

VARIATIONAL STUDY OF S-SHELL LAMBDA HYPERNUCLEAR SYSTEM

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Abstract:: Theoretical study on hypernuclear systems is important to know the nature of hyperon-nucleon and hyperon-hyperon interaction as only hypernuclear systems give the scope of knowing these interactions. A hypernucleus, in addition to the nucleons contains at least one hyperon which is a strange particle composed of quarks. A hypernucleus is produced mostly in heavy ion collisions and it undergoes weak decay. Experimental detection of hypernuclear events are rare and this makes the study of hypernuclear physics more challenging. Hypernuclear physics has a close association with astrophysics as hyperon-nucleon and hyperon-hyperon interactions are found to play important role in the interiors of neutron stars. The core of neutron stars contains strange quark matter and therefore, study of hyperon involved potentials are essential for the determination of the composition of neutron star matter. But, there is a scarcity of data from hyperon-nucleon scattering experiments. Also, since it is impossible to have hyperon-hyperon scattering experiments, the direct determination of the baryon-baryon interaction strength is extremely difficult. Therefore theoretical models play important role in unfolding the mysteries of hyperon-nucleon and hyperon-hyperon interaction. In this study, binding energies of hypernuclear systems calculated using different two-body lambda-nucleon and three-body lambda-nucleon interactions have been analysed. Also effect of lambda-lambda potential on the binding energy of hypernuclear system have been analysed. In this few-body study, we have employed Variational Monte Carlo technique for calculation of the binding energies of different hyperclear systems.

Keywords: hypernuclear system; few-body study; Variational Monte Carlo technique

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1. Introduction:

A hypernucleus, in addition to the nucleons contains at least one hyperon which is a strange particle. Hypernucleus is produced mostly in heavy ion collisions and it undergoes weak decay. In general, a single and a double lambda hypernucleus is represented by, ${}^{A}_{\Lambda}Z$ and ${}^{A}_{\Lambda\Lambda}Z$ where 'Z' and 'A' denote respectively the atomic number and mass number of the parent atom. A lambda hypernucleus has one or more lambda particle occupying the same quantum state occupied already by the nucleons. The strange hyperon 'A' has a lifetime of 10⁻¹⁰s and therefore a hypernucleus decays very rapidly. This makes experimental detection of hypernuclear events very rare and therefore the study of hypernuclear physics is very challenging. Study of hyperon-hyperon interactions, to understand baryon-baryon interaction in general, enriching our knowledge about role of strangeness in a nuclear medium of different densities in physical observables starting from deuteron to neutron stars etc. Study of hyperon involved potentials are essential for the determination of the composition and properties of neutron stars. Since the first report of hypernuclear event by M. Danysz and J. Pniewski[1] in 1953, lots of theoretical as well as experimental work has been done.

Earlier we have performed variational Monte-Carlo studies on different hypernuclear systems with different potential models to study the effect of parameters of the potential models on the binding energy of the hypernuclear systems [2,3,4]. In the present study we revisit the hypernuclear system ${}_{\Lambda\Lambda}^{4}H$ with the potential model Λ N4 of ref[8]. We discuss the role of Λ NN interaction parameters on the stability of the double hypernuclear system ${}_{\Lambda\Lambda}^{4}H$ From the study we find that the stability of the hypernuclear system depends on the three-body Λ NN potential crucially.



2. Potentials and wavefunction:

NN and NNN potential :

For the nuclear part of the Hamiltonian, we use ArgonneV₁₈ NN[6] and Urbana IX NNN[7] potentials .

<u>**AN**</u> and <u>ANN</u> potential :

We use phenomenological potential consisting of central, Majorana space-exchange and spin-spin ΛN components for ΛN potential[5], and it is given by,

$$V_{\Lambda N} = (V_c(r) - \overline{V}T_{\pi}^2(r))(1 - \varepsilon + \varepsilon P_x) + \frac{1}{4}V_{\sigma}T_{\pi}^2(r)\sigma_{\Lambda}\sigma_N$$
(1)

Here P_x is the majorana space-exchange operator and ε is the space exchange parameter which is taken as 0.2[8]. $V_c(r)$, \overline{V} and V_{σ} are respectively Wood-saxon core, spin-average and spin-dependent strength and $T_{\pi}^2(r)$ is one-pion tensor shape factor.

In the ANN potential, there are two terms, a two-pion exchange part and a dispersive part[5]. The two-pion exchange part of the interaction is given by

$$W_{p} = -\frac{1}{6} C_{p}(\tau_{i} \cdot \tau_{j}) \{ X_{i\Lambda} \cdot X_{j\Lambda} \} Y_{\pi}(r_{i\Lambda}) Y_{\pi}(r_{j\Lambda})$$
(2)

Here $X_{k\Lambda}$ is the one-pion exchange operator given by,

$$X_{k\Lambda} = (\sigma_k . \sigma_{\Lambda}) + S_{k\Lambda}(r_{k\Lambda}) T_{\pi}(r_{k\Lambda})$$

with

$$S_{k\Lambda} = \frac{3(\sigma_k.r_{k\Lambda})(\sigma_{\Lambda}.r_{k\Lambda})}{r_{k\Lambda}^2} - (\sigma_k.\sigma_{\Lambda})$$

The dispersive part of the ANN potential is given by,

$$V_{\Lambda NN}^{DS} = W_0 T_{\pi}^2(r_{i\Lambda}) T_{\pi}^2(r_{j\Lambda}) [1 + \frac{1}{6} \sigma_{\Lambda} . (\sigma_i . \sigma_j)]$$
(3)

Here $Y_{\pi}(r_{kA})$ and $T_{\pi}(r_{kA})$ are the usual Yukawa and tensor functions with pion mass, $\mu = 0.7 \text{ fm}^{-1}$; C_p and W_0 are ANN interaction parameters.

We revisit the hypernuclear system ${}_{\Lambda\Lambda}^{4}H$ using the potential model given in Table 1. The values of the ΛN and ΛNN interaction parameters, viz. \mathcal{E} , $C_{p\&}W_0$ selected with the criterion of giving bound state for ${}_{\Lambda}^{3}H$. For the potential model $\Lambda N4$, the spin-average and spin-dependent strength of the ΛN potential are kept same with spin-average strength $\overline{V} = 6.150$ Mev and spin-dependent strength $V_{\sigma} = 0.176$ Mev, same as in $\Lambda N1[2,3]$. The values of the interaction parameter of our earlier potential model $\Lambda N4$ [8] are listed in Table 2. The ΛN and ΛNN potential parameters for our preferred model denoted by $\Lambda N4$ [8] are listed in Table 1. C_p and W_0 are the strength parameters of the two-pion and dispersive parts of the ΛNN potential. The values of the three body ΛNN interaction parameters have been reduced in this preferred potential model $\Lambda N4$.

Table 1: ΛN and ΛNN interaction parameters for $\Lambda N4$. Except for ϵ , all other quantities are in MeV.

ΛN	\overline{V}	V_{σ}	ε	C_{p}	W ₀
$\Lambda N4$	6.150	0.176	0.2	0.7	0.012

Table 2: ΛN and ΛNN interaction parameters for $\Lambda N1$. Except for ϵ , all other quantities are in MeV.

ΛN	V	V_{σ}	8	$C_{ m p}$	W_0
ΛNI	6.150	0.176	0.2	0.15	0.028



<u>ΛΛ potential :</u>

For $\Lambda\Lambda$ potential, we use low-energy phase equivalent Nijmegen interactions represented by a sum of the three Gaussians[9,10,11],

$V_{\Lambda\Lambda} = v^{(1)} \exp(-r^2 / \beta_{(1)}^2) + \mathcal{W}^{(2)} \exp(-r^2 / \beta_{(1)}^2) + v^{(3)} e^{-r^2 / \beta_{(1)}^2}$	$\exp(-r^2/\beta_{(3)}^2)$)
trength parameters v^i and the range parameters β are		

Here the strength parameters v^i and the range parameters β_i are,

Sl. No.	$\beta_i(fm)$	$v^i(Mev)$
1	1.342	-21.49
2	0.777	-379.10
3	0.350	9324.00

The variational wave function is represented by,

$$|\Psi_{\mathbf{v}}\rangle = \left[1 + \sum_{i < j < k} \left(U_{ijk} + U_{ijk}^{TNI}\right) + \sum_{i < j,\Lambda} U_{ij,\Lambda} + \sum_{i < j} U_{ij}^{LS}\right] \prod_{i < j < k} f_c^{ijk} |\Psi_{\mathbf{p}}\rangle$$
(4)

Here, $|\Psi_p\rangle$ is the pair wave function[2,3] given by

$$|\Psi_{\rm p}\rangle = S \prod_{i < j} (1 + U_{ij}) S \prod_{i < \Lambda} (1 + U_{i\Lambda}) |\Psi_{\rm J}\rangle$$
(5)

The Jastrow wave function for lambda hypernuclei is given by,

$$|\Psi_{\rm J}\rangle = \left[\prod_{i < j < k} f_c^{ijk} \prod_{i < \Lambda} f_c^{i\Lambda} \prod_{i < j} f_c^{ij} \right] |\Psi_{\rm JT}\rangle |\varphi\rangle$$

Here f's are the central correlation functions and $|\phi\rangle$ is an antisymmetric wave function of the lambda particle. $|\Psi_{\pi}\rangle$ is the spin and isospin wavefunction of the s-shell nucleus.

3. Variational Monte Carlo technique:

Variational Monte Carlo method is used to find the ground state energy and binding energy of different hypernuclear systems. This technique is based on variational principle and is used to find ground state energy of a system by varying the different parameters variationally.

Variational principle states that, the approximate value of a Hamiltonian, calculated using trial wave-function is never lower in value than the true ground state energy

$$E = \frac{\left\langle \Psi | H | \Psi \right\rangle}{\left\langle \Psi | \Psi \right\rangle} \ge E_0 \tag{7}$$

where E_0 is the true ground state energy of the system.

To find the true ground state energy, a suitably parametrized trial wave function is selected which is a function of position, spin, isospin and other intrinsic variables and parameters. This trial wave function is used to find the upper bound to the energy using metropolis algorithm. The minimum energy is searched by calculating energy differences for wave functions using configurations generated by random walk. Energy expectation values are calculated by varying variational parameters one or two at a time. The minimum energy so obtained is taken as the true ground state energy of the system.

The binding energy (B_{Λ}) formulae for single and double hypernuclear system are given by,

$$-B_{\Lambda}({}^{A}_{\Lambda}Z) = E({}^{A}_{\Lambda}Z) - E({}^{A-1}Z)$$
(8)

$$-B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z) = E({}^{A}_{\Lambda\Lambda}Z) - E({}^{A-2}Z)$$
(9)

4. Results and discussion:

The binding energy results for the hypernuclear systems ${}_{\Lambda\Lambda}{}^{4}H$ with the potential model Λ N4 are tabulated in Table 3. We have also presented the results for ${}_{\Lambda}{}^{3}H$ the two potentials. We have compared the present result with experimental value and our earlier result on ${}_{\Lambda\Lambda}{}^{4}H$ with the potential model Λ N1 [Ref. 3].

Table3: Binding energy (B_{Λ} and $B_{\Lambda\Lambda}$) results for ${}^{3}_{\Lambda}H$ and ${}^{4}_{\Lambda\Lambda}H$. All quantities are in MeV.

Potential	Term	$^{3}_{\Lambda}H$	${}^4_{\Lambda\Lambda} H$
ЛN4	Е	-2.39(01)	-2.30(01)
	$V_{\Lambda N}$	-2.86(09)	-2.12(04)
	V _{ANN}	-0.15(01)	-0.07(01)
	SEC	0.05(00)	0.07(00)
	\mathbf{B}_{Λ}	0.17(01)	
	$\mathbf{B}_{\Lambda\Lambda}$		0.08(01)
Experimental		0.13	
ЛN1 [Ref. 3]		0.34(01)[3]	0.38(03)

The potential model $\Lambda N4$ contains both space exchange part of ΛN potential and non zero values of the parameters $C_p \& W_0$ of ΛNN potential but with reduced values. The binding energy for ${}_{\Lambda\Lambda}{}^4H$ is found to be very small, 0.08(01) with this potential model. Therefore higher values of these two parameters give more stable ${}_{\Lambda\Lambda}{}^4H$.

5. Conclusions:

The results show that the decreased values of ΛNN interaction parameters reduces the calculated binding energy. Thus, in addition to the two-body ΛN interaction parameters and $\Lambda\Lambda$ interaction parameters, the three-body ΛNN interaction parameters are important for bound state of ${}_{\Lambda\Lambda}{}^{4}H$.

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