BOUNDS ON FINITE EXTRA-DIMENSION FROM THE MASSES OF LIGHT-HEAVY MESONS IN A QCD POTENTIAL MODEL

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Abstract: Recently, the problem of stability of the H atom has been reported in extra-finite dimension, and found out that it is stable in extra-finite dimension of size $R \leq \frac{a_0}{4}$ where,a_0 is the Bohr radius. Motivated by this, we assume that the light-heavy mesons also have such stability controlled by the scale of coupling constant, and we obtain corresponding QCD Bohr radius, $R_{QCD} \leq \frac{a_0|_{QCD}}{4}$, where $a_0 = \frac{4\alpha_s}{3}$, and α_s is the strong coupling constant and μ is the reduced mass of mesons, and it is found to be well within the present theoretical and experimental limit of higher dimension. We then, obtain the wave function of light-heavy mesons considering the well known linear plus coulomb, Cornell potential in a space with one finite extra-dimension. Specifically, we consider one finite extra dimension and calculate the masses of few light-heavy mesons. Comparing our results with the well-known experimental data,we obtain the bound on the size of extra- dimension, which is well within the present theoretical and experimental bound.

Keywords: compact extra dimension; QCD; mesons

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1. Introduction:

Quantum Chromo-dynamics in a space with extra-spatial dimension has become a topical interest of research. The standard model(SM) of particle physics although finds immense success in explaining most of the physics of particles and fields,but still there exist certain limitations. The fundamental limitations are :

(1) Grand Unification and Quantum gravity[1, 2].
(2) Hierarchy between Planck scale and electroweak scale[3].
(3) Dark matter and Dark energy candidates[4].

Therefore physics beyond SM (BSM) finds very important applicability. The two most popular aspects of BSM physics are :
(1) Super-symmetry[5],
(2) Extra-dimension[6, 7, 8, 9, 43].
Recently,BSM theories with extra-dimension has got lot of interest from theoretical as well as experimental prospective after the development of Universal Extra Dimension model by Applequist etal.,[9],which allows the well known standard model particles to propagate in extra-dimension.

Most of the works deal with a general case of d-dimensional hydrogen atoms with the potential proportional to $\frac{1}{r}$, irrespective of the number of spatial dimensions[45,46]. More recently, a more physically relevant potential proportional to $\frac{1}{r^2}$ is suggested [47, 48] instead of inverse distance potential. Specifically it has been shown first, while there is no stable H-atom in a space with extra-dimension of infinite length, but in a space with compact extra-dimension, the stability of H-atom is restored.

In the present work, we assume that the potential is proportional to $\frac{1}{r}$ and consider one finite extra-dimension. This will facilitate solving the Schrodinger equation in the standard manner unlike the $\frac{1}{r^2}$ potential.

The result of [10] implies a critical size above which H-atom is not stable, $R \leq a_0/4$, where a_0 is the Bohr radius[10]. Motivated by this, we applied the same argument that heavy flavour mesons are also stable in a space with one finite extra-dimension of size less than the QCD Bohr radius,$R_{QCD} \leq \frac{a_0|_{QCD}}{4}$,where $a_0|_{QCD} = \frac{3}{16\mu\alpha_s}$,$\alpha_s$ is the strong coupling constant and μ is the reduced mass of mesons . For calculation, in this work we consider $\frac{1}{r}$ potential instead of $\frac{1}{r^2}$ potential and see its consequences in the masses of a few heavy flavour mesons .

However, the dynamics of Heavy flavour mesons are governed by the inter-quark potential. The properties of Heavy meson are in rough approximation as described by the linear plus coulomb Cornell potential given by$V(r) = -\frac{4\alpha_s}{3r} + br$. These two potentials play important role in the quark dynamics and in generally speaking, their separation is not possible. Recently, perturbation theory was employed to study several properties of heavy flavour mesons[20],[24],[33]. In [37],[38],we employed Dalgarno's perturbation theory with linear cum coulomb potential in a space with D-dimensions, where all the dimensions have infinite extent, and computed the Isgur-Wise function of a few heavy-light mesons. Further in the ref.[13] also, Dalgarno's perturbation technique was employed to estimate the masses of heavy flavour mesons in a space which has extra-dimension of infinite extent, even though its interest is not theoretically appealing at present. However, while using perturbation theory we have to choose one potential as parent and other as perturbation. This was done with the argument that at short distance coulomb plays parent and at long distance linear is parent. However, there are intermediate distances between the quark and anti-quark, where both potential is equally effective. Hence, it will be of topical interest to explore alternative method where such divide is not necessary. Recently in [14],a new quantum mechanical scheme is reported,where separation between the short range coulomb and the long range linear is not required. In this method the solution is the product of asymptotic solution of linear and coulomb potentials, which is reasonable for ground states(l=0)of heavy flavour mesons.

The aim of the present work is to use this method to estimate the masses of a few heavy flavour mesons in a space with one finite extra-dimension using the generalized linear plus inverse distance potential. Another aim is to put theoretical bounds on the size of extra-dimension from the experimental uncertainties of the measured masses of heavy flavour mesons in 3 D. Detailed comparison will be done with the bounds obtained experimentally as well as other theoretical models. For comparison, we also calculate the masses of a few heavy flavour mesons in standard 3 dimension and compare with the previous results obtained with Variational method [31] and Variationally Improved Perturbation Theory[30].

The paper is arranged as, $Section2$ is the formalism, in $Section3$ we discuss the result and $Section4$ is the summary and conclusion.

2. Formalism:

2.1. Linear plus inverse distance potential in a space with one finite extra-dimension:

The linear plus inverse distance Cornell potential in a space with one finite extra-dimension can be written as,

$$V(r_D) = -\frac{A}{r_D} + br_D \tag{1}$$

where,$A = \frac{4\alpha_s}{3}$, α_s is the strong coupling constant and b is the confinement parameter. We consider that behaviour of strong coupling constant and confinement parameter is independent of the dimensions and r_D is defined as,

$$r_D^2 = r_1^2 + r_2^2 + r_3^2 + y^2 \tag{2}$$

$$= r^2 + y^2 \tag{3}$$

where $r^2 = r_1^2 + r_2^2 + r_3^2$, y is the size of finite extra dimension. For $r >> y$ we get,

$$r_D \simeq r + \frac{y^2}{2r} \tag{4}$$

Substituting equation (4) in equation (1) we ultimately get,

$$V(r_D) \simeq r(b - A) + \frac{y^2}{2r}(b + A) \tag{5}$$

We substitute this potential, in the D-dimensional Schrodinger equation and solve it in the Quantum mechanical approximation scheme reported in[14]. Here, the total wave-function is assumed to be the product of the wave-function at short distance ($r \longrightarrow 0$) and at long distance ($r \longrightarrow \infty$). It is well known that, the short distance behaviour is ($\simeq e^{-\mu\alpha r}$) whereas the long distance behaviour is ($A_i[r]$). The scheme is however more appropriate for ground state ($l = 0$) state only, as can be seen by comparing with standard H-atom wave-function. For $r = 0$,the wave-function is purely controlled by the asymptotic behaviour ($e^{-\mu\alpha r}$) whereas, for $r \neq 0$ additional multiplicative factor r^l appears [13].

We apply the same technique to solve the D-dimensioal Schrodinger equation for $l = 0$ state .

2.2. Schrodinger equation in a space with one finite extra-dimension and it's solution with linear plus inverse distance potential:

The D-dimensional Schrodinger equation is[14,15,16],

$$[\frac{d^2}{dr_D^2} + \frac{D-1}{r_D}\frac{d}{dr_D} - \frac{l(l+D-2)}{r_D^2} + \frac{2\mu}{\hbar^2}(E - V_0)]R(r_D) = 0 \tag{6}$$

where r_D is as defined in equation (4) .The equation is solved by considering two extreme conditions as in[14] ,

CaseI:(Inverse distance potential:)

When $r_D \longrightarrow 0$,the linear term vanishes($br_D = 0$),for l=0,taking$\hbar = 1$,we get

$$\ddot{R}(r_D) + \frac{D-1}{r_D}\dot{R}(r_D) + 2\mu(E + \frac{A}{r_D})R(r_D) = 0 \tag{7}$$

Let,$R(r_D) = F(r_D)e^{-\mu A_D r_D}$, Now putting $R(r_D)$ in equation (7) we get,

$$\ddot{F}(r_D) + (\frac{D-1}{r_D} - 2\mu A)\dot{F}(r_D) + (\mu^2 A^2 - \frac{D-1}{r_D}\mu A + 2\mu E + \frac{2\mu A r_D)}{F}(r_D) = 0 \tag{8}$$

Now,we consider the series expansion of $F(r_D)$ as, $F(r_D) = \sum_{n=0}^{\infty} a_n r_D^n f(r_D, D)$, such that $f(r_D) = 1$ at $D = 3$.Let us consider, $f(r_D) = r^{\frac{\sigma(D-3)}{2}}$,which satisfies this condition. Then the radial wave function can be expressed as, $R(r_D) = \sum_{n=0}^{\infty} a_n r_D^{n+\frac{\sigma(D-3)}{2}} e^{-\mu A r_D}$. For ground state,$n = 0$,we get the unperturbed wave function,

$$\psi(r_D) \simeq (r_D)^{\sigma(D-3)} e^{-\mu A(r+\frac{y^2}{2r})} \tag{9}$$

σ is related to the normalization constant [11]. In 3-dimension σ do not occur. For any given value of $'\sigma'$ one can find $'N_D'$ at $D = 4, 5, 6,$ .... etc. For definiteness, $\sigma = 1$ and for $D = 4$ we get,

$$\psi(r_D) \simeq r_D e^{-\mu A r_D} \tag{10}$$

It should be noted that,at $D = 4$ the r_D term survives in equation(9),but for $D = 3$ it vanishes.

Now,at $D = 3, y = 0$ and we get from above equation(10),

$$\psi(r) \simeq e^{-\mu\frac{4\alpha_s}{3}r} \tag{11}$$

which is consistent with standard H-atom wave function [17] at $D = 3$.

Case-II:(Linear)

When $r_D \longrightarrow \infty$, the coulomb term vanishes and the D-dimensional Schrodinger equation (for $l = 0$, $\hbar = 1$),as,

$$\ddot{R}(r_D) + \frac{D-1}{r_D}\dot{R}(r_D) + 2\mu(E - br_D)R(r_D) = 0 \tag{12}$$

Let us consider ,

$$R(r_D) = \frac{U(r_D)}{2\sqrt{\pi}r_D} \tag{13}$$

And we introduce a dimensionless variable $\varrho(r_D)$,where,

$$\varrho = (2\mu b)^{\frac{1}{3}} r_D - (\frac{2\mu}{b^2})^{\frac{1}{3}} E \tag{14}$$

Substituting equations $(13), (14)$ in (12),we get,

$$\frac{d^2u}{d\varrho^2} - \varrho u = 0 \tag{15}$$

The solution of this equation contains linear combination of two types of Airy's function [18],$A_i[r_D]$ and $B_i[r_D]$. But as $r_D \longrightarrow \infty$,$A_i[r_D] \longrightarrow 0$ and $B_i[r_D] \longrightarrow \infty$. Therefore, we consider only $A_i[r_D]$ part, and the radial wave-function then can be expressed as:

$$U(r_D) = A_i[(2\mu b)^{\frac{1}{3}} r_D - (\frac{2\mu}{b^2})^{\frac{1}{3}} E] \tag{16}$$

From the boundary condition $U(0) = 0$,we get the ground state energy [19],

$$W_0 = E = -(\frac{b^2}{2\mu})^{\frac{1}{3}} \varrho_0 \tag{17}$$

here,ϱ_0 is the zero of Airy function, and $A_i[\varrho_0] = 0$,and ϱ_0 has the explicit form,

$$\varrho_0 = -[\frac{3\pi(4n-1)}{8}]^{\frac{2}{3}} \tag{18}$$

For ground state, $n = 1$,and we get the radial wave-function for the ground state as,

$$\psi(r_D) \simeq \frac{1}{2\sqrt{\pi}r_D} A_i[(2\mu b)^{\frac{1}{3}} r_D - (\frac{9\pi}{8})^{\frac{2}{3}}] = \frac{1}{2\sqrt{\pi}r_D} A_i[\varrho] \tag{19}$$

As the Airy's fuction ,$A_i[\varrho]$ is an infinite series in ϱ ,in this work we consider terms only upto first order,$A_i[\varrho] = a_0 - b_0\varrho$,where,$a_0 = \frac{1}{3^{\frac{2}{3}}\Gamma(2/3)}$ and $b_0 = \frac{1}{3^{\frac{1}{3}}\Gamma(1/3)}$. Physically, adding higher order polynomials of Airy Function will basically give rise to terms L^3,L^4,L^5,L^6 etc., (L is the size of finite extra-dimension) which can be neglected due to $L \ll r$.

2.3. Total wave-function:

As suggested in [14],we now construct a purely analytic solution for ground state (l=0)as the multiplication of the solutions of the two extreme conditions equations $(10), (19)$:

$$\psi(r_D) = \frac{N}{2\sqrt{\pi}} A_i[\varrho] e^{-\mu A r_D} \tag{20}$$

And ultimately we get,

$$\psi(r_D) = \frac{N}{2\sqrt{\pi}}[a_0 - b_0((2\mu b)^{\frac{1}{3}} r_D - 2.3194)]e^{-\mu A r_D} \tag{21}$$

The normalization condition is,

$$\int_0^\infty \int_0^L DC_D(r_D)^{(D-1)}|\psi(r_D)|^2 dr dy = 1 \tag{22}$$

where, $C_D = \frac{(\pi)^{\frac{D}{2}}}{\Gamma(\frac{D}{2}+1)}$. The wave-function at the origin is

$$|\psi(0)| = \frac{N}{2\sqrt{\pi}} a_0 \tag{23}$$

2.4. Masses of Light Heavy mesons:

The masses of heavy flavour mesons is given by [20],[21],[44], For pseudo-scalar mesons,

$$M_P = M + m - \frac{8\pi\alpha_s}{3Mm}|\ \psi(0)\ |^2 \tag{24}$$

Similarly,for vector mesons[23],

$$M_V = M + m + \frac{8\pi\alpha_s}{9Mm}|\ \psi(0)\ |^2 \tag{25}$$

where, M and m are the masses of Heavy quark/anti-quark and light quark/anti-quark respectively and α_s is the strong coupling constant. It is to be noted that unlike previous works [11, 12],the WFO is well defined here.

3. Result:

3.1. Masses of Pseudo-scalar and vector mesons:

With the above discussed formalism, we calculate the masses of a few heavy flavour mesons,which are shown in $Table1$. The input parameters are $m_{u/d} = 0.336GeV$,$m_b = 4.95GeV$, $m_c = 1.55GeV$, $m_s = 0.483GeV$ and $b = 0.183GeV^2$[23],[24]. We take $\alpha_s = 0.39$ for C-scale and $\alpha_s = 0.22$ for b-scale. Table 1 shows that our results for masses of heavy flavour mesons in a space with one finite extra dimension of size $0.001GeV^{-1}$,as a representative case, which is in well agreement with those of experimental values [25] and also within the QCD Bohr radii.

Table 1: Masses of heavy flavour pseudo-scalar mesons (for $L = 0.001GeV^{-1} = 10^{-18}m$).

Meson	WFO	$M_P(GeV)$	Exp.Mass(GeV)[25]
$D(c\bar{u}/cd)$	0.036	1.85	1.869 ± 0.0016
$D(c\bar{s})$	0.075	1.958	1.968 ± 0.0033
$B(u\bar{b}/d\bar{b})$	0.005	5.28	5.279 ± 0.0017
$B_s(s\bar{b})$	0.0192	5.418	5.366 ± 0.0024

In Table 2, we calculate the masses of a few vector heavy flavour mesons for $L = 0.001GeV^{-1}$, which also agrees with exp.data[25].

Table 2: Masses of heavy flavour vector mesons (for $L = 0.001GeV^{-1} = 10^{-18}m$).

Meson	WFO	$M_V(GeV)$	Exp.Mass(GeV)[25]
$D(c\bar{u}/cd)$	0.036	1.889	2.006 ± 0.0016
$D(c\bar{s})$	0.075	2.041	2.106 ± 0.0033
$B(u\bar{b}/d\bar{b})$	0.005	5.286	5.324 ± 0.0017
$B_s(s\bar{b})$	0.0192	5.433	5.415 ± 0.0024

Table 3: Different experimental and theoretical limit on the size of extra dimension.

Experiment and Models	Limit on the size of extra-dimension (m)
Fermi-LAT[27]	$8 \times 10^{-9}m$ (LED)
LEP-I[28]	$4.5 \times 10^{-14}m$
ADD [7]	$\sim 10^{-3}m$
Martin Bures[10]	$\leq \frac{a_0}{4}(0.13225 \times 10^{-10})m$
ALEPH,DELPHI,OPAL[28]	$\sim 6 \times 10^{-18}m$
RS[8]	$2 \times 10^{-9}m$
I.Antoniadis[29]	$6.2 \times 10^{-19}m$
LHC[26]	$2.06 \times 10^{-18}m$

3.2. Masses in $L = 0$ limit:

Here,we calculate the masses of both pseudo-scalar and vector mesons in the $L = 0$ limit and compare with the results obtained in our previous approaches[30],[31],[32]. Our results are smaller than the previous results. This may be presumably due to the limitation of the Quantum mechanical method [14] employed here.

Table 4: Masses of heavy flavour pseudo-scalar mesons (for $L = 0$).

Meson	Mass	[31]	[32]	[30]
$D(c\bar{u}/cd)$	1.835	1.94	1.878	1.841
$D(c\bar{s})$	1.87	2.032	2.01	1.969
$B(u\bar{b}/d\bar{b})$	5.13	5.35	5.28	5.16
$B_s(s\bar{b})$	5.18	5.48	5.4	5.35

Let us compare the present result with the results of previous works [11] and [12]. In [11],with inverse distance potential in a space with one finite extra-dimension, it was observed that masses increase with size of extra-dimension. The pattern is similar in [12] and in this work. While, the allowed size of extra-dimension in [11] and [12] are, $L \leq 13 \times 10^{-17}m$, $L \leq 10^{-7}GeV^{-1}(2 \times 10^{-23}m)$respectively, in this work, it is $L \leq 10^{-18}m(0.001GeV^{-1})$,which agrees with[11] and is well within the different theoretical and experimental limits of extra-dimension as summarised in Table.3. However, it is interesting to note that, the allowed size of extra-dimension obtained with linear potential[12] is several order small in magnitude ($\simeq 10^5$) than these values.

3.3. Graphical representation:

The variation of mass of D meson and B meson with size of finite extra-dimension is shown in Fig.1, 2, 3 and 4 respectively. From the graphs,it is clear that mass of mesons increases with size of extra-dimension, similar to the results obtained in the previous works[11,12].

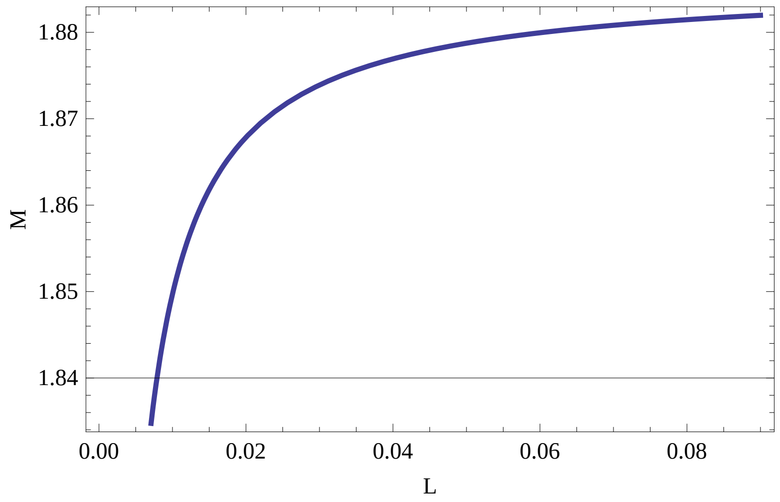

Figure 1: Mass vs size of extra-dimension of D meson(L in GeV^{-1},M in GeV)

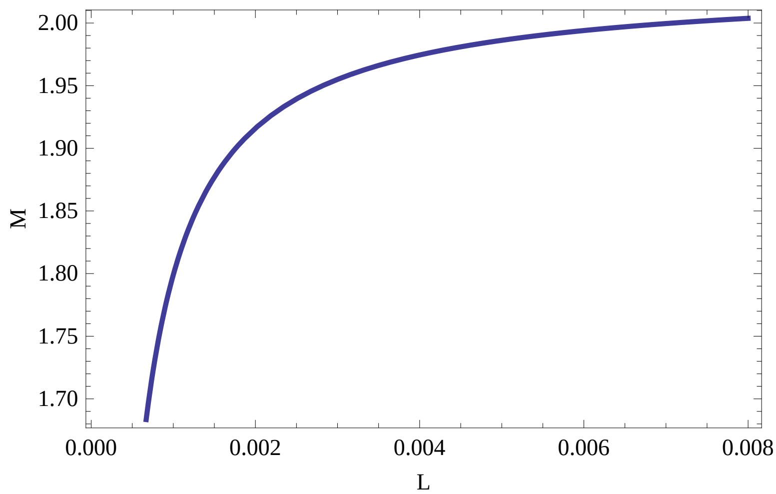

Figure 2: Mass vs size of extra-dimension of D_s meson(L in GeV^{-1},M in GeV)

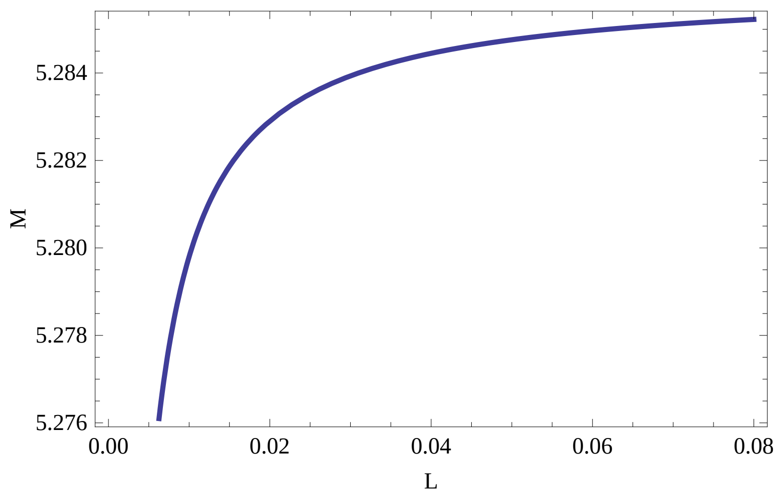

Figure 3: Mass vs size of extra-dimension of B meson (L in GeV^{-1},M in GeV)

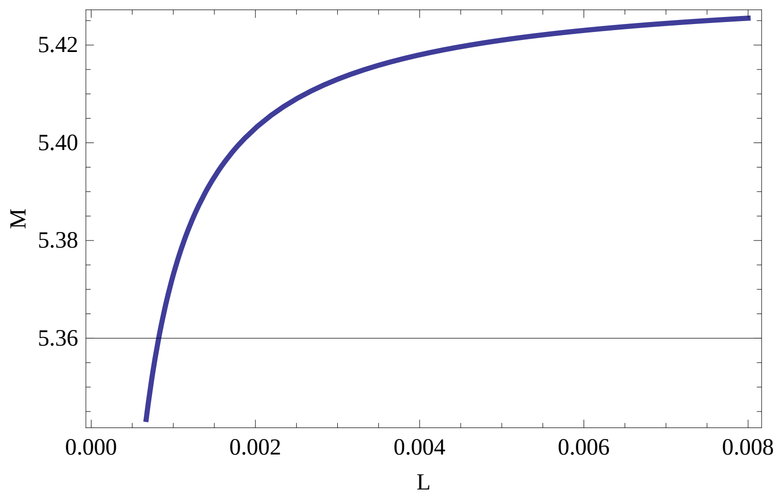

Figure 4: Mass vs size of extra-dimension of B_s meson (L in GeV^{-1},M in GeV)

4. Conclusion:

In this paper, we have considered heavy flavour mesons are stable in finite extra-dimension whose scale is less than the estimated QCD Bohr radii[11]. Then we consider linear plus inverse distance potential in a space with one finite extra-dimension and find the wave-function of the heavy flavour mesons in 4 spatial dimension(3 non-compact+1 compact) and used it to calculate their masses. Our results agrees well with experimental data. Comparison with experimental mass gives allowed range of upper bound on the size of extra-dimension as $\leq 10^{-19}m - 10^{-18}m()$. This is well within the different theoretical and experimental bounds on the size of extra-dimension as given in Table (3). It is also noted that in a space with one extra-dimension ,only for linear plus inverse distance potential the wave-function at the origin is well defined, while in the absence of inverse distance or linear potential, it is not so. The model can be generalised to take into account more than one finite extra-dimension as well, as has been suggested in more recent literature [34],[35],[36].

Let us now conclude the paper with a few comments:

In this work, we evaluate the masses of few heavy-light mesons with one finite extra-dimension and extracted bound on the size of extra-dimension by comparing with data on these mesons. The treatment of the extra-dimension is reduced to using an extra-coordinate which is required to be much smaller than the others and then compute the results using Schrodinger equation.

However, in this work we neglect the fact that the extra-dimensions might be compact and warped, having a definite curvature factor [8], [39], [41], [42]. Specifically,5-dimensional de-Sitter [] and anti de Sitter space[] having positive and negative curvature has got attention in current literature. Further, heavy quark theory for heavy mesons is not well approximated by a non-relativistic potential, Heavy quark effective theory [40] instead will make more sense. Both the above aspects are currently under study.

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