INDIRECT NATURAL CONVECTION FOR TRANSIENT HYDROMAGNETIC GAS FLOW ALONG AN INCLINED PLANE IN A POROUS MEDIA: LAPLACE TECHNIQUE

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Abstract: The present flow model deals with analytical solution for unsteady gravity-driven thermal convection flow of a viscous incompressible, absorbing-emitting, electrically-conducting, optically-thick gray gas along an inclined plane in saturated porous medium in presence of a transverse magnetic field. To simulate thermal radiation effects the Rosseland diffusion flux model is employed. Moreover, for some of physical parameters numerical investigations have been made for the flow velocity, flow temperature, skin-friction coefficient and surface heat transfer rate. With increasing inclination of the plane the flow is found to be accelerated. With progression of porosity and greater inclination of the plate velocity gradients at the plate are found to be enhanced. Applications of the model arise in astrophysics, high temperature materials operations exploiting magnetic fields and MHD (Magneto-Hydro-Dynamic) energy generators.

Keywords: Gravity-Driven; Thermal Radiation; Absorbing-Emitting; Electrically-conducting; Magneto-Hydro-Dynamic.

1. Introduction:

Radiative convective flows are encountered in countless industrial and environment processes in various solar power technologies, fossil fuel combustion energy processes, heating and cooling chambers, evaporation from large open water reservoirs, and space vehicle re-entry. Radiative heat transfer plays an important role in manufacturing industries for the design of reliable equipment. Examples of such engineering applications are Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles. For numerous applications in physics and engineering the hydrodynamic rotating flow of an electrically conducting viscous incompressible fluid has gained considerable attention. The free convective flow in channels formed by vertical plates has received attention among the researchers in last few decades due to its widespread importance in engineering applications like cooling of electronic equipments, design of passive solar systems for energy conversion, design of heat exchangers, human comfort in buildings, thermal regulation processes and many more.

Helliwell [1] investigated a flux model to analyzed radiation Magnetohydrodynamic channel flow. Hydromagnetic thermal convection in a vertical conduit with significant thermal radiation effects have been studied by Gupta and Gupta [2]. The steady Radiative Magnetogasdynamic Couette flow using temperature dependent coefficients of viscosity and electrical conductivity had discussed by Helliwell [3], together with a density-dependent absorption coefficient. Mandal et al. [4] investigated combined radiative-hydromagnetic flow, heat and mass transfer in a vertical channel. Raptis and Masslas [5] also investigated the unsteady Magnetohydrodynamic convection non-scattering fluid regime using the Rosseland radiation model and presented analytical solutions with graphical representation for the mean temperature, velocity and the induced magnetic field.

Azzam [6] have been considered the thermal radiation flux influence on hydromagnetic mixed free- forced convective steady flow also using the Rosseland approximation. Gbadeyan and Idowu [7] investigated the Magnetohydrodynamic heat transfer between two concentric rotating spheres employing the optically thin limit case for radiative heat flux. Ogulu and Prakash [8] discussed analytically free convection magneto-heat transfer using the differential approximation for optically-thin radiative flux in the energy equation and incorporating viscous dissipation effects. Temperature-dependent viscosity effects in transient dissipative radiation-hydromagnetic convection, showing that an increase in Eckert number and decrease in air viscosity accelerate the flow have been investigated Mahmoud [9]. Jang and Hsu [10] discussed numerically the vortex instability of a horizontal Magnetohydrodynamic natural convection boundary layer flow in a saturated porous medium including thermal radiation.

In the above discussion have not assess however the flow from an inclined surface, a regime of considerable importance in glass manufacturing [11,12], solar energy collectors, film cooling chemical engineering systems, and electronic circuit cooling mechanisms. The transient convection flow past an inclined plane in presence of magnetic field with thermal radiation investigated Ghosh et al. [13].

The our present studies aims to consider a Darcian flow model for the unsteady convection flow of an electrically-conducting, absorbing-emitting, optically-thick gray gas along an inclined plane in a saturated porous medium in the presence of a transverse magnetic field with thermal radiation flux. The effects of the significant physical parameters on the flow regime are discussed in detail and Analytical solutions are developed through Laplace Transform Technique.

2. Mathematical Formulation:

We consider the transient hydromagnetic flow of a viscous, incompressible, electrically-conducting, absorbing-emitting, non-scattering, optically-thick gas along an infinite plate inclined at α to the horizontal immersed in a saturated porous medium, the plate is moving with constant velocity, u_0 as shown below in Figure 1. Refractive index of the gas medium is constant.

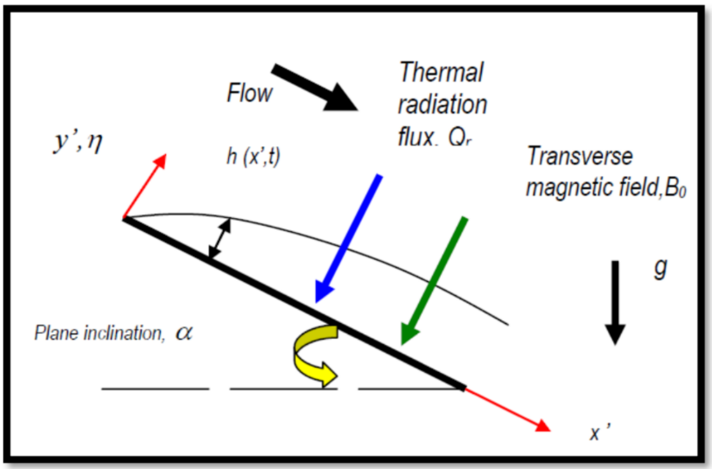

Figure 1: Flow configuration

A uniform magnetic field, B0, is applied perpendicular to the plate. The x- axis is orientated along the plate and the y- axis perpendicular to the plate. The Maxwell field equations, as described by Sutton and Sherman [14] comprise five vector equations-the Ampere law, magnetic field continuity, Faraday's law, Kirchoff's law and finally Ohm's law. The generalized equations in vectorial form, for flow of an electrically-conducting gas are the Maxwell equations:

$$curl\,(\vec{B}) = \mu\vec{J} \quad Ampere's\ Law \tag{1}$$

$$div\,(\vec{B}) = 0 \quad Magnetic\ field\ continuity \tag{2}$$

$$curl\,(\vec{E}) = -\frac{\partial\vec{B}}{\partial t} \quad Faraday's\quad Law \tag{3}$$

$$div\,(\vec{J}) = -\frac{\partial\vec{B}}{\partial t} \quad Kirchoff's\quad Law \tag{4}$$

$$\vec{J} = \sigma[\vec{E} + \vec{v}\times\vec{B}] \quad Ohm's\quad Law \tag{5}$$

where $\vec{J}$ is the current density, $\vec{B}$ is the magnetic field vector, σ is the electrical conductivity, $\vec{E}$ is the electrical field intensity vector, ρ is density, $\vec{v}$ v is the velocity vector, μ is co-efficient of viscosity, t is time.

From an order of magnitude analysis, it can be shown, Sutton and Sherman [14] that for two-dimensional $(x-y)$ magneto-hydrodynamic gas dynamic flows, the hydromagnetic retarding force (Lorentz body force) acts only parallel to the flow and has the form:

$$F_{magnetic} \approx -\sigma B_y^2 u, \tag{6}$$

where B_y is the component of magnetic field in the y- direction.

We consider an aerodynamic viscous flow where the magnetic field is sufficiently weak to sustain a small magnetic Reynolds number such that induced magnetic field effects can be neglected. Joule electro-heating and Hall current/ion-slip effects are also neglected. The temperature of the gas in the regime is $\overline{T}$ and an induced pressure gradient generated by indirect natural convection acts along the $\overline{x}-$ direction. All fluid properties are constant. The plate temperature is prescribed $\overline{T}_w$and is of sufficiently high magnitude that thermal radiation effects are significant. In accordance with the Boussinesq approximation, all fluid properties are constant with the exception of the density variation in the buoyancy term. Unidirectional radiation flux, Q_r is considered and it is assumed that

$$\frac{\partial Q_r}{\partial \overline{y}} \gg \frac{\partial Q_r}{\partial \overline{x}}.$$

Under these simplification, the mass, momentum and energy conservation equations for the regime with regard to indirect natural convection, may be presented as follows:

$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} = 0 \tag{7}$$

$$\frac{\partial \overline{u}}{\partial \overline{t}} + \overline{u}\frac{\partial \overline{u}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{u}}{\partial \overline{y}} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial \overline{x}} + \nu\frac{\partial^2 \overline{u}}{\partial \overline{y}^2} + g\beta\left(\overline{T} - \overline{T}_\infty\right) sin\alpha - \frac{\sigma B_0^2}{\rho}\overline{u} - \frac{\nu}{\overline{K}_p}\overline{u} \tag{8}$$

$$0 = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial \overline{y}} - g\beta\left(\overline{T} - \overline{T}_\infty\right) cos\alpha \tag{9}$$

$$\rho C_p \frac{\partial \overline{T}}{\partial \overline{t}} = \kappa \frac{\partial^2 \overline{T}}{\partial \overline{y}^2} - \frac{\partial q_r}{\partial \overline{y}} \tag{10}$$

The initial and boundary conditions are

$$\begin{Bmatrix} u = 0, \overline{T} = \overline{T}_\infty \forall \overline{y}, \overline{t} \le 0 \\ \overline{u} = u_0, \overline{T} = \overline{T}_w at \overline{y} = 0, \overline{t} > 0 \\ u \to 0, \overline{T} \to \overline{T}_\infty \overline{t} > 0 \end{Bmatrix}, \tag{11}$$

where $\overline{u}$ is the velocity in the $\overline{x}-$ direction, $\overline{v}$ is velocity in the $\overline{y}-$ direction, $\overline{t}$ denotes time, g the acceleration due to gravity, ν is the kinematic viscosity of the optically-dense gas, β is volumetric coefficient of thermal expansion, $\overline{T}$ the temperature of the fluid, $\overline{T}_\infty$ is free stream temperature of the fluid, $\overline{T}_w$ is the plate surface temperature, ρ the density, C_p is the specific heat at constant pressure, κ is thermal conductivity of the optically-dense fluid, σ is electrical conductivity of the gas and B_0 is magnetic field and Q_r is the radiative heat flux.

In transient flow, the frictional (viscous) and gravitational forces do not balance exactly and the discrepancy is proportional to the acceleration of the fluid. The deviation between the free surface of the gas and the plate inclination also contributes to this and an instability mechanism arises in the inclined plane flow. There is a pressure distribution in the flow with a gradient defined as:

$$\frac{\partial \overline{p}}{\partial \overline{y}} = \rho g \tag{12}$$

After integration, Eq. (9) becomes:

$$\overline{p} = \rho g y(h - \overline{y})\left(\overline{T} - \overline{T}_\infty\right) cos\alpha, \tag{13}$$

where h denotes free surface elevation.

Differentiating (13) with respect to $\overline{x}$, yields:

$$\frac{\partial \overline{p}}{\partial \overline{x}} = \rho g y\left(\overline{T} - \overline{T}_\infty\right)\frac{\partial h}{\partial \overline{x}} cos\alpha \tag{14}$$

Above the leading edge of the plate ($\overline{x} = 0$), the density variation with depth is constant i.e. will remain unchanged for all $\frac{\partial h}{\partial \overline{x}}$. We therefore prescribe the following condition:

$$\frac{\partial h}{\partial \overline{x}} = cos\alpha tant = F_1 \tag{15}$$

Introducing the following non-dimensional quantities for the solution of the two-point boundary value problem defined by Eqs (7) to (10) under the boundary conditions (11):

where u denotes dimensionless $\overline{x}-$ direction velocity, t is non-dimensional time, $\overline{y}$ is dimensionless transverse coordinate, θ is the dimensionless temperature function, Gr is the Grashof (free convection) number, Pr is the Prandtl number and M_d denotes the square root of the Hartmann hydromagnetic number.

On using Eq. (15) in Eqs (8) and (9), and neglecting convective acceleration terms, the dimensionless form of the momentum equation is:

$$\frac{\partial u}{\partial t}=Gr_h(Sin\alpha-F_1cos\alpha)\theta+\frac{\partial^2 u}{\partial y^2}-(M_d+K^{-1})u \tag{17}$$

The radiation heat flux vector is addressed using the approach outlined by Isachenko et al. [15]. The Rosseland diffusion flux approximation is therefore used leading to a Fourier-type gradient function, viz:

$$Q_r=\frac{-4\overline{\sigma}}{3\kappa^*}\frac{\partial\overline{T}^4}{\partial\overline{y}}, \tag{18}$$

where $\overline{\sigma}$ is the Stefan-Boltzmann constant and κ* is the spectral mean absorption coefficient of the medium.

This model is valid for optically-thick media in which thermal radiation propagates only a limited distance prior to experiencing scattering or absorption. The local thermal radiation intensity is due to radiation emanating from proximate locations in the vicinity of which emission and scattering are comparable to the location of interest. For zones where conditions are appreciably different thermal radiation has been shown to be greatly attenuated before arriving at the location under consideration. The energy transfer depends on conditions only in the area adjacent to the plate regime i.e. the boundary layer regime. Rosseland's model yields accurate results for intensive absorption i.e. optically-thick flows which are optically far from the bounding surface. Implicit in this approximation is also the existence of wavelength regions where the optical thickness may exceed a value of five. As such the Rosseland model, while limited compared with other flux models, can simulate to a reasonable degree of accuracy thermal radiation effects in engineering flow problems, as elaborated upon by Modest [16]. We further assume that the temperature differences within the flow are sufficiently small such that $\overline{T}^4$ can be expressed as a linear function of the temperature, $\overline{T}$. Expansion of $\overline{T}^4$ as a Taylor series about the free stream temperature, $\overline{T}_\infty$, neglecting higher order terms, generates a result of the form:

$$\overline{T}^4\cong 4\overline{T}_\infty^3\overline{T}-3\overline{T}_\infty^4 \tag{19}$$

Proceeding with the analysis, the energy Eq. (10) may be orchestrated in dimensionless form subject to Eq. (16) as follows:

$$(1+R_a)\frac{\partial^2\theta}{\partial y^2}-Pr\frac{\partial\theta}{\partial t}=0, \tag{20}$$

where R_a denotes the Boltzmann – Rosseland radiation-conduction number.

The first term in Eq. (20) is an augmented diffusion term i.e. with R_a=0, thermal radiation vanishes and Eq. (20) reduces to the familiar unsteady one-dimensional conduction-convection equation.

The boundary conditions (11) are transformed to:

$$\begin{Bmatrix} \forall\overline{y},\overline{t}\le 0: u=0,\theta=0 \\ t>0: u=1,\theta=0 aty=0 \\ t>0: u\to 0,\theta\to 0 asy\to\infty \end{Bmatrix} \tag{21}$$

3. Analytical Solutions:

The two-point boundary value problem defined by Eqs (17) and (20) along with their boundary conditions (21) have been solved analytically using Laplace transforms technique and their exact solutions are as follows:

$$u(y,t)=\begin{bmatrix} \frac{1}{2}\left\{1-\frac{Gr}{N}(sin\alpha-F_1cos\alpha)\right\}\left\{e^{-Ny}erfc\left(\frac{y-2Nt}{\sqrt{t}}\right)+e^{Ny}erfc\left(\frac{y+2Nt}{\sqrt{t}}\right)\right\} \\ \frac{+Gr}{N}(sin\alpha-F_1cos\alpha)erfc\left(\frac{y}{2}\sqrt{\frac{Pr}{(1+R_a)t}}\right)+(sin\alpha-F_1cos\alpha)\frac{y}{\sqrt{4\pi}}\frac{Gr_h}{N}\frac{\sqrt{2}-1}{\sqrt{t}} \\ \left[exp\left\{-\left(\frac{y^2}{4}+Nt\right)\right\}-\sqrt{\frac{Pr}{1+R_a}}exp\left\{-\left(\frac{y^2Pr}{4(1+R_a)t}\right)\right\}\right] \end{bmatrix} \tag{22}$$

$$\theta(y,t) = erfc\left\{\frac{y}{2}\sqrt{\frac{Pr}{(1+R_a)t}}\right\}, \tag{23}$$

where,

$$N = M_d^2 + K^{-1}.$$

In order to interest in engineering design, the frictional shear stress at the plate surface (y = 0) and the surface temperature gradient, are defined respectively as:

$$\tau = -\frac{\partial u(y,t)}{\partial y}\bigg|_{y=0}$$

$$\begin{bmatrix} \frac{1}{2}\left\{1-\frac{Gr}{N}(sin\alpha - F_1 cos\alpha)\right\}\left\{Nerfc\left(N\sqrt{t}\right) - Nerfc\left(-N\sqrt{t}\right) - e^{-Nt}\right\} \\ \frac{-Gr}{N}(sin\alpha - F_1 cos\alpha)\frac{1}{\sqrt{\pi}}erfc\left(\frac{y}{2}\sqrt{\frac{Pr}{(1+R_a)t}}\right) \\ +(sin\alpha - F_1 cos\alpha)\frac{1}{\sqrt{4\pi}}\frac{Gr_h}{N}\frac{\sqrt{2}-1}{\sqrt{t}}\left[exp(-Nt) - \sqrt{\frac{Pr}{1+R_a}}\right] \\ \left[exp(-Nt) - \sqrt{\frac{Pr}{1+R_a}}\right] \end{bmatrix} \tag{24}$$

$$Nu = -\frac{\partial\theta(y,t)}{\partial y}\bigg|_{y=0} = \sqrt{\frac{1}{\pi(1+R_a)t}} \tag{25}$$

Some special cases have studied during the investigation.

Case I: Non-Radiative Hydromagnetic Inclined Plate Flow –
In the absence of thermal radiation effects, $R_a \to 0$ and the velocity and temperature solutions (22) and (23) reduce to the case for hydromagnetic free convection flow from an inclined plane:

$$u(y,t) = \begin{bmatrix} \frac{1}{2}\left\{1-\frac{Gr}{N}(sin\alpha - F_1 cos\alpha)\right\}\left\{e^{-Ny}erfc\left(\frac{y-2Nt}{\sqrt{t}}\right) + e^{Ny}erfc\left(\frac{y+2Nt}{\sqrt{t}}\right)\right\} \\ \frac{+Gr}{N}(sin\alpha - F_1 cos\alpha)erfc\left(\frac{y}{2}\sqrt{\frac{Pr}{t}}\right) \\ +(sin\alpha - F_1 cos\alpha)\frac{y}{\sqrt{4\pi}}\frac{Gr}{N}\frac{\sqrt{2}-1}{\sqrt{t}}\left[exp\left\{-\left(\frac{y^2}{4}+Nt\right)\right\} - \sqrt{Pr}exp\left\{-\left(\frac{y^2 Pr}{4t}\right)\right\}\right] \end{bmatrix} \tag{26}$$

$$\theta(y,t) = erfc\left\{\frac{y}{2}\sqrt{\frac{Pr}{t}}\right\} \tag{27}$$

Case II: Magnetohydrodynamic Radiative-Convection from a Horizontal Plane –
With $\alpha \to 0$, $sin\alpha \to 0$ and $cos\alpha \to 1$, the plate becomes horizontal and the velocity solution (22) reduces to:

$$u(y,t) = \begin{bmatrix} \frac{1}{2}\left\{1+\frac{F_1 Gr_h}{N}\right\}\left\{e^{-Ny} erfc\left(\frac{y-2Nt}{\sqrt{t}}\right)+e^{Ny} erfc\left(\frac{y+2Nt}{\sqrt{t}}\right)\right\} \\ \frac{-F_1 Gr}{N} erfc\left(\frac{y}{2}\sqrt{\frac{Pr}{(1+R_a)t}}\right) \\ -F_1 \frac{y}{\sqrt{4\pi}}\frac{Gr}{N}\frac{\sqrt{2}-1}{\sqrt{t}}\left[exp\left\{-\left(\frac{y^2}{4}+Nt\right)\right\}-\sqrt{\frac{Pr}{1+R_d}}exp\left\{-\left(\frac{y^2 Pr}{4(1+R_d)t}\right)\right\}\right] \end{bmatrix} \quad (28)$$

Case III: Magnetohydrodynamic Radiative-Convection from a Vertical Plane –
With $\alpha \to \pi/2$, $sin\alpha \to 1$ and $cos\alpha \to 0$, the plate becomes vertical and the velocity solution (22) will reduce to:

$$u(y,t) = \begin{bmatrix} \frac{1}{2}\left\{1-\frac{Gr}{N}\right\}\left\{e^{-Ny} erfc\left(\frac{y-2Nt}{\sqrt{t}}\right)+e^{Ny} erfc\left(\frac{y+2Nt}{\sqrt{t}}\right)\right\} \\ \frac{+Gr}{N} erfc\left(\frac{y}{2}\sqrt{\frac{Pr}{(1+R_a)t}}\right) \\ \frac{+y}{\sqrt{4\pi}}\frac{Gr}{N}\frac{\sqrt{2}-1}{\sqrt{t}}\left[exp\left\{-\left(\frac{y^2}{4}+Nt\right)\right\}-\sqrt{\frac{Pr}{1+R_a}}exp\left\{-\left(\frac{y^2 Pr}{4(1+R_a)t}\right)\right\}\right] \end{bmatrix} \quad (29)$$

4. Validity:

The effects of α and R_a on skin-friction distributions at the wall ($y = 0$) are compared with the available solution of Ghosh et al. [13] for accuracy has been presented in Table – 1 and it is found to be in excellent agreement and hence the proposed flow model is validated and stable for investigation.

Table 1: Skin friction, $\left(\frac{\partial u}{\partial y}\right)_{y=0}$, at $y = 0$ for $F_1 = 2.0$, $Pr = 0.71$, $Gr = 5.0$, $t = 2$ with various plate inclinations (α) and Boltzmann-Rosseland numbers (R_a) without porosity.

	Present work			Ghosh *et al.* (2010)		
α	$R_a = 0.0$	$R_a = 2.0$	$R_a = 5.0$	$R_a = 0.0$	$R_a = 2.0$	$R_a = 5.0$
0	–6.536174	–6.915851	–7.260521	–6.536142	–6.915804	–7.260507
π/6	–6.351610	–6.396273	–6.472885	–6.351590	–6.396255	–6.472835
π/4	–6.092735	–5.978556	–5.997309	–6.092711	–5.978523	–5.997283
π/3	–5.658501	–5.643792	–5.637782	–5.658423	–5.643715	–5.637752
π/2	–5.361291	–5.299105	–5.258132	–5.361270	–5.298371	–5.258072

The results through the Table 1 shows that the velocity gradients, $\left(\frac{\partial u}{\partial y}\right)_{y=0}$, at the wall y = 0 enhanced by the influence of various plate inclinations (α) and Boltzmann-Rosseland numbers (R_a). Back flow may also observed due to negativity of the values of $\left(\frac{\partial u}{\partial y}\right)_{y=0}$.

5. Results and Discussion:

All the numerical computations have been done from the analytical solutions given in Eqs (22) and (23) with respect to air (Pr = 0.71) and the arbitrary constant, $F_1 = 1$. The entire boundary layer regime has controlled by five thermo physical parameters, R_a, Pr, Gr, M_d, K, a single geometric parameter α and time t.

The influence of magnetic parameter (M_d) and plane inclinations (α) on the flow velocity distributions with transverse distance (y) normal to the cooled plate ($Gr > 0$) for conduction air ($Pr = 0.71$) have been presented in Fig. 2. The hydromagnetic term in the dimensionless Eq. (17), is a linear drag force term. With increasing magnetic field strength, B_0, M_d is increased and this serves to decelerate the flow along the inclined plate and therefore all the velocity profiles are strongly reduced with increasing M_d. It is seen that the velocity profiles are elevated near the surface and decay to zero progressively for all values of M_d, α and y. Moreover, an increase in the plane inclination, the flow velocity is significantly increased. When plane inclination is zero i.e. for the horizontal plane, velocity is minimized since gravitational acceleration effects are neglected. Also the flow is strongly accelerated for maximum plane inclination (α = π/3) and these profiles correspond to strong buoyancy (free convection) force.

Figure 3 depicts the temperature response for different thermal radiations (R_d) and time parameter (t) with distance transverse to the surface (i.e. with y-coordinate). The parameter R_a defines the ratio of thermal conduction contribution relative to thermal radiation. For $R_a = 1$, the thermal radiation and the thermal conduction contributions are equivalent. For $R_a > 1$, the thermal radiation effect is dominant over the thermal conduction effect and vice versa for $R_a < 1$. An increase in the value of R_d from 0 (non-radiating) through 2.0, to 4.0 (radiation is dominant over thermal conduction), causes a significant increase in the velocity with distance into the boundary layer i.e. accelerates the flow. The velocities in all cases ascend from the surface, peak close to the wall and then decay smoothly to zero in the free stream. As expected, the temperature values are also significantly enhanced with an growth in the value of t as there is a progressive increase in the time parameter contribution accompanying this. Rosseland's radiation diffusion model effectively enhances the thermal diffusivity, as described by Siegel and Howell (1972). In Eq. (20) this is apparent in the coefficient of the spatial temperature gradient i.e. diffusion term, $(1 + R_a)\frac{\partial^2\theta}{\partial y^2}$. The temperature profiles all decay monotonically from the maximum at the plate surface to the free stream.

Figure 4 illustrates the behaviour of the porosity (K) and free convection parameter i.e. Grashof number (Gr) on the boundary layer variable u. Increasing the porosity of the porous medium clearly serves to enhance the flow velocity i.e. accelerates the flow. This effect is accentuated close to the surface where a peak in the velocity profile arises. With further distance transverse to the surface, the velocity profiles are all found to decay into the free stream. An increased porosity clearly corresponds to a reduced presence of matrix fibers in the flow regime which, therefore, provides a lower resistance to the flow and in turn, boosts the momentum. However, the free convection currents as simulated with the buoyancy term, $Gr(Sin\alpha - F_1 cos\alpha)\theta$, in Eq. (17), clearly serves to accelerate the flow along the inclined plate and maximum peaks have been attained at higher free convection $(Gr = 10)$.

The rate of heat transfer in terms of Nusselt number (Nu) at the plate $y = 0$ for various effects of Boltzmann-Rosseland number (R_a) and time parameter (t) is shown in Fig. 5. An increase in R_a serves to reduction in rate of heat transfer throughout the regime, but maximum effect has observed at the plate. Also a substantial decrease has seen in the rate of heat transfer for increasing time parameter. The negative values of Nu assert that the heat is diffused through fluid region to the plate.

In Fig. 6 we have plotted the behaviour of velocity gradients at the plate $y = 0$ for the variance of porosity of the porous medium (K) and inclinations of the plate (α) against time (t). With increasing porosity and inclination values, the velocity gradients in the regime are found to increases. Also the velocity gradients have a reverse behaviour for the time parameter i.e. substantial decrease is observed in velocity gradients. Insignificant effect has occurred in the velocity gradients when the plane becomes vertical $(\alpha = \pi/2)$.

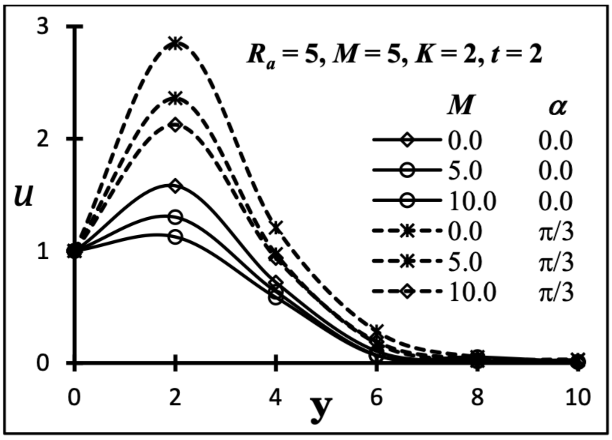

Figure2: Velocity distribution for magnetic number (M_d) and inclinations of the plate (α).

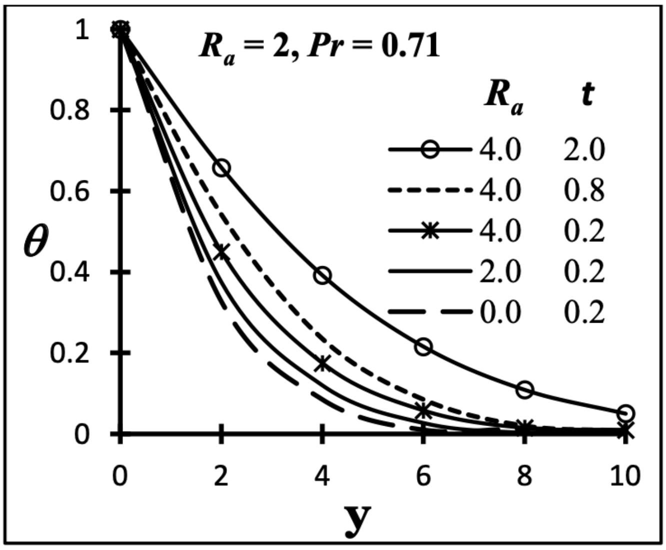

Figure3: Temperature distribution for Boltzmann-Rosseland radiation-convection parameter (R_d) and time (t).

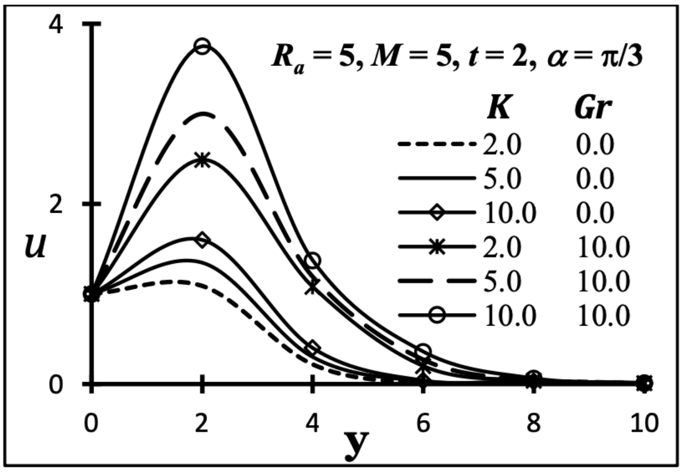

Figure4: Velocity distribution for porosity parameter (K) and Grashof number (Gr).

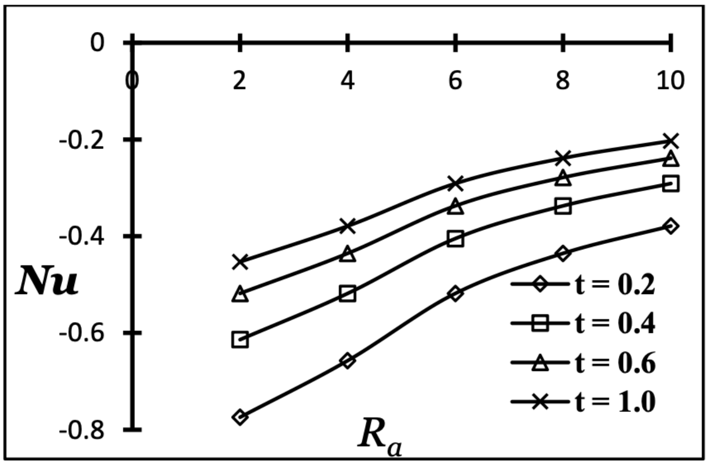

Figure5: Rate of heat transfer distribution for Boltzmann-Rosseland radiation-convection parameter (R_d) and time (t).

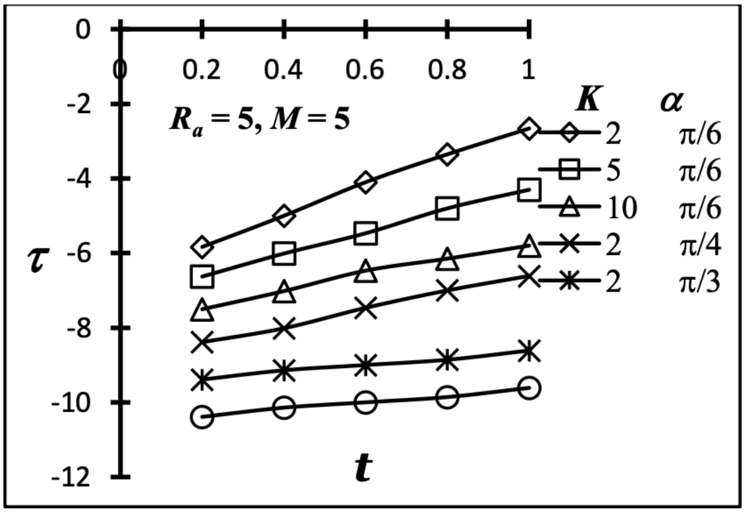

Figure 6: Skin-friction distribution for porosity (K) and inclinations of the plate (α) against time (t).

6. Conclusions:

A Darcian flow model is developed for the transient hydromagnetic free convection-radiation flow of an incompressible viscous fluid along an inclined plane in a saturated porous medium under a transverse magnetic field. The Rosseland diffusion approximation, valid for optically-thick gases has been utilized. The governing partial differential equations are transformed into a system of ordinary differential equations using similarity transformations. The transformed ordinary differential equations are then solved analytically by Laplace transform Technique. Few significant results are summarized:

- Enhancement of the magnetic parameter (M) and inclination of the plane (α) accelerated the flow velocity;
- Increasing the porosity (K) and free convection parameter (G) serves to boosts the flow velocity and velocity gradients at the plate in the boundary layer regime;
- Increasing Boltzmann- Rosseland number (R_a) i.e. greater thermal radiation heat transfer contribution serves to enhanced the fluid temperature, but a converse behaviour has occurred in the rate of heat transfer at the plate;
- Increasing time parameter escalated the fluid temperature and the rate of heat transfer, whereas an opposite character is sustained for the velocity gradients.

Nomenclature:

$\bar{u}$ Fluid velocity in $\bar{x}$ direction $\bar{w}$ Fluid velocity in $\bar{z}$ direction
υ Kinematic coefficient of viscosity σ Electrical conductivity
ρ Density $\bar{K}_1$ Permeability of porous medium
g Acceleration due to gravity $\bar{T}$ Fluid temperature
$\bar{C}$ Fluid concentration β_T Coefficient of thermal expansion
β_C Coefficient of volume expansion m Hall current parameter
κ Thermal conductivity C_p Specific heat at constant pressure
Q_0 Heat absorption coefficient T Non-dimensional Fluid temperature
u Non-dimensional fluid velocity x direction v Non-dimensional fluid velocity y direction
C Non-dimensional Fluid concentration M^2 Magnetic parameter
K^2 Rotation or Ekman number K_1 Permeability parameter
Gr Grashof Number for heat transfer Gm Grashof Number for mass transfer
Pr Prandtl Number Sc Schmidt Number
D mass diffusivity $\bar{C}_r$ Chemical reaction parameter
ϕ Non dimensional heat absorption coefficient
C_r Non dimensional chemical reaction parameter

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