A Comparative study on Ferro-fluid Lubricated Porous Journal Bearing

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Abstract*: *This research considered hydrodynamic theory on ferro-fluid flow models such as R. E. Rosensweig model and Jenkins model for axially undefined journal bearing with porous attached at the inner surface (i.e. on the journal). In this study, the variable external magnetic field is assumed. The expressions of (non-dimensional) pressure and load-carrying capacity are obtained. The values of load capacity for both models are calculated for various parameters like porous thickness, slip velocity, squeeze velocity, permeability, and eccentricity. Also, calculated load capacity by considering porous structure models like the globular sphere model and capillary fissures model. Based on the obtained results, the globular sphere model gives much better performance for load capacity rather than the capillary fissures model. From the study, it is suggested that to design the journal bearing, globular sphere model of porous structure should be preferred over the capillary fissures model. Also, for the better performance of the journal bearing, the choice of fluid-flow model depends on the values of parameters.

***Keywords*: Magnetic Fluid, Journal Bearing, Neuringer-Rosensweig Model, Jenkins Model.**

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I. INTRODUCTION

In machine mechanism point of view, where vibrations are there like engines of motor vehicles, turbines, printers, gear, pumps, etc., hydrodynamic journal bearings are broadly used. In all such types of machinery, there is a circular shaft (inner solid reference of bearing) known as a journal (shaft) sometimes called a shafting, which rotates inside a circular outer. It is notable that most journal bearings have common applications that use a lubricant fluid to diminish the friction between the moving surfaces. During the operational mode, smooth conduction of bearing and proper lubrication is required for the bearing. This improves the life of bearings and enhances the performance.

The magnetic fluid is made with fine iron-oxide particles covered with surfactant and added to the base fluid. These fluids can be placed, designed, and controlled at a specific location that prompts the use of magnetic fluid in the lubrication of bearings. Many authors [3-9] have considered magnetic fluid as a lubricant in their research on bearing problems.

Neuringer and Rosensweig [3] have given a straightforward model to explain the flow of magnetic fluids in the occurrence of fluctuating external magnetic fields in 1964. By considering various types of bearings, many authors have done a good amount of research by considering Neuringer-Rosensweig model (NR Model). In view of Maugin's [1] modifications, Jenkins [4] modified the NR model flow model in 1972. It was concluded that the Jenkins model (JE Model) modifies fluid velocity and pressure both whereas NR model modifies pressure only. Using JE Model, Ram and Verma [7] studied a porous inclined slider bearing and generalized the analysis done by Agrawal [9] in the case of the NR model.

Up to this, no one has taken the slip condition into consideration. Beavers and Joseph [11] and sparrow [21] found that such a condition was false at the minimal edge of porous material. Prakash and Vij [10] analyzed the performance of journal bearings, and their analysis considered the velocity slip at the surface of the porous medium with Beavers and Joseph [11] criterion.

Many authors [16-18] have done their studies on magnetic fluid lubricated journal bearing problems, but none of the authors has attached the porous layer on the journal (shaft) which means the inner surface of the bearing.

The aim of the study is to carried out the comparison between the NR Model and the JE Model in the case of axially undefined journal bearing by using a magnetic fluid as a lubricant. The expressions of pressure and load-carrying capacity are obtained for axially undefined journal bearing by using a magnetic fluid as a lubricant in which various parameters have been studied and concluded which model is more suitable in such cases. Also, various porous structure models such as a globular sphere model [19] (introduced by Kozeny-Carman) and a capillary fissures model [20] (Irmay) are taken in to consideration for the study. The Reynolds's type equation is derived. For the Bearing under study, dimensionless load-carrying capacity is calculated for its extreme performance.

II. Flow Models for magnetic fluids

Neuringer and Rosensweig [3] have given a straightforward model to explain the flow of magnetic fluids in the existence of changing external magnetic fields in 1964. The basic flow equations of NR Model [3] are:

$$\rho[\bar{q}\bullet\nabla]\bar{q}=-\nabla p+\eta\nabla^2\bar{q}+\mu_0(\bar{M}\bullet\nabla)\bar{H}, \quad (1)$$

$$\nabla\bullet\bar{q}=0, \quad (2)$$

$$\nabla\times\bar{H}=0, \quad (3)$$

$$\nabla\bullet(\bar{H}+\bar{M})=0, \quad (4)$$

$$\bar{M}=\bar{\mu}\bar{H}, \quad (5)$$

where ρ is the density of fluid, $\bar{q}$ is the fluid velocity vector in film region, p is film pressure, η is viscosity of fluid, μ_0 is free space permeability, $\bar{M}$ is the magnetization vector, $\bar{H}$ is the applied magnetic field, $\bar{\mu}$is magnetic susceptibility of the fluid. In 1972, Jenkins modified the NR flow Model and presented the term which depends on material parameter α^2(the SI unit of α^2 is$m^3s^{-1}A^{-1}$) and gave rise to a force

$$\left(-\frac{\rho\alpha^2\bar{\mu}}{2}\frac{\partial}{\partial z}\left(H\frac{\partial u}{\partial z}\right),0,\frac{\rho\alpha^2\bar{\mu}}{2}\frac{\partial}{\partial x}\left(H\frac{\partial u}{\partial z}\right)\right), \quad (6)$$

in the domain $0\le x\le 2\pi$ (i.e., in the film region.), where $H$is magnitude of $\bar{H}$ and u is fluid velocity component of $\bar{q}$in the directions of x-direction.

Due to the force as in (6), the modified NR Model is known as JE Model and flow equations (1), (4) and (5) becomes (Jenkins [4] and Ram and Verma [7]):

$$\rho[\bar{q}\bullet\nabla]\bar{q}=-\nabla p+\eta\nabla^2\bar{q}+\mu_0(\bar{M}\bullet\nabla)\bar{H}+\rho\alpha^2\nabla\times\left(\frac{\bar{M}}{M}\times\bar{M}^*\right), \quad (7)$$

$$\nabla\bullet(\bar{H}+4\pi\bar{M})=0, \quad (8)$$

$$\bar{M}=\bar{\mu}H, \quad (9)$$

and

$$\bar{M}^*=\frac{1}{2}(\nabla\times\bar{q})\times\bar{M}, \quad (10)$$

III. ANALYSIS

There are various types of journal bearings used in machines. One of the types of such bearing system shown in Fig. 1. The cross-segment of Fig. 1 is presented in Fig. 2, which is the configurations of journal bearing taken for this study. It is considered that R is the radius of journal, l is uniform thickness of attached porous layer with journal. Choose the geometry by taking origin O as shown in Fig. 2, the x-axis along with circumference and z-axis is perpendicular to it. The slip velocity s is generated between film region and porous layer. Also $\dot{h}$ is squeeze velocity which is generated during the operating mode (*i.e.*, when outer surface approaches to the journal). Fig. 3 shows the bearing is opened up at the origin O as shown in Fig. 2. The journal circumference is on the θ-axis which lies on $[0,2\pi]$.

Fig. 1. Journal Bearing

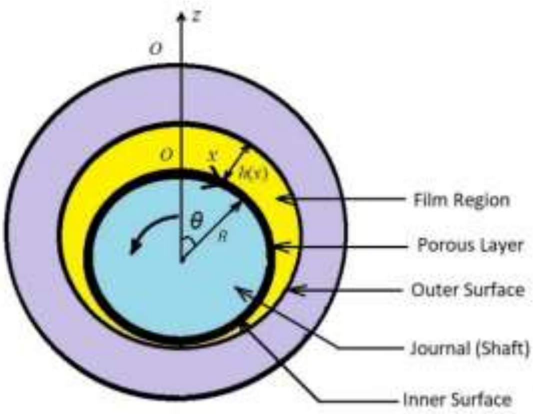

Fig. 2. Configurations of journal bearing

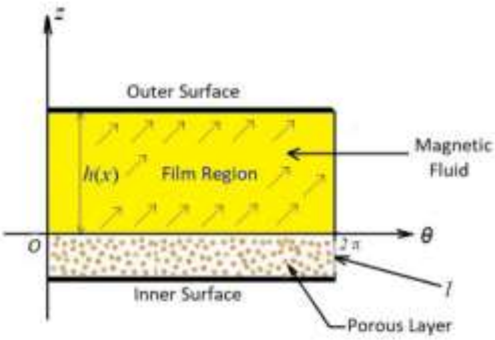

Fig. 3. Journal bearing opened up at O

Making use of the conventions of hydrodynamic lubrication and the flow is steady and axially symmetric, (2), (3), (7)-(10) take the form

$$\frac{\partial^2 u}{\partial z^2}=\frac{1}{\eta\left(1-\frac{\rho\alpha^2\bar{\mu}H}{2\eta}\right)}\frac{d}{dx}\left(p-\frac{1}{2}\mu_0\bar{\mu}H^2\right), \quad (11)$$

where expression of H is

$$H^2=Kx(2\pi R-x), \quad (12)$$

where K is a quantity such that the dimensions of (12) and the field strength. φ_x and $\eta_x$denotes the permeability in lower porous matrix in x-direction and the porosity of the lower porous region in x-direction respectively. Solving (11) under the boundary conditions [8]

$$u = \frac{1}{s}\frac{\partial u}{\partial z}, \ \ when\ z = 0$$

$$and$$

$$u = 0, when\ z = h,$$

where h is film thickness, s is a slip defined by $\frac{1}{s} = \frac{\sqrt{\varphi_x \eta_x}}{5}$, one obtains

$$u = \frac{(1+hs)z^2 - szh^2 - h^2}{2\eta(1+hs)\left(1-\frac{\rho\alpha^2\bar{\mu}H}{2\eta}\right)}\frac{\partial}{\partial x}\left(p - \frac{1}{2}\mu_0\bar{\mu}H^2\right) \tag{13}$$

The velocity components of the fluid in the porous region in x-direction and z-direction are

$$\bar{u} = -\frac{\varphi_x}{\eta}\left[\frac{\partial}{\partial x}\left(P - \frac{1}{2}\mu_0\bar{\mu}H^2\right) + \frac{\rho\alpha^2\bar{\mu}}{2}\frac{\partial}{\partial z}\left(H\frac{\partial u}{\partial z}\right)\right] \tag{14}$$

and

$$\bar{w} = -\frac{\varphi_z}{\eta}\left[\frac{\partial}{\partial z}\left(P - \frac{1}{2}\mu_0\bar{\mu}H^2\right) - \frac{\rho\alpha^2\bar{\mu}}{2}\frac{\partial}{\partial x}\left(H\frac{\partial u}{\partial z}\right)\right] \tag{15}$$

respectively. In (14) and (15), P denotes fluid pressure in both the porous region and in (15), φ_z represents permeability in lower porous matrix in z – direction. Now the continuity equation in porous matrix is

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{w}}{\partial z} = 0. \tag{16}$$

By using (14)-(15) and integrating (16) with respect to z over $(-l, 0)$ yields

$$\varphi_x l\frac{\partial^2}{\partial x^2}\left(P - \frac{1}{2}\mu_0\bar{\mu}H^2\right) + \varphi_z\frac{\partial}{\partial z}\left(P - \frac{1}{2}\mu_0\bar{\mu}H^2\right)\bigg|_{z=0} + (\varphi_x - \varphi_z)\frac{\rho\alpha^2\bar{\mu}}{2}\frac{\partial}{\partial x}\left(H\frac{\partial u}{\partial z}\right)\bigg|_{z=-l}^{z=0} = 0 \tag{17}$$

by Morgan–Cameron approximation [10] and the surface is solid at $z = -l$. Equation (17) is equivalent to

$$\frac{\partial}{\partial z}\left(P - \frac{1}{2}\mu_0\bar{\mu}H^2\right)\bigg|_{z=0} = -\frac{\varphi_x l}{\varphi_z}\frac{\partial^2}{\partial x^2}\left(P - \frac{1}{2}\mu_0\bar{\mu}H^2\right) - \frac{(\varphi_x - \varphi_z)\rho\alpha^2\bar{\mu}}{2\varphi_z}\frac{\partial}{\partial x}\left(H\frac{\partial u}{\partial z}\right)\bigg|_{z=-l}^{z=0} \tag{18}$$

The film thickness is $h = c(1 + \varepsilon cos\theta)$, where ε and c being the eccentricity ratio and the radial clearance respectively. The derivative of h with respect to time t is known as squeeze velocity and is $\dot{h} = c\dot{\varepsilon}cos\theta$, where $\dot{\varepsilon} = \frac{d\varepsilon}{dt}$. As the fluid velocity components are continuous across the surface $z = 0$, $w|_{z=0} = \dot{h} - \bar{w}|_{z=0}$.

Hence

$$w|_{z=0} = \dot{h} + \frac{\varphi_z}{\eta}\frac{\partial}{\partial z}\left(P - \frac{1}{2}\mu_0\bar{\mu}H^2\right)\bigg|_{z=0} - \frac{\varphi_z\rho\alpha^2\bar{\mu}}{2\eta}\frac{\partial}{\partial x}\left(\frac{-sh^2H}{2\eta(1+hs)\left(1-\frac{\rho\alpha^2\bar{\mu}H}{2\eta}\right)}\frac{\partial}{\partial x}\left(p - \frac{1}{2}\mu_0\bar{\mu}H^2\right)\right) \tag{19}$$

The integral form of continuity equation over the film region $(0, h)$ is

$$\frac{\partial}{\partial x}\int_0^h u dz + w_h - w_0 = 0. \tag{20}$$

Here $w_h = 0$ due to solid surface at $z = h$. Therefore (20) rewrites as

$$\frac{\partial}{\partial x}\int_0^h u dz = w_0 \tag{21}$$

Substituting the expressions as in (13) and (19) in (21), one obtains the Reynold's Type equation as

$$\frac{d}{d\theta}\left[g\frac{d}{d\theta}\left(p - \frac{1}{2}\mu_0\bar{\mu}H^2\right)\right] = -12\eta R^2 c\dot{\varepsilon}cos\theta, \tag{22}$$

using $x = R\theta$, where

$$g = -12\varphi_x l + \frac{(4+hs)h^3 + \left(3s\rho\alpha^2\bar{\mu}\varphi_z h^2 H/\eta\right)}{(1+hs)\left(1 - \frac{\rho\alpha^2\bar{\mu}H}{2\eta}\right)} - \frac{6\rho\alpha^2\bar{\mu}(\varphi_x - \varphi_z)lh}{\eta\left(1 - \frac{\rho\alpha^2\bar{\mu}H}{2\eta}\right)}$$

Using the dimension less quantities

$$\psi_x = -\frac{\varphi_x l}{c^3}, \ \ \bar{s} = sc, \ \ \beta = \frac{\rho\alpha^2\bar{\mu}\sqrt{K}R}{2\eta}, \ \ \ \gamma^* = \frac{6\varphi_z}{c^2},$$

$$\psi_z = \frac{\varphi_z l}{c^3}, \ \ \bar{h} = \frac{h}{c}, \ \ \mu^* = \frac{K\mu_0\bar{\mu}c^2}{2\eta\dot{\varepsilon}}, \ \bar{p} = \frac{c^2 p}{\eta R^2\dot{\varepsilon}} \tag{23}$$

and using (12), (22) can be written as

$$\frac{d}{d\theta}\left[G\frac{d}{d\theta}\left(\bar{p} - \mu^*\theta(2\pi - \theta)\right)\right] = -12cos\theta, \tag{24}$$

where

$$G = \frac{-12\left(\psi_x - \beta\psi_z\sqrt{\theta(2\pi - \theta)}\right)}{1 - \beta\sqrt{\theta(2\pi - \theta)}} + \frac{\bar{h}^3(4 + \bar{h}\bar{s}) + \left(\bar{s}\beta\gamma^*\bar{h}^2\sqrt{\theta(2\pi - \theta)}\right)}{(1 + \bar{h}\bar{s})\left(1 - \beta\sqrt{\theta(2\pi - \theta)}\right)}.$$

Equation (24) is known as the non-dimensional form of Reynolds type equation.

IV. Solution

Solving (24) under the pressure boundary conditions

$\frac{d\bar{p}}{d\theta} = 0$, when $\theta = \pi$

and

$\bar{p} = 0$, when $\theta = 0$, (25)

one obtains the expression of non-dimensional film pressure $\bar{p}$ as

$$\bar{p} = \mu^*(2\pi\theta - \theta^2) - 12\int_0^\theta \frac{\sin\theta}{G} d\theta. \quad (26)$$

Now $W_x = LR\int_0^{2\pi} p\sin\theta d\theta$ and $W_z = LR\int_0^{2\pi} p\cos\theta d\theta$ are the components of load capacity in x and z-directions respectively, where L is the length of bearing. Here $W_x = 0$ as pressure is generated only in the direction of z-axis. Hence the average load carrying capacity $W = \sqrt{{W_x}^2 + {W_z}^2}$ is $W = W_z = LR\int_0^{2\pi} p\cos\theta d\theta$.

The non-dimensional form of W can be expressed as

$$\bar{W} = \frac{c^2 W}{\eta l R^3 \dot{\varepsilon}} = 4\pi\mu^* + 12\int_0^{2\pi} \frac{\sin^2\theta}{G} d\theta \quad (27)$$

V. Results and Discussions

Using Simpson's one third Rule with step size $11/14$ and for water based magnetic fluid having density $\rho = 1400\ kg/m^3$ and viscosity $\eta = 0.012\ kg/ms$ and for fixed values of $c = 2.5 \times 0.00001$, $\bar{\mu} = 0.05$, $\eta_x = 0.81$, $\alpha^2 = 0.001, \varepsilon = 0.3, \dot{\varepsilon} = 0.02, \varphi_z = 10^{-10}, l = 0.002$, the length of bearing $L = 2\pi R = 0.05\ m$, the results for non-dimensional load carrying capacity (using (27)) are computed which are reflected in tables below. Moreover, when $K = 0$; FF effect without magnetic field and $K \neq 0$; FF effect with applied magnetic field. Similarly, $\alpha^2 = 0$; represents NR model and $\alpha^2 = 0.001$ represents JE model.

Many authors [12-15] concluded in their study that better performance is observed when Ferro-fluid is used as a lubricant in bearing systems in presence of applied magnetic field effect.

TABLE-1: $\bar{W}$ in both models for various values of slip parameter s and magnetization parameter K

s→	5.56×10^4	$5.56 \times 10^{4.5}$	5.56×10^5	$5.56 \times 10^{5.5}$	5.56×10^6	K↓
NR	0.0025578	0.0246991	0.2494422	2.7605813	27.9920845	10^8
JE	0.2950114	0.2488331	0.2446306	0.2439117	0.2436991	
NR	0.0127441	0.0348853	0.2596284	2.7707675	28.0022717	10^{10}
JE	0.2597015	0.2556163	0.2548245	0.2544377	0.2542568	
NR	1.0313665	1.0535077	1.2782509	3.7893898	29.0208931	10^{12}
JE	1.2745618	1.2739044	1.2734478	1.2730941	1.2729164	
NR	102.8936081	102.9157486	103.1404953	105.6516342	130.8831329	10^{14}
JE	103.1364365	103.136116	103.1356888	103.1353378	103.1351624	

- As shown in table-1, the data confirms that when the slip parameter is $s < 5.56 \times 10^5$, the values of $\bar{W}$ are better in the JE Model as compared to NR model. But when $s > 5.56 \times 10^5$, NR model should be preferred over JE model. Also in NR model, $\bar{W}$ is incresed with the increase of slip, but there is reverse trend in case of JE model.
- In both the models, increase in magnetization parameter K results in to increase in $\bar{W}$.
- It is clear from the table-1, that when we take the value of $K = 10^{12}$ and $K = 10^{14}$, there is high jump in the values of $\bar{W}$ in both the models.

TABLE-**2**: $\bar{W}$ for various values of porous thickness l and magnetization parameter K for fixed value of $s = 5.56 \times 10^5$

l→ K↓	0.001	0.002	0.004	0.008	0.016	Model Type
0	0.5067978	0.2493393	0.1236837	0.0615988	0.0307391	NR
10^8	0.5069007	0.2494422	0.1237865	0.0617017	0.030842	
10^{10}	0.5170869	0.2596284	0.1339728	0.0718879	0.0410282	

10^{12}	1.5357094	1.2782509	1.1525953	1.0905104	1.0596507	
10^{14}	103.397949	103.140495	103.014839	102.9527512	102.92189	
0	0.4873634	0.2446306	0.1225924	0.0614042	0.0307677	JE
10^{8}	0.4975799	0.2548245	0.1327806	0.071591	0.0409541	
10^{10}	1.5162054	1.2734478	1.1514032	1.0902134	1.0595765	
10^{12}	1.5162054	1.2734478	1.1514032	1.0902134	1.0595765	
10^{14}	103.378449	103.135689	103.013649	102.9524536	102.921822	

As it is shown in table-2, for the fixed value of $s = 5.56 \times 10^5$, same scenario is found in both the models. i.e., the $\bar{W}$ increase with the decrease in uniform porous thickness l. Moreover, the drastic change can be seen in $\bar{W}$ when the magnetization parameter $K = 10^{14}$.

TABLE- 3: $\bar{W}$for various values of porous thickness l and slip parameter s for fixed value of magnetization parameter $K = 10^{12}$

s→ l↓	5.56×10^4	$5.56 \times 10^{4.5}$	5.56×10^5	$5.56 \times 10^{5.5}$	5.56×10^6	Model Type
0.001	1.0338228	1.0782104	1.5357094	7.3940372	42.6111832	NR
0.002	1.0313665	1.0535077	1.2782509	3.7893898	29.0208931	
0.004	1.0301389	1.0411963	1.1525953	2.3271511	25.4978104	
0.008	1.0295252	1.0350506	1.0905104	1.6596582	8.9325218	
0.016	1.0292183	1.0319803	1.0596507	1.3399222	4.4691591	
0.001	1.5198261	1.5179479	1.5162054	1.5148108	1.5141087	JE
0.002	1.2745618	1.2739044	1.2734478	1.2730941	1.2729164	
0.004	1.151785	1.151527	1.1514032	1.1513135	1.1512687	
0.008	1.0903604	1.0902492	1.0902134	1.0901905	1.0901792	
0.016	1.059639	1.0595878	1.0595765	1.0595706	1.0595677	

From table-3, it is clear that when there is decrease in porous thickness, the value of $\bar{W}$ is increased in both the models.Also, when the value of slip parameter$s < 5.56 \times 10^5$, JE model gives better results as compared to NR model and when$s > 5.56 \times 10^5$, NR model gives better results as compared to JE model.

TABLE-4: $\bar{W}$for various values of $\dot{\varepsilon}$ and magnetization parameter K for fixed value of $s = 5.56 \times 10^5$

$\dot{\varepsilon}$ → K↓	0.2	0.02	0.002	0.0002	Model Type
0	0.2493393	0.2493393	0.2493393	0.2493393	NR
10^{8}	0.2493496	0.2494422	0.2503682	0.2596284	
10^{10}	0.2503682	0.2596284	0.3522305	1.2782509	
10^{12}	0.3522305	1.2782509	10.5384550	103.1404953	
10^{14}	10.5384550	103.1404953	1029.1608887	10289.3652344	
0	0.2493393	0.2493393	0.2493393	0.2493393	JE
10^{8}	0.244538	0.2446306	0.2455566	0.2548168	
10^{10}	0.2455643	0.2548245	0.3474265	1.2734469	
10^{12}	0.3474273	1.2734478	10.5336514	103.1356888	
10^{14}	10.5336514	103.1356888	1029.156128	10289.36035	

From table-4, it can be seen that when the value of $\dot{\varepsilon}$ decreases, the load carrying capacity increases in both models. However, when the value of K is more than 10^{12}, there is sudden high jump in load carrying capacity.

TABLE-5: $\overline{W}$ for various values of $\dot{\boldsymbol{\varepsilon}}$ and slip parameter s for fixed $K = 10^{12}$

s→ $\dot{\varepsilon}$↓	5.56×10^4	$5.56 \times 10^{4.5}$	5.56×10^5	$5.56 \times 10^{5.5}$	5.56×10^6	Model Type
0.2	0.1053461	0.1274873	0.3522305	2.8633695	28.0948734	NR
0.02	1.0313665	1.0535077	1.2782509	3.7893898	29.0208931	
0.002	10.2915707	10.3137112	10.5384550	13.0495939	38.2810974	
0.0002	102.8936081	102.9157486	103.1404953	105.6516342	130.8831329	
0.2	0.3485414	0.3478841	0.3474273	0.3470736	0.346896	JE
0.02	1.2745618	1.2739044	1.2734478	1.2730941	1.2729164	
0.002	10.5347652	10.5341082	10.5336514	10.5332975	10.5331202	
0.0002	103.1368027	103.1361465	103.1356888	103.1353378	103.1351547	

Table-5 shows that when the value of slip parameter s increases, the load carrying capacity increases in case of NR model. But there is an opposite scenario in case of JE model. Moreover, when the value of $\dot{\varepsilon}$ is less than 0.002, there is sudden high jump in load carrying capacity in both the models.

TABLE-6: $\overline{W}$ for various values of φ_z and magnetization parameter K for fixed value of $s = 5.56 \times 10^5$

φ_z→ K↓	10^{-8}	10^{-9}	10^{-10}	10^{-11}	10^{-12}	Model Type
0	0.2493393	0.2493393	0.2493393	0.2493393	0.2493393	NR
10^8	0.2494422	0.2494422	0.2494422	0.2494422	0.2494422	
10^{10}	0.2596284	0.2596284	0.2596284	0.2596284	0.2596284	
10^{12}	1.2782509	1.2782509	1.2782509	1.2782509	1.2782509	
10^{14}	103.1404953	103.1404953	103.1404953	103.1404953	103.1404953	
0	0.2493393	0.2493393	0.2493393	0.2493393	0.2493393	JE
10^8	0.0025442	0.0245191	0.2446306	2.4824531	29.2859936	
10^{10}	0.0127341	0.034739	0.2548245	2.4592919	24.8716373	
10^{12}	1.0313569	1.0533649	1.2734478	3.4746382	25.5226669	
10^{14}	102.8936005	102.9156036	103.1356888	105.3365555	127.348793	

Tabular values shown in table-6 confirms that change in φ_z does not affect the load carrying capacity in NR model, whereas the better load carrying capacity is obtained in case of JE Model according to increase in φ_z.

TABLE-7: $\overline{W}$ for various values of φ_z and slip parameter s for fixed $K = 10^{12}$

s→ φ_z↓	5.56×10^4	$5.56 \times 10^{4.5}$	5.56×10^5	$5.56 \times 10^{5.5}$	5.56×10^6	Model Type
10^{-8}	1.0313665	1.0535077	1.2782509	3.7893898	29.0208931	NR
10^{-9}	1.0313665	1.0535077	1.2782509	3.7893898	29.0208931	
10^{-10}	1.0313665	1.0535077	1.2782509	3.7893898	29.0208931	
10^{-11}	1.0313665	1.0535077	1.2782509	3.7893898	29.0208931	
10^{-12}	1.0313665	1.0535077	1.2782509	3.7893898	29.0208931	
10^{-8}	1.031364	1.0313611	1.0313569	1.0313534	1.0313516	JE
10^{-9}	1.0534393	1.0534072	1.0533649	1.0533298	1.0533121	
10^{-10}	1.2745618	1.2739044	1.2734478	1.2730941	1.2729164	
10^{-11}	3.5234149	3.4825563	3.4746382	3.4707692	3.4689603	
10^{-12}	30.5694904	25.9436493	25.5226669	25.4507637	25.4294319	

Tabular values shown in table-7 confirms that in case of JE Model, when there is increase in values of φ_z, better $\overline{W}$ is obtained. Also, in JE model, when the value of φ_z is more than or equal to 10^{-11}, the better load carrying capacity is obtained as compared to NR Model.

DISCUSSIONS ON POROUS STRUCTURES

The Globular Sphere Model (refer Fig. 5) proposed by Kozeny-Carman [19], in which the permeability of the assorted porous matrix is described as

$$k = \frac{{D_c}^2\zeta^3}{180(1-\zeta)^2},$$

where D_c denotes the mean particle size and ζ denotes the porosity of the porous matrix.

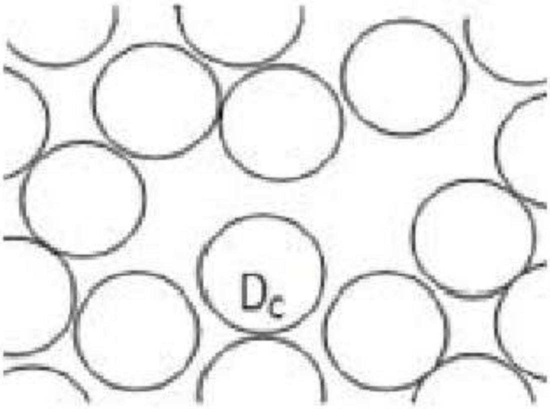

Fig. 4 Structure of Globular Sphere Model

Another Model namely capillary fissures model (refer Fig. 5)proposed by Irmay [19, 20], in which the permeability of the assorted porous matrix is described as

$$k = \frac{\left(1-m^{2/3}\right){D_S}^2}{12m},$$

where D_s denotes the average diameter particle size and $m = 1 - \zeta$. Both the Models are well described in [22].

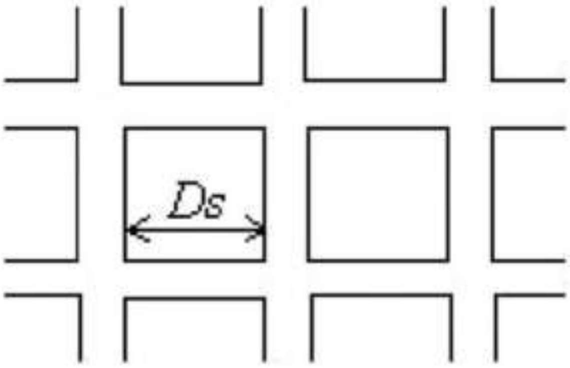

Fig. 5 Structure of Capillary Fissures Model

The following results and discussion are for the above discussed porous structures.

TABLE-8: $\bar{W}$ for fixed value of $K = 10^{12}$ and $D_c = 0.001$ for Globular Sphere Model

Flow Model	$\zeta = 0.1$	$\zeta = 0.2$	$\zeta = 0.3$	$\zeta = 0.4$	$\zeta = 0.5$
NR Model	109.7494507	103.7930145	103.231987	103.085495	103.0340881
JE Model	108.3157425	103.7501068	103.2237778	103.0822296	103.0320587

As per the tabulated values in Table-8, it is clear that for fixed $K = 10^{12}$ and $D_c = 0.001$, better $\bar{W}$ is obtained when ζ decreases.

TABLE-9: $\bar{W}$ for fixed value of of $K = 10^{12}$ and $D_s = 0.001$ for Capillary fissures Model

Flow Model	$\zeta = 0.1$	$\zeta = 0.2$	$\zeta = 0.3$	$\zeta = 0.4$	$\zeta = 0.5$
NR Model	102.8959808	102.8938141	102.8931427	102.8928528	102.892746
JE Model	102.8959732	102.8938141	102.8931427	102.8928528	102.892746

Tabular values in Table-9 confirms that for fixed $K = 10^{12}$ and $D_s = 0.001$, there is no much change in $\overline{W}$ is observed with the change in ζ. Also, in comparison point of view, better $\overline{W}$ is obtained in case of the Globular Sphere Model rather than the Capillary fissures Model (from Table-8 and Table-9).

TABLE-10: $\overline{W}$ for fixed value of of $K = 10^{12}$ and $\zeta = 0.1$ for Globular Sphere Model

Flow Model	$D_c = 0.1$	$D_c = 0.01$	$D_c = 0.001$
NR Model	102.8917007	102.9459152	109.7494507
JE Model	102.8917007	102.9455566	108.3157425

As per the results shown in Table-10 one can see that, $\overline{W}$ increases with decrease in D_c. Also, one can observe sudden increase in $\overline{W}$ after $D_c = 0.01$ for Globular Sphere Model.

TABLE-11: $\overline{W}$ for fixed value of of $K = 10^{12}$ and $\zeta = 0.1$ for Capillary fissures Model

Flow Model	$D_s = 0.1$	$D_s = 0.01$	$D_s = 0.001$
NR Model	102.8917007	102.9459152	102.8959808
JE Model	102.8911514	102.8912048	102.8959732

As per the Table-11 results it is clear that, no much change in $\overline{W}$observed as decrease in D_s when Capillary fissures Model is considered. Also, from Table-10 and Table-11, one can observe that for very small size of solid particles gives better performance when the Globular Sphere Model is considered as compared to Capillary fissures Model.

VI. CONCLUSIONS

The present study deliberates NR model and JE model for magnetic fluid flow by aspect of hydrodynamic theory of lubrication for axially undefined porous journal bearing having porous attached on the outer surface of journal. The results of non-dimensional load carrying capacity are shown for various parameters like slip velocity, porous thickness, squeeze velocity, eccentricity, permeability in tables. Based on the results and discussions, the following conclusions can be made:

- In case of NR model, better $\overline{W}$ is obtained as soon as the value of slip starts with 5.56×10^5.
- $\overline{W}$ is much better in both the models once magnetic field parameter K is more than 10^{12}.
- $\overline{W}$ is better observed in case of the uniform porous layer thickness l is smaller.
- $\overline{W}$is better obtained when squeeze velocity $\dot{\varepsilon}$ is decreased in both the models and it is observed that when $\dot{\varepsilon} = 0.002$, $\overline{W}$ is rapidly increased.
- Increase in $\varphi_z$results increase in $\overline{W}$ in case of JE model and to get better $\overline{W}$, one should choose $\varphi_z \geq 10^{-11}$.

In the presence of applied (variable) external magnetic field (having magnetic field parameter $K \geq 10^{12}$), this study concludes that, to design a magnetic fluid lubricated axially undefined porous journal bearing with porous layer of smaller thickness attached on journal havingsmallest amount of squeeze velocity,NR model is more preferable over JE model with $s = 5.56 \times 10^5$or more.If one wants to consider the JE model to design such bearing, the values of parameters should be taken in to consideration. Moreover, the Globular Sphere Model is more suitable than the Capillary fissures Model for porous structure.

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