissidence Number
Dissidence exists[7] if $\frac{dD}{dt} > 0 => \frac{\beta C}{\eta} > 1$

Therefore, Fundamental Dissidence Number is $D_0 = \frac{\beta C}{\eta}$

**3.4.** Stability of the model
3.4.1. Local stability of the equilibrium:
The Jacobian matrix around the equilibrium point $E\left(\frac{\eta}{\beta},\; \frac{A}{\eta}\right)$ is

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial C} & \frac{\partial f_1}{\partial D} \\ \frac{\partial f_2}{\partial C} & \frac{\partial f_2}{\partial D} \end{bmatrix}$$

$$= \begin{bmatrix} -\beta D & -\beta C \\ \beta D & -\beta - \eta \end{bmatrix}$$

$$(f_1 = A - \beta CD,\; f_2 = \beta CD - \eta D$$

Eigen values of $\;J\; are\; \lambda_1 and\; \lambda_2$ which are proved to be negative. Hence the system is locally stable.

Assume a Lyapunov function $L(\mu) = \mu_1 C + \mu_2 D$

Then $\frac{\partial L}{\partial t} = \frac{\partial L}{\partial C}.\frac{dC}{dt} + \frac{\partial L}{\partial D}.\frac{dD}{dt}$

$$= \mu_1(A - \beta CD) + \mu_2(\beta CD - \eta D)$$

$$= -\beta CD(\mu_1 - \mu_2) - (\mu_2 \eta D - \mu_1 A)$$

If $\mu_1 - \mu_2 \geq 0$ and $\mu_2 \eta D - \mu_1 A > 0$, then $\frac{\partial L}{\partial t}$ is negative definite.

3.4.2. Global stability for the equilibrium:
Assume a Lyapunov function$L(\omega) = \frac{1}{2}\omega_1 C^2 + \frac{1}{2}\omega_2 D^2$

Then $\frac{\partial L}{\partial t} = \frac{\partial L}{\partial C}.\frac{dC}{dt} + \frac{\partial L}{\partial D}.\frac{dD}{dt} = \omega_1 C(A - \beta CD) + \omega_2 D(\beta CD - \eta D)$

$$= -\beta CD(\omega_1 C - \omega_2 D) - (\omega_2 \eta D^2 - \omega_1 CA)$$

The Lyapunov function $L(\omega)$ will be negative definite under the conditions

(i) $\omega_1 C - \omega_2 D > 0$

(ii) $\omega_2 \eta D^2 - \omega_1 CA > 0$

**3.5.** Sensitivity Analysis:

Sensitivity index[8] [9] of the system is given as: $S_A^{D_0} = \frac{\partial D_0}{\partial A} \cdot \frac{A}{D_0}$

The index table is shown below:

Table 1: Sensitivity index table

| Parameters | Sensitivity index | Sensitivity index values |
|---|---|---|
| $\beta$ | 1 | 1 |
| $\eta$ | -1 | -1 |
| $C$ | 1 | 1 |

Interpretation: From table 1, it is seen that that the fundamental dissidence number is directly proportional to rate of contact between vulnerable human being and dissident people whereas the fundamental dissidence number is inversely proportional to date rate of dissident people.

**3.6.** Numerical simulation

On the basis of the above discussed model (2.3.1), numerical simulations have been done with the help of Matlab 2016a to see the graphical behaviour for the unofficial data of a city

Time in days = [1:13]
Common men = [500 490 480 460 455 450 444 438 435 430 420 415 400]
Dissidents = [1 2 4 6 10 14 17 20 28 40 60 80 100]

as follows:

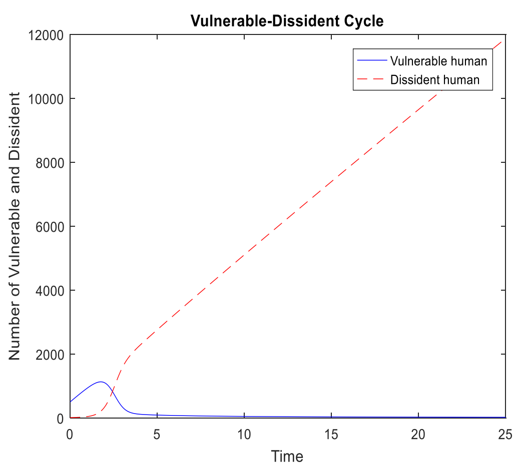


Fig. 1: Common men -Dissident Cycle

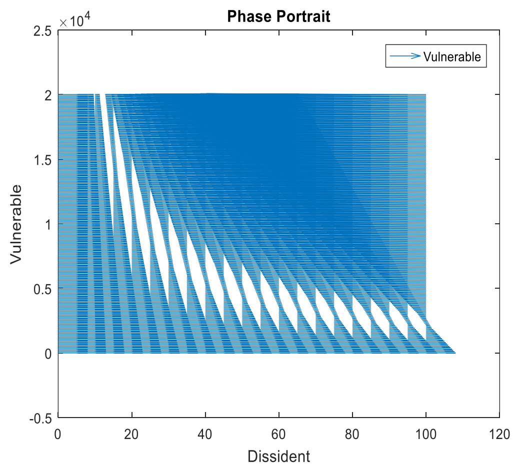


Fig. 2: Phase portrait (Dissident vs Common

Interpretation: In Figure 1, it is seen that the graph of vulnerable common has come down and that of dissident has gone up. This is approximately true in reality too. In figure 2, the model shows the asymptotic stability w.r.t. the equilibrium point.

Then the equilibrium point is:

- $E\left(\frac{\eta}{\beta}, \frac{A}{\eta}\right) = E(1.2,\ 196670)$

Eventually the dissidence number is greater than 1, which indicates the existence of the revolution.

Parameter Estimation: Using the method of least squares[10], [11] in Python[12], the parameters of the model (2.3.1) are obtained for some of the synthetic data based on the model. The parameters so obtained are:

A=1.95651751, β=0.58060017 and η= 0.16154568

Thus, the Equilibrium point is: E (0.278,12.111).

Based on these parameters and the unofficial data, the fundamental dissidence number can be depicted in Figure 3 as follows:

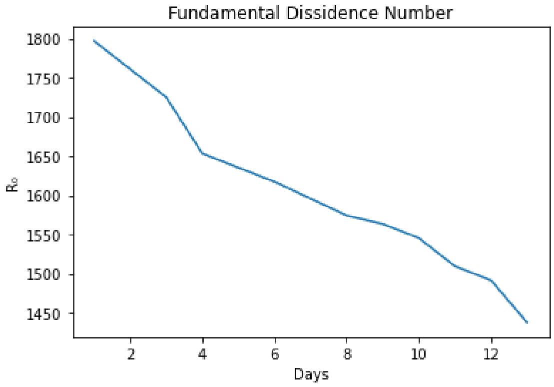


Fig. 3: Fundamental Dissidence Number

4. Conclusion

In this paper, a study has been conducted to examine the mathematical behaviour of public revolution. Here, deterministic model is utilized. Jacobian of the system shows the local stability. Global stability of the model exists under the condition:

(i) $\omega_1 C - \omega_2 D > 0$

(ii) $\omega_2 \eta D^2 - \omega_1 CA > 0$

Fundamental dissidence number is: $D_0 = \frac{\beta C}{\eta}$

and is greater than 1. This shows the existence of the dissidence as well as revolution. Sensitivity index of $\beta$ and C show that these are directly proportionate to Dissidence number. η is inversely proportionate to the dissidence number.

Phase portrait shows the asymptotic stability of the model.

After implementing the estimated parameters, we get the proposed model as:

$$\left.\begin{aligned}\frac{dC}{dt} &= 1.95651751 - 0.58060017CD \\ \frac{dD}{dt} &= 0.58060017CD - 0.16154568D\end{aligned}\right\}$$

And the Equilibrium point of the model is: E (0.278,12.111).

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