

# A comparative analysis of optimization techniques for optimal control of two-level inverter considering low switching frequency

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**Abstract:** Selective Harmonic Elimination (SHE) technique is popularly implemented for precise elimination of eminent low-order voltage harmonics while precisely maintaining the preferred value of fundamental voltage. This technique involves derivation of optimal switching angles by solving transcendental non-linear equations. This paper presents a comparative study of Hybrid Genetic Algorithm Method (HGAM), Teaching Learning Based Optimization (TLBO) and Particle Swarm Optimization (PSO) technique on the basis of the weighted total harmonic distortion, effective harmonic elimination and fitness function. All the techniques have been programmed in Matlab software platform and simulation results are attained considering two-level inverter with  $N=2$  switching angles..

**Keywords:** Hybrid technique, Particle swarm algorithm, Voltage source Inverter, Teaching learning-based algorithm.

(Article history: Received: 18<sup>th</sup> March 2022 and accepted 26<sup>th</sup> June 2022)

## I. INTRODUCTION (HEADING 1)

The main purpose of voltage source inverters (VSI) is to essentially produce pure sinusoidal voltage with variable or fixed magnitude and frequency. Since the introduction of multilevel inverters in 1981, it has gained great deal of popularity in medium as well as high power applications [1]. To achieve reduced switching loss, conventionally low device switching frequency is adopted. However, the operation of VSI under such low switching frequency introduces harmonics of lower order in the output voltage waveform and significantly effects the operation of the inverter load [2]. The use of filtering equipment to remove such low order harmonics increases the cost and overall inverter size.

Numerous modulation strategies developed and proposed to achieve optimal inverter performance such as sinusoidal PWM, Space vector PWM and widely employed switching technique Selective Harmonic Elimination method. Proposed in 1973, SHE technique is superior compared to former two techniques based on the power quality, reduced switching loss, desired voltage control and also the elimination of triplen-harmonics due to the three-phase symmetry [3]. In order to achieve optimal angles of switching for inverter power devices, it is essential to solve a set of non-linear equations which confirms the fundamental voltage regulation and specific harmonic elimination [4]. Several algebraic, iterative and optimization techniques have been

recommended to derive solutions of SHE equation set [8]-[12].

In case of algebraic method such as Resultant theory, the non-linear trigonometric equations are reduced to elementary algebraic equations to obtain true switching angle values [5]. Iterative method such as Newton Raphson (NR) method has also been suggested for precise calculation of optimal solution [6]. The NR method offers fast convergence towards global minimum. However, such iterative methods require good selection of initial angles to converge towards optimal solution. On account of large number of switching angles and unwanted harmonics, the complexity of the equations increases which leads to difficulty in deriving the true solutions.

However, in order to overcome the limitations associated with iterative methods such as inability to determine multiple solutions and also resulting in no solution for certain regions of modulation index [7], many researchers have suggested the utilization of modern optimization search techniques such as Teaching Learning Based Optimization (TLBO) [12], Particle Swarm Optimization (PSO) [8], [9] Genetic Algorithm (GA) [10], [11] and many other optimization techniques for solving SHE equations.

In [8], PSO technique is proposed to solve SHE transcendental equations to find solution sets considering non-equal DC sources. The major constraint related to PSO technique is the lack of inherent ability to tune its velocity step-size in the search space resulting in poor rate of

convergence. Moreover, increasing the number of unknown switching angle variables simultaneously increases the search space complexity and the tendency that the solutions will be confined to local optima increases [9]. In order to address the aforementioned problem, a combination of mesh adaptive direct search (MADS) and PSO is suggested to improve local search for optimal results in [9]. GA technique was adopted in [10] to derive switching angles for 11-level inverter for specific subduing of low-order harmonics with simultaneous regulation of fundamental voltage. A combination of GA and ANN has also been suggested to improve the potential of the algorithm to generate switching angles for real time application [11]. Recently, many researchers have also examined techniques such as TLBO for solving SHE problems with improved exploration capability [12].

In this paper, a combination of GA and NR based solving technique is employed in two-level inverter with only two switching angles within a quarter-period to accomplish minimum harmonic distortion. The performance of the proposed technique is evaluated and compared against PSO and TLBO base SHE methods in finding the global optimal solutions at faster rate of convergence. Simulation results of voltage waveform along with related harmonic spectrum for two-level inverter are also presented.

### II. MATHEMATICAL FORMULATION

Fig. 1 presents the fundamental configuration of a two-level voltage source inverter. A midpoint 'O' is considered in every inverter bridge leg in order to measure the corresponding phase voltages. A minimum dead time is allotted in between the switching of the power devices in the same leg to avoid any short circuit condition of the DC source. For example, in fig. 1 the switches SR1 and SR2 are switched in complementary fashion and the pole voltage obtained is  $V_{dc}/2$  and  $-V_{dc}/2$  respectively when SR1 and SR2 is ON and OFF sequentially. The pole voltage waveform with two instants of switching in a quarter period is illustrated in fig. 2. The pulse number  $P=5$  for two switching angles results in switching frequency of 250Hz and fundamental frequency of 50Hz.

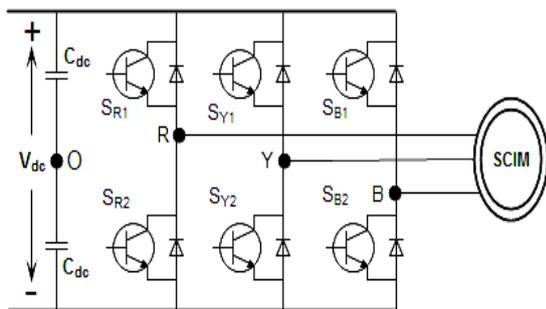


Fig. 1. A three-phase two-level VSI driving an induction-motor.

The waveform considered in fig. 2 follows both quarter (QWS) and half wave (HWS) symmetric properties as indicated in (1) and (2):

$$V_{RO}(\theta) = V_{RO}(\theta + 180^\circ) \quad (1)$$

$$V_{RO}(\theta p - \theta) = V_{RO}(\theta p + \theta) \quad (2)$$

where,  $\theta_p$  denotes the maximum angle point of the corresponding voltage waveform both at positive and negative and negative half cycle.

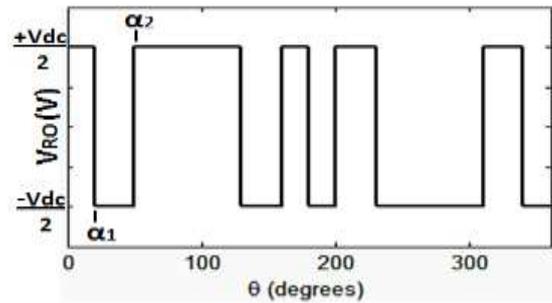


Fig. 2. Voltage waveform for two-level inverter with  $N=2$ .

As per the principle of Selective Harmonic Elimination technique, a total of  $(N-1)$  odd non triplen voltage harmonics can be abolished completely from the inverter line voltage waveform, where  $N$  indicates the predetermined number of independent switching angles. In this paper two switching instants are considered for complete 5th harmonic elimination and preserving the desired value of fundamental voltage. In order to determine the objective function, the initial step is to derive the Fourier series expansion of the voltage waveform shown in fig. 2 which can be further expressed as given in (3).

$$V_{out} = \sum_{n=1}^{\infty} [ a_n \cos(n\theta) + b_n \sin(n\theta) ] \quad (3)$$

where,  $a_n$  and  $b_n$  denotes the Fourier coefficients.

The cosine-series component and also the even harmonics present in the sine-series are not considered for further computation due to the odd and half-wave symmetric properties followed by the two-level voltage waveform under evaluation. Hence, the modified pole voltage expression with two independent switching angles ( $\alpha_1$  and  $\alpha_2$ ) is stated in (4). Here,  $V_{dc}$  denotes the supply source connected to the two-level h-bridge circuit.

$$b_n = \frac{2V_{dc}}{n\pi} [ 1 - 2 \cos(n\alpha_1) + 2 \cos(n\alpha_2) ] \quad (4)$$

In case of three-phase system, the triplen voltage harmonics are absent in the line voltage. Therefore, in accordance to the 5th voltage harmonic elimination and also fundamental voltage regulation, expression (4) is re-arranged and expressed accordingly in (5) and (6). However, the values of  $\alpha_1$  and  $\alpha_2$  obtained by solving equations (5) and (6) are considered as true solutions only if it follows the boundary conditions stated in (7).

$$V_1 = \frac{2V_{dc}}{\pi} [ 1 - 2 \cos(\alpha_1) + 2 \cos(\alpha_2) ] \quad (5)$$

$$V_5 = \frac{2V_{dc}}{5\pi} [ 1 - 2 \cos(5\alpha_1) + 2 \cos(5\alpha_2) ] \quad (6)$$

$$0 \leq \alpha_1 \leq \alpha_2 \leq \pi/2 \quad (7)$$

### III. HYBRID GENETIC ALGORITHM

A combination of two methods is employed to derive optimum solutions for non-linear set of equations i.e.,

Newton Raphson (NR) and Genetic Algorithm (GA), hence the name Hybrid Genetic Algorithm Method (HGAM). The implementation of only NR method to solves non-linear equations require a good starting guess to start the iteration process and reach the exact true solution. However, the complexity to achieve a good starting guess increases as the number of voltage levels and also the switching instants increases. In HGAM technique, GA is used to obtain a set of solution for a particular value of modulation index. This value of switching angles is used as initial guess to find the complete range of solution from  $0 < M < 1$  using NR method. The fitness function constructed for minimization of overall THD and also regularization of proper fundamental voltage component is defined in (8).

$$f(\alpha_1, \alpha_2, \dots, N) = \min \left[ \left( 100 \frac{V_1^* - V_1}{V_1} \right)^4 + \sum_{n=5,7,\dots}^{3N-1} \frac{1}{n} \left( \frac{V_n}{V_1} \right) \right] \quad (8)$$

$$\text{restricted to: } 0 \leq \alpha_1 \leq \alpha_2 \leq \pi/2 \quad (9)$$

where,  $V_1$  is the desired fundamental component and  $n$  is the harmonic order.

The solution obtained using the fitness function mentioned in (8) is considered valid if the following three conditions are satisfied:

- (i) The desired value of fundamental voltage component ( $V_1$ ) is achieved i.e. ( $V_1 = V_1^*$ )
- (ii) The value of THD is below the threshold value.
- (iii) The restriction stated in (9) are satisfied by the derived switching angles.

Once the result is obtained using GA technique, the process of evaluation for the switching angles corresponding to the entire range of modulation index is initiated. The equations defined in (10) and (11) corresponds to the elimination of 5th voltage harmonic and proper fundamental voltage control respectively.

$$1 - 2\cos(\alpha_1) + 2\cos(\alpha_2) = M^* \quad (10)$$

$$1 - 2\cos(5\alpha_1) + 2\cos(5\alpha_2) = 0 \quad (11)$$

here,  $M^*$  refers the desired value of  $M$  which must be within the range  $[0 \ 1]$ .

Expressions defined in (10) and (11) can be re-worked and presented in matrix format as shown in (12).

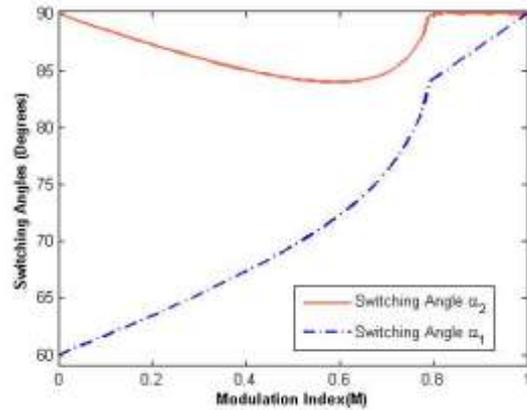
$$F(\alpha) = H \quad (12)$$

where,

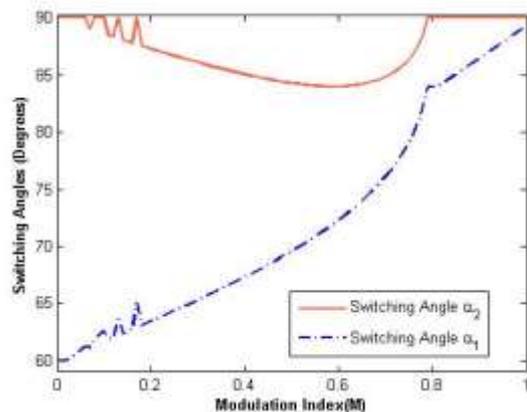
$$F(\alpha) = \begin{bmatrix} 1 - 2\cos(\alpha_1) + 2\cos(\alpha_2) \\ 1 - 2\cos(5\alpha_1) + 2\cos(5\alpha_2) \end{bmatrix}, \quad (13)$$

$$H = [M^* \ 0], \text{ and} \quad (14)$$

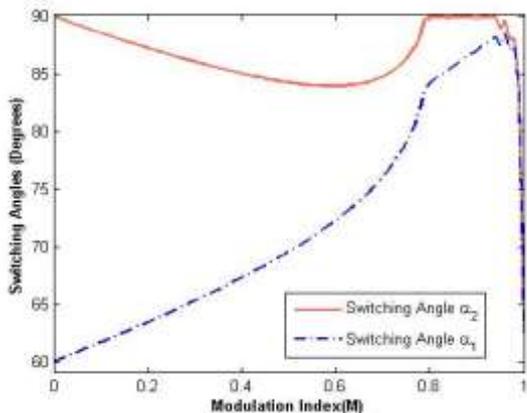
$$\alpha = [\alpha_1 \ \alpha_2] \quad (15)$$



(a)



(b)



(c)

Fig. 3. Switching angles derived using (a) HGAM (b) PSO and (c) TLBO.

The Jacobian matrix formulated using the above mentioned non-linear trigonometric equation set is presented in Eq. (16).

$$J^i(\alpha) = \begin{bmatrix} \frac{\partial F_1^i(\alpha)}{\partial \alpha_1} & \frac{\partial F_1^i(\alpha)}{\partial \alpha_2} \\ \frac{\partial F_2^i(\alpha)}{\partial \alpha_1} & \frac{\partial F_2^i(\alpha)}{\partial \alpha_2} \end{bmatrix}$$

$$= \begin{bmatrix} 2\sin(\alpha_1) & -2\sin(\alpha_2) \\ 10\sin(5\alpha_1) & -10\sin(5\alpha_2) \end{bmatrix} \quad (16)$$

The complete set of steps are outlined to determine globally optimal solutions using HGAM:

1. The initial step is to evaluate the value of  $f_{new}$  and  $J_{new}$  for starting values of switching angles ( $\alpha_{old}$ ).

2. Calculate the latest values of switching angles by finding the solution of the expression:  $\alpha_{new} = \alpha_{old} + J_{new}/F_{new}$ .

3. Once the optimum solution is achieved, the next step is to increment the value of  $M$  by 0.01 and repeat the steps outlined in 3 and 4.

4. Follow similar steps to acquire solutions for the other half of the initial  $M$  value for complete solution set.

5. In case of higher number of independent angles of switching, there are possibilities to obtain multiple solution also. Therefore, repeat the steps for different initial values of switching angles. For a particular value of  $M$ , a minimum of 100 iterations is performed to achieve globally optimum solution.

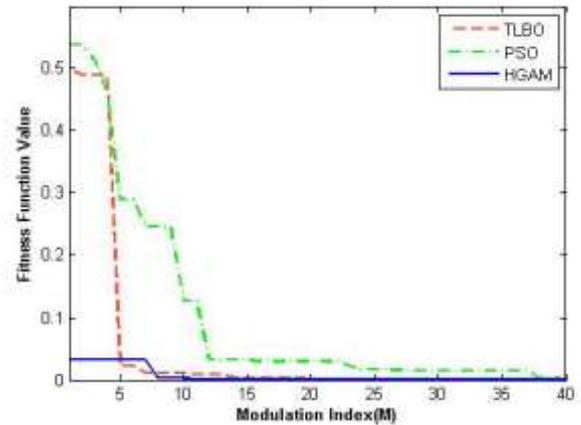
In this section, the solutions related to the three methods HGAM, PSO and TLBO under comparison are also presented along with performance comparison corresponding to the rate of convergence of fitness function and THD value for the entire range of  $M$ . The simulation parameters corresponding to each method are presented in Table 1. Here,  $N$  and  $T$  denotes population size and the maximum value of iteration.  $D_c$  and  $D_m$  for HGAM method denote the distribution index for crossover and mutation. Also, the parameter  $P_c$  and  $P_m$  indicate the mutation and crossover probability. Next, for PSO method the terms  $w$ ,  $C_1$  and  $C_2$  represents the inertia of the particles and acceleration coefficient probability respectively. The upper and lower bound set for switching angles is  $60^\circ$  to  $90^\circ$ .

TABLE I. SIMULATION PARAMETERS FOR OPTIMIZATION TECHNIQUES

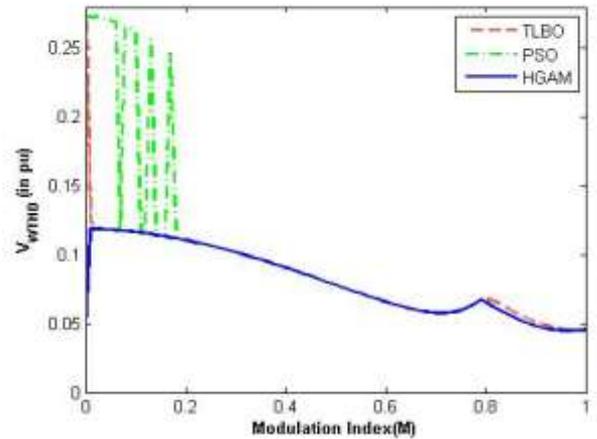
Methods	Simulation Parameters					
	$N$ (Population Size)	$T$ (Iteration Count)	$D_c$	$D_m$	$P_c$	$P_m$
HGAM	50	200	50	50	0.8	0.2
	-	-	$w$	$C_1$	$C_2$	
PSO	50	200	0.8	1.5	1.2	-
	-	-				
TLBO	50	200				

Figs. 3(a), (b) and (c) illustrates the solutions corresponding to the three methods. The range of solution for  $\alpha_1$  and  $\alpha_2$  extends within the range from  $60^\circ$  to  $90^\circ$ . It can be observed that a smoother and even variation of switching angles is obtained using the proposed HGAM technique over the entire  $M$  range. The performance comparison between HGAM, PSO and TLBO based SHE techniques in terms of rate of convergence with respect to the number iterations is shown in fig. 4(a). As seen in fig. 4(a), the proposed technique using the hybrid GA and NR approach results in faster convergence compared to PSO and TLBO techniques.

In fig. 4(b), simulated overall THD value pertaining to the three methods under evaluation is shown. Due to the uneven nature of the switching angles due to PSO and TLBO based SHE technique, it is evident that HGAM results in superior THD performance.



(a)



(b)

Fig. 4. (a) Comparison of fitness value and (b) overall THD value derived using HGAM, PSO and TLBO.

#### IV. SIMULATION RESULTS AND DISCUSSION

In this section, simulation results for two-level inverter using HGAM, PSO and TLBO based SHE techniques are presented. The two-level three-phase simulation model and the optimization programs are executed using MATLAB/Simulink software-package. The inverter in simulation model is powered using 30V DC source and the THD calculation is performed considering harmonics up to 17<sup>th</sup> order.

Fig. 5 illustrates the Simulink model developed to generate the three-phase gate signals to drive the power devices in two-level inverter. Sub-systems 1, 2 and 3 generates the reference signals 'thetaR', 'thetaY' and 'thetaB' required for generation of gate signals and is fed with real time and frequency input for optimum operation for any duration of simulation. The predefined values for two switching angles  $\alpha_1$  and  $\alpha_2$  are also required as input from the user. The output from sub-systems 4, 5 and 6 are

normalized and fed to the gate terminals of the R, Y and B phase devices for production of two level three phase output. The complementary signals for the devices at the lower end

of the h-bridge are also generated with the help of 'NOT' gate.

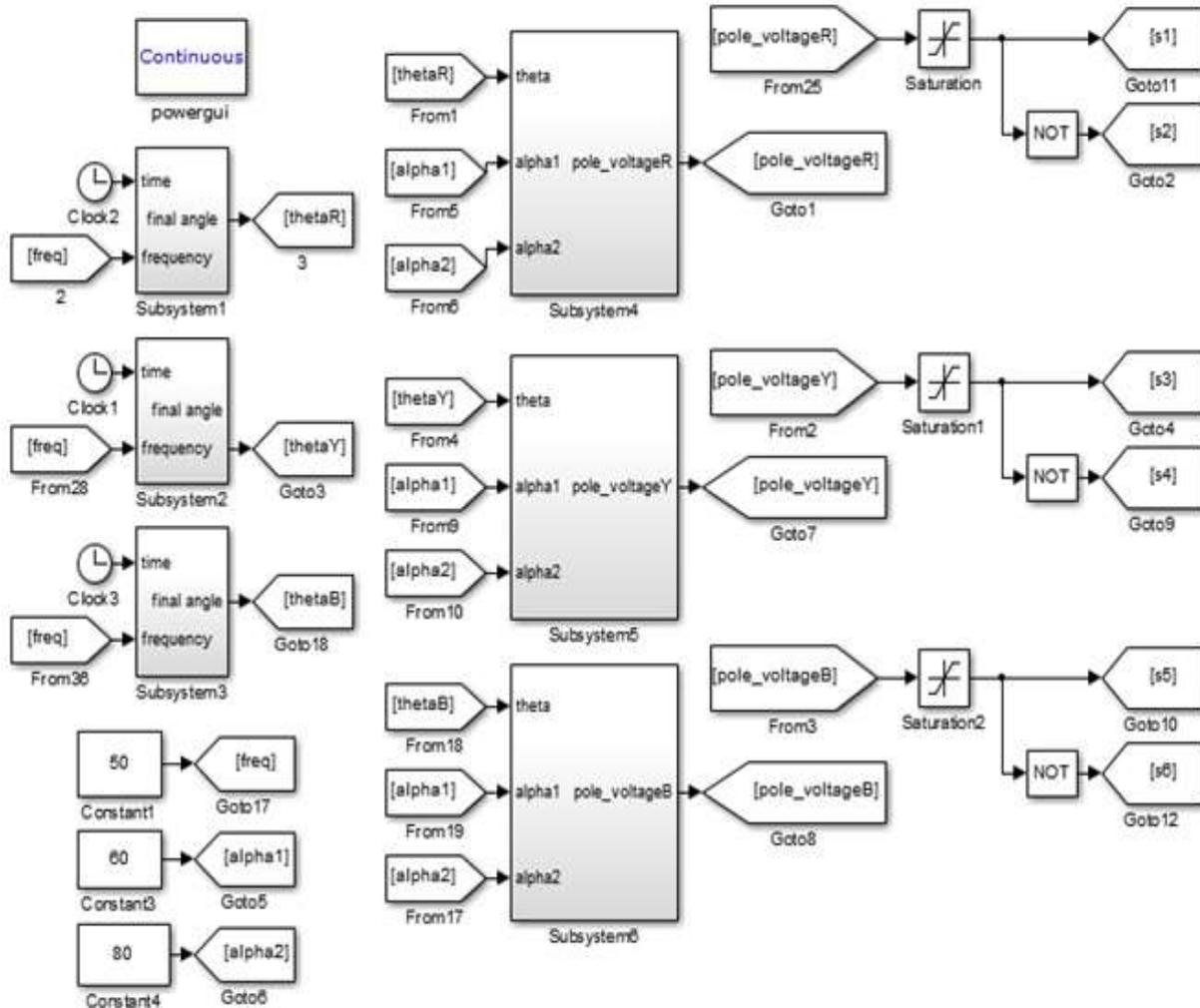


Fig. 5. The overall Simulink model for Gate signal generation.

**Case1 (M=0.18):**

The line voltage and harmonic spectrum for HGAM and PSO methods are illustrated in figs. 6 (a) and (c) respectively. Fig. 6(b) and (d) exhibits the FFT spectrum pertaining to HGAM and PSO respectively which confirms a lower THD of 0.1127 compared to 0.2466 for PSO method and simultaneous lower order 5th harmonic elimination. Moreover, the dominant 7th and 13th harmonics are also minimized in the output waveform using HGAM. For the lower range of M, the performance of TLBO is similar to HGAM and hence have not been presented in the results.

**Case2 (M=0.82):**

The line voltage waveform for HGAM and TLBO are shown in fig. 7(a) and (c) at M=0.82. In case of HGAM technique, the magnitude of the fundamental component at 50Hz is nearer to 27V with overall THD equal to 0.0633 along with complete abolishment of 5th voltage harmonic.

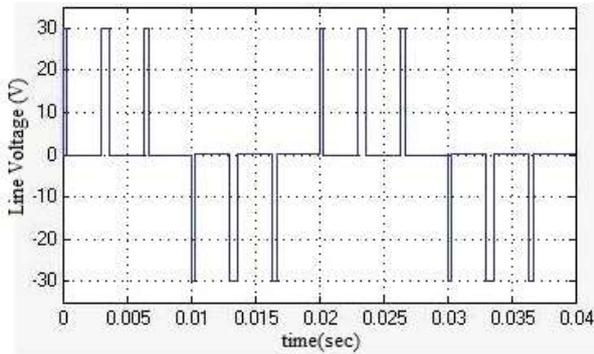
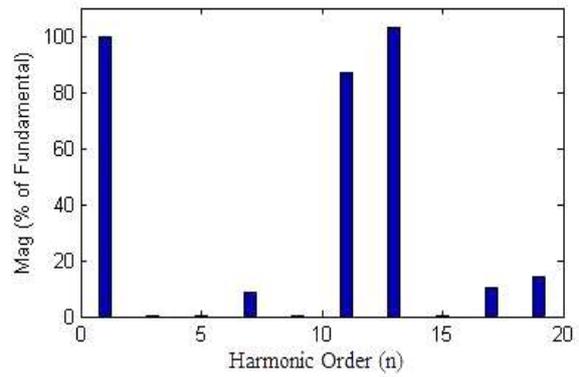
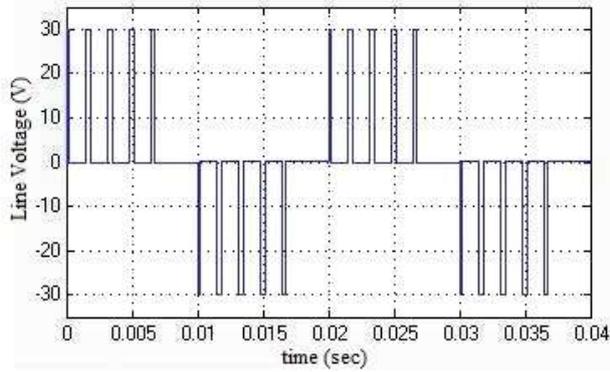
However, the PSO technique resulted in lesser fundamental voltage equal to 24V and higher THD value of 0.0709. Also,

the complete elimination of 5th voltage harmonic could not be achieved using PSO technique as shown in fig. 7(b) and (d). Table 2 shows the switching angle and THD values for the corresponding methods under comparison at M=0.18 and M=0.82.

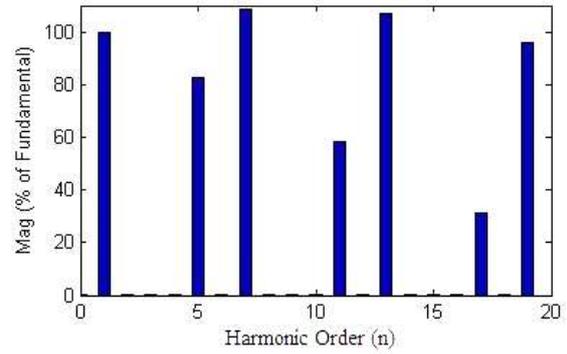
TABLE II. SWITCHING ANGLES AND THD VALUES

Methods	Switching Angle ( $\alpha_1$ )	Switching Angle ( $\alpha_2$ )	Simulated THD (pu)
<i>Modulation Index (M=0.18)</i>			
HGAM	62.9	87.67	0.1126
PSO	64.98	90	0.2466
<i>Modulation Index (M=0.82)</i>			

Methods	Switching Angle ( $\alpha_1$ )	Switching Angle ( $\alpha_2$ )	Simulated THD (pu)
HGAM	84.55	90	0.0633
TLBO	81.56	89.99	0.0709



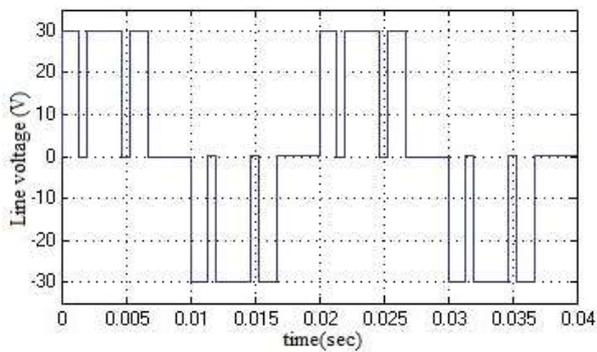
(b)



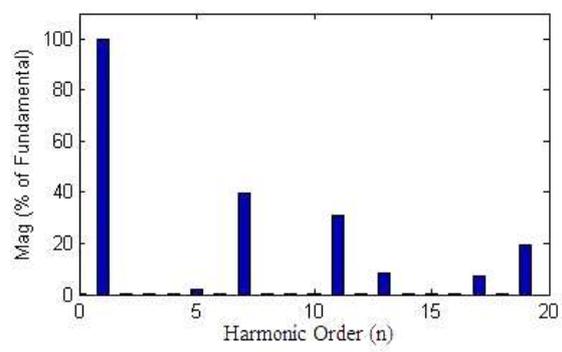
(c)

(d)

Fig. 6. Line-voltage waveform corresponding to (a) HGAM (c) PSO and FFT spectrum of the corresponding waveform pertaining to (b) HGAM (d) PSO at  $M=0.18$ .



(a)



(b)

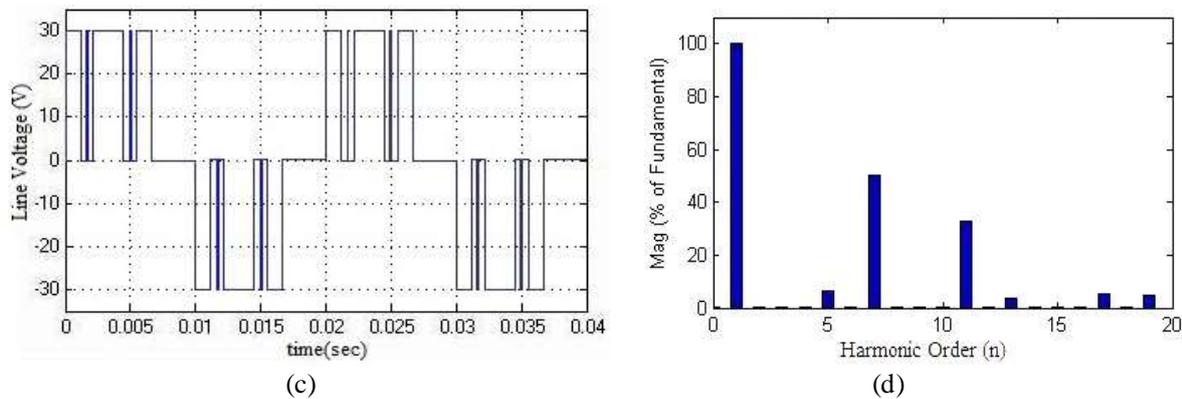


Fig. 7 Line-voltage waveform corresponding to (a) HGAM (c) TLBO and FFT spectrum of the corresponding waveform pertaining to (b) HGAM (d) TLBO at  $M=0.82$ .

### V. CONCLUSIONS

This paper reports the methodology of HGAM based SHE-technique for the optimal switching angle computation in two-level inverter. The proposed hybrid technique offers advantages such as reaching the global minimum value of the fitness function with least number of iterations. The superiority of HGAM was also validated, as it is able to effectively eliminate 5th harmonic of the output voltage as observed using FFT analysis and simultaneous minimization of overall THD. Also, based on the comparative FFT analysis for particular modulation index values between HGAM, PSO and TLBO further confirms the effective computation of ideal switching angle values over complete modulation-index range.

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