# Modeling and Analysis of Artificial Arm

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***Abstract*: A precise modeling and an analysis on an artificial arm have a very significant role in the control action applications. The control activities are useful to identify the system performance and design requirements. The efforts have been made by the researchers to initiate different approaches to direct the best effective model. The state-space analysis has been introduced to analyze different control performances such as controllability, observability and stability testing of the standard artificial arm model after producing the closed loop representation by the simulation approach using Jury stability and Lyapunov stability analysis. The control action analysis shows the relevancy and the precision of the proposed mathematical model.***

***Keywords*: System modeling, Control performance, The state space analysis, Jury stability, Lyapunov stability.**

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## I. INTRODUCTION

An establishment of a mathematical model is an essential requirement as per the applications of artificial arms [1]. An exact control mechanism is also an essential criterion to have the effective system performance. The design process can be made simplified with some approximation over the modelling[4]. A chart regarding different types of control system performance analysis is presented in Fig. 1. Here, this comparative analysis is established to signify the opted method for the betterment of the present discussion. The present approach, motivated by the research work of Salem [2], is selected for further analysis to apply on an Otto bock inspired Arm model shown in fig.2. The DC motor mathematical background is considered so that the control parameters of the actuator and the end effectors can be realized and related better. At first, closed loop model of geared DC motors is achieved by Proportional-Integral-Derivative (PID) tuning through Simulink considering an open loop transfer function for the angular positions of the motor for a standard robotic arm model [2]. The study of control parameters through closed loop model is set up to select the further methodology. The state-space analysis is performed to show the system controllability and observability. The primary objective is to determine the controllability and observability of composite systems which are formed by the interconnection of several multi-variable sub-systems. The discrete domain stability analysis is attempted by Jury testing in Jury Simulator for the system stability checking. Finally, Lyapunov stability method is established. This can extend to provide a strategy for constructing a stabilizing feedback controller.

In the work of Salman et al.[2], the error calculation for the required position of the robotic hand and the consumed time was discussed. Robotic arm control using discrete PID controller technology was conversed in the paper of Agrawal et al.[5]. In the study of SHEWALE S. et al.[7], the DC motor and the pulse width modulation control of DC motor performance is compared for a artificial gripper. The 3DOF articulated manipulator's dynamic modelling consist of actuator and link model was achieved to get accurate design of robust controller by Agbaraji et al[10].

In the present study, the contributions can be summarized as follows:

1. This work enlightens the path to choose control parameter range in which a system can be analysed to show the performance.

2. System modelling and simulation can create a resource place for designing a controlled and tuned system.

## II. PREVIEW OF PRE-REQUISITE OF AN ARTIFICIAL UPPER LIMB MODEL

The open loop transfer function of DC motor without load can be expressed as

$$G_{shaft\ angle}(s) = \frac{\theta(s)}{V(s)} = \frac{\rho}{\{(L_w J_m s^3 + (R_w J_m + b_m L_w)s^2 + (R_w b_m + \rho k_b)s\}}, \quad (1)$$

where, $\rho$ = torque constant,
$\theta(s)$ = output angle of motor shaft,
$V(s)$ = input voltage given,
$L_w$= inductance of the motor armature,
$J_m$ = moment of inertia of the motor,
$R_w$ = resistance of the motor armature,
$b_m$ = viscous damping constant,

$k_b$ = electromotive force constant

The robot arm system has some nominal parameters which are shown in a tabular form in Table 1 [2].

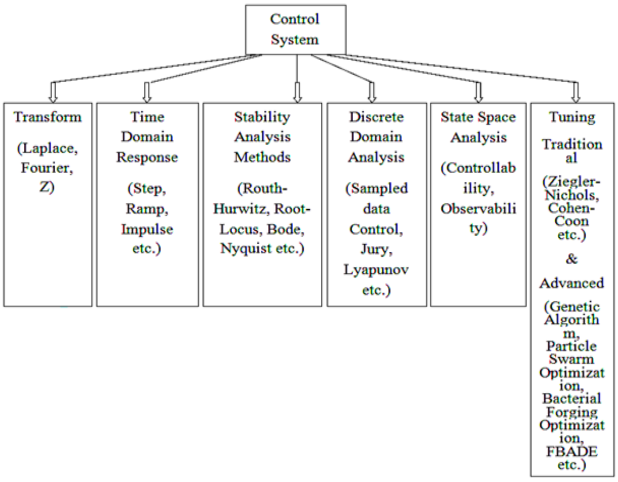


Fig. 1. Flow Diagram of different types of Control System Performance Analysis

TABLE I. NOMINAL PARAMETERS OF ROBOT ARM SYSTEM

| Arm Mass (M) | Arm Length (L) | Viscous Damping Constant (b) |
|---|---|---|
| 8 kg. | 0.4 m | 0.09 N.sec/m |

Other nominal values of electric DC motor on robotic arm system are given in a tabular form in Table 2 [2].

TABLE II. NOMINAL VALUES OF ELECTRIC MOTOR ON ROBOTIC ARM SYSTEM

| Input Voltage ($V_{in}$) | Moment of inertia of the motor ($J_m$) | viscous damping constant ($b_m$) | torque constant ($\rho$) | electromotive force constant ($k_b$) | resistance of the motor armature ($R_w$) | inductance of the motor armature ($L_w$) | Gear Ratio (n) |
|---|---|---|---|---|---|---|---|
| 12 volt | 0.02 kg./m 2 | 0.03 | 0.023 N-m/A | 0.023 V-s/radian | 1 ohm | 0.23 Henry | 1 |

## III. RETUNING AND PERFORMANCE STUDY ON STANDARD ARTIFICIAL ARM MODEL

The requirement of controller is to reduce the error by adjusting control variables. The most important reason behind a proportional-integral-derivative (PID) controller is due to its simple control structure and satisfactory results. As the constants of proportionality, integration, and derivation are very useful metrics in PID controller, therefore precise and finest control is possible by its applications only. At first estimated values of the constants can be taken significantly as per application, and then they are generally developed by monitoring the response of the system. Though the PID algorithm may not assure optimized control but it simply relies on the response obtained from the calculated process [3,4]. Here, an open loop transfer function [2] of an robotic arm-Load output followed by its angular position and this can be expressed as :

$$G(s) = \frac{0.023}{0.02913s^3 + 0.1543s^2 + 0.1205s}, (2)$$

Closed loop mathematical model and controlled output graph is formed by MATLAB shown in equation (3)

$$G_p(s) = \frac{0.023}{0.02913s^3 + 0.1543s^2 + 0.1205s + 0.023} \quad (3)$$

Where, Gp(s) = Process transfer function

Final closed loop Transfer Function = $\frac{G(s)}{1+G(s)H(s)}$ and as per control theory where, G(s) = System transfer function and H(s) = Unity feedback transfer function. Then the generated transfer function is presented in equation (4)

$$G(s) = \frac{230}{0.006s^3 + 30.78s^2 + 3905.29s} \quad (4)$$

Closed loop mathematical model and controlled output graph is formed by MATLAB shown in equation 5.

$$\boldsymbol{G(s) = \frac{230}{0.006s^3 + 30.78s^2 + 3905.29s + 230}} \quad \mathbf{(5)}$$

Where, Gp(s) = Process transfer function

Final closed loop Transfer Function = $\frac{G(s)}{1+G(s)H(s)}$ and as per control theory where, G(s) = System transfer function and H(s) = Unity feedback transfer function.

## IV. DISCRETE DOMAIN ANALYSIS OF STANDARD ARTIFICIAL ARM MODEL TOWARDS STABILITY

The stability of a linear discrete time system can be determined by Jury stability criterion. Computation requirements in Jury test are simpler than other techniques for physical systems with real co-efficient [8,9]. Here this technique is adopted to serve the purpose. In the beginning, the Z domain transfer function is established and in the continuation of the process, Jury criterion is tested through simulation.

Discrete-time transfer function of Zero order hold Transfer Function is given below in equation (6) where the sampling time is 0.8 seconds as per equation 12 & 27:

$$G(z) = \frac{0.1121z^3 + 0.05832z^2 + 0.09306z + 0.0746}{z^4 - 3.426z^3 + 4.539z^2 - 2.768z + 0.6546} \quad (6)$$

Jury Test is an algebraic test. For a polynomial,

If $F(1) > 0$ and $(-1)^n F(-1) > 0$ ,

Then the system in z domain is stable. Now, the sufficient conditions for stability are obtained by forming a table shown in Fig. 12 using Jury simulator. The sufficient conditions for stability are given by:

$$|a_0| < a_n$$

$$|b_0| > b_{n-1}$$

$$|c_0| > c_{n-2}$$

Through equation 7, 8 & 9 primaries Jury Stability Output has been demonstrated using Jury simulator [8].

$$F(1) = 0.33808, \quad (7)$$

$$(-1)^n F(-1) = 0.07224 \quad (8)$$

$$0.0746 < 0.1121 \quad (9)$$

Since, the above conditions are satisfied through Table 3, Jury Stability Output has been demonstrated with the generated Iteration table using Jury simulator.

TABLE III. JURY STABILITY OUTPUT ANALYSIS THROUGH JURY SIMULATOR WITH THE GENERATED ITERATION TABLE

| | $z^0$ | $z^1$ | $z^2$ | $z^3$ |
|---|---|---|---|---|
| $a$ | 0.0746 | 0.09306 | 0.05832 | 0.1121 |
| $a_n$ | 0.1121 | 0.05832 | 0.09306 | 0.0746 |
| $b$ | 0.007001 | 0.000405 | 0.006082 | |
| $b_n$ | 0.006082 | 0.000405 | 0.007001 | |
| $c$ | 0.000013 | 0.00000037 | | |
| $c_n$ | 0.00000037 | 0.000013 | | |

Through equation 10 & 11 final Jury Stability Output has been demonstrated using Jury simulator.

From Table 3 it is shown that $|b_0| > |b_2|$ (10)

$$|c_0| > |c_1| \quad (11)$$

Since, these conditions are satisfied. Therefore, the system can be declared as stable.

## V. LYAPUNOV STABILITY APPROACH OF STANDARD ARTIFICIAL ARM MODEL

In robotics, consistently asymptotically stable equilibria are a great concern. The theory of Lyapunov stability is a standard theory for non-linear systems and one of the most important mathematical tools in the analysis of non-linear systems. The method makes use of the system linearization to establish the original system stability. As the Jury test is successful to define the stability, the Lyapunov stability analysis is proposed hereafter [11].

The Controlled output is shown in equation (12)

$$G(s) = \frac{0.1034s^2 + 0.05163s + 0.03150}{0.2913s^4 + 0.1543s^3 + 0.1205s^2 + 0.023s} \quad (12)$$

Since, $e(s)G_c(s).G_p(s) = \mathrm{Y(s)}$ (13)

And $r(s) - y(s) = e(s)$ and assuming $r(s) = 0$, it can be stated that $y(s) = -e(s)$

Where, r(s) = reference input, y(s) = system output and e(s) = error value, $G_c(s)$ = Controller transfer function, $G_p(s)$ = Process transfer function.

Now, the mathematical model is shown in equation (14) according to equation (13).

$$e(s)(0.1034s^2 + 0.05163s + 0.03150) = -e(s)(s)(0.2913s^4 + 0.1543s^3 + 0.1205s^2 + 0.023s) \quad (14)$$

Or, $e(s)(0.2913s^4 + 0.1543s^3 + 0.2239s^2 + 0.07463s + 0.0315) = 0$ (15)

Using Inverse Laplace Transform, the transfer function is shown in equation (16)

$$\left(0.2913\ddddot{e} + 0.1543\dddot{e} + 0.2239\ddot{e} + 0.07463\dot{e} + 0.0315e\right) = 0 \quad (16)$$

Let, $x_1 = e, x_2 = \dot{e}, x_3 = \ddot{e}, x_4 = \dddot{e}, x_5 = \ddddot{e}$ (17)

$\dot{x_1} = x_2$ , $\dot{x_2} = x_3$ , $\dot{x_3} = x_4$ , $\dot{x_4} = x_5$ (18)

$$\dot{x_4}.a + \dot{x_3}.b + \dot{x_2}.c + \dot{x_1}.d + x_1.f = 0 \quad (19)$$

Here,

$$a = 0.2913, b = 0.1543, c = 0.2239,$$

$$d = 0.07463, f = 0.03150$$

Equation (19) can be written as

$$\dot{x_4} = -\{x_4.a_1 + x_3.b_1 + x_2.c_1 + x_1.d_1\} \quad (20)$$

Here,

$$a_1 = 0.5297, b_1 = 0.7686, c_1 = 0.2562, d_1 = 0.1081$$

Now taking scalar positive definite function is given by,

$$V(x) = \frac{1}{2}S_1.x_1^2 + \frac{1}{2}S_2.x_2^2 + \frac{1}{2}S_3.x_3^2 + \frac{1}{2}S_4.x_4^2 + \frac{1}{2}S_5.x_5^2 \quad (21)$$

Where, $S_1 > 0$, $S_2 > 0$

Now we take the derivative with respect to time t, yields

$$V(x) = S_1.x_1.\dot{x_1} + S_2.x_2.\dot{x_2} + S_2.x_3.\dot{x_3} + S_2.x_4.\dot{x_4} \quad (22)$$

$$= S_1.x_1.x_2 + S_2.x_2.x_3 + S_3.x_3.x_4 + S_4.x_4.x_5$$

$$= S_1.x_1.x_2 + S_2.x_2.x_3 + S_3.x_3.x_4 + S_4.x_4.(a_1x_4 + b_1x_3 + c_1x_2 + d_1x_1)$$

$= S_1 \,.\, x_1 \,.\, x_2 \; + S_2 \,.\, x_2 \,.\, x_3 + S_4 \,.c_1 \,.\, x_2 \,.\, x_4 \; + x_3 \,.\, x_4 \,.\, (S_3 \; - b_1 \,.S_4 \,) - S_4 \,.\, (a_1 \,.x_4^2 + d.x_1 \,.\, x_4 \;)$ (23)

For the positive definite function V we need another positive definite function U such that $\dot{V}(x) = -U(x)$

Now we take the coefficients in such a manner that

$\dot{V}(x) = -U(x)$

Taking $(S_3 - b \,.\, S_4\,) = 0$ (24)

And $a \,.\, S_4 \; = 0$ (25)

Therefore, $S_4 \; = 0$, $S_3 \; = 0$

Now, substituting equation (24 & 25) in equation (23) and also $\dot{V}(x) = 0$ , as from equation (22) $V(x) \to \infty$ as $x \to \infty$ and $\dot{V}(x) = 0$ A way of showing that $\dot{V}(x)$ being negative semi-definite is sufficient for asymptotic stability is to show that $x_1$ axis is not a trajectory of the system.

For $\dot{x_1} \; = x_2 = 0$ and $\dot{x_2} \; = x_3 = 0$ this shows that $x_1 = m$ (constant). The equilibrium state at the origin of the system is asymptotically stable.

Therefore, the mentioned system in this work is asymptotically stable.

## VI. GRAPHICAL RESULTS AND DISCUSSIONS

The main parameters of the simulation framework are set as per equation (3). In Fig. 2, amplitude of tuned response in regard to the discussed model is a function of time for plant and system, respectively where plant is referred as reference input of the system and sys is referred as the tuned output graph of equation 3.

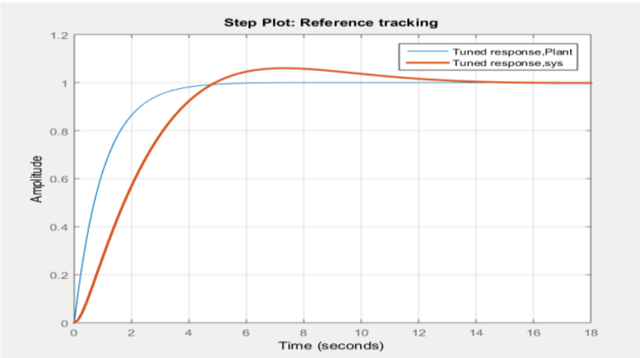


Fig. 2. FStep response of closed loop system with unity Feedback

Here, Proportional constant $(k_p)$ =2.245,

Integral constant $(k_i)$ =0.6102,

Derivative constant $(k_d)$ =2.002 for the output graph of controlled output with unity feedback is given in the Fig. 2.

From the above-mentioned graph, it can be shown that the transient response is good in terms of Performance and Robustness parameters which is the prior need to achieve a stable and an efficient system for better performance. The steady state error is zero also. Through PID controller, system performance can be presented with justified characteristics of the discussed system.

Performance and Robustness parameters of PID tuned transfer function are given in Table 4:

TABLE IV. TABLE 4: PERFORMANCE AND ROBUSTNESS PARAMETERS OF PID TUNED TRANSFER FUNCTION

| Rise Time | Settling Time | Overshoot | Peak | Gain Margin | Phase Margin |
|---|---|---|---|---|---|
| 3.32 seconds | 11.5 seconds | 6% | 1.06 | Inf dB @Inf radian/ second | 69.4 degree @0.453 radian/ second |

The input disturbance rejection analysis of the mathematical model in equation (3) is shown in Fig. 3. In this graph, the input disturbance has been rejected and compared with the reference signal.

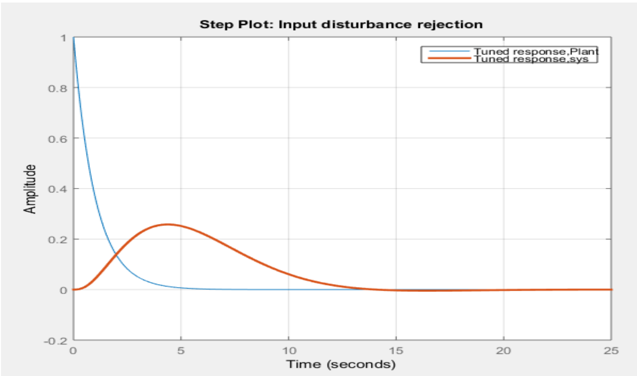


Fig. 3. Input disturbance rejection response

The output disturbance rejection analysis of the mathematical model in equation (3) is shown in Fig. 4. In this graph, the output disturbance has been rejected and compared with the reference signal.

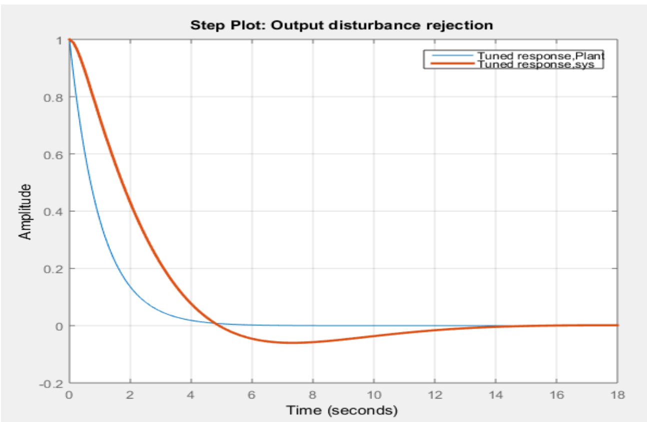


Fig. 5: Output disturbance rejection response

## VII. Controllability And Observability Testing of The Model

The concepts of controllability and observability were introduced by Kalman and have been employed primarily in the study of optimal control. Key advantages of the state-space approach were that a time-domain formulation exploited the advances in digital computer technology and the analysis and design methods were well-suited to multiple-input, multiple-output systems [9].

The PID equation is given below in equation (26) for the above mentioned Closed loop output shown in equation (3).

$$G_c(s) = k_p\left(1 + \frac{k_i}{s} + k_d.s\right)$$

$$= 2.245\left(1 + \frac{0.6102}{s} + 2.002s\right) \qquad (26)$$

where, $\mathrm{Gc}(s) =$ Controller transfer function

Now, the tuned output is shown in equation (27)

where, $\mathrm{G}(s) = \mathrm{G_c}(s).\mathrm{G_p}(s)$

$$G(s) = \frac{0.1034s^2 + 0.05163s + 0.03150}{0.2913s^4 + 0.1543s^3 + 0.1205s^2 + 0.023s} \qquad (27)$$

A dynamic system has to be controllable for further processing with control input. The system is identified as controllable if the states of the system can be configured by altering the system input [6].

State Space matrices are given below for tuned transfer function mentioned as equation (27):

$$A = \begin{bmatrix} -0.5297 & -0.4137 & -0.0790 & 0 \\ 1.0000 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix}$$

$$B = \begin{matrix} 1 \\ 0 \\ 0 \\ 0 \end{matrix}$$

$$C = 0 \quad 0.3550 \quad 0.1772 \quad 0.1081$$

$$D = 0$$

Now, controllability testing has been performed.

Rank of the controllability matrix is 4.

It is feasible that the output performance of the system is observed to establish the internal system states. This type of system is called observable system [6]. Observability testing has been performed. Rank of the observability matrix is 4.

The rank of both the controllability and observability is 4. So, it is controllable and observable also.

## VIII. Conclusion:

In this research work the closed loop modeling and retuning are performed with standard artificial arm model. The state space model is formed to achieve controllable and observable system with rank determination. Jury stability testing is attempted to establish the discrete aspect. Then Lyapunov stability process is done to show the asymptotically stable system. A significant area of discrete domain control system analysis has been incorporated in this work for better efficacy and digital presentation. This analysis will help to design the hardware model. In future, there are some other deterministic approaches which can be involved for control system analysis.

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