

Cosine Based Non-Linear Frequency Modulation Waveforms with Low Sidelobes

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Abstract: *Suppression of sidelobes is critical in most radar applications. The sidelobes of around -30 dB to -60 dB are of primary interest in several radar applications. Several studies focused on the design of Non-linear frequency modulated (NLFM) waveforms. Cosine-based NLFM waveforms, NLFM I, II and III, are designed and investigated for their performance for sidelobe level for different time-bandwidth (BT) products. The designed waveforms achieved sidelobe levels of about -63.93 dB, -75.47dB and -76.32 dB at BT product 1000. The reduction in sidelobes increased with an increase in BT product. For overall performance, the designed waveforms were investigated for doppler tolerance and signal to noise ratio (SNR). An increase in SNR caused sidelobe levels to decrease. Like other NLFM waveforms, they exhibited doppler intolerance.*

Keywords: NLFM, SNR, Peak Side Lobe, Main lobe width.

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I. INTRODUCTION

Pulse compression is a technique used in radar systems for fine range resolution and long-range reduction. Digital radars perform pulse compression using a matched filter. The autocorrelation of an input signal is equivalent to matched filtering in the time domain. After pulse compression, the radar signal pulse main lobe is accompanied by higher side lobes. Suppression of side lobes is critical in some applications like meteorological radar. Low sidelobes, roughly around -30 dB to -60 dB, are of interest because of the reduction in radar and communication intercepts probability, radar clutter, jammer vulnerability, and increasing spectrum congestion in satellite transmission [1].

Pulse compression uses different coding schemes like Linear Frequency Modulation (LFM) and Non-Linear Frequency Modulation (NLFM). LFM and NLFM signals are characterized by the time-bandwidth product (BT). LFM signals produce sidelobes of around -13 dB. These values are not satisfying for target detection. If sidelobe is too high, there will be severe range ambiguity. NLFM signals can produce low range sidelobes for BT product more than 50.

Simplicity and doppler tolerance are the main advantages of using LFM in radar systems. However, the major drawback is high PSL values, which reduce radar detectability. To overcome this, NLFM waveforms are used. Several studies focused on designing new NLFM waveforms that can yield low peak side lobe (PSL) values without any weighting techniques [2-5]

Yee Kit Chan et al. in 2009 [6] proposed a NLFM waveform that provides a PSL value of -19 dB. They designed the waveform by using two/Tri stage LFM signals. Later, tangent based, S-shaped and curve-shaped waveforms [7-9] were developed and investigated for sidelobe levels. A 10 dB reduction in sidelobe level is achieved by tangent based NLFM. Curve shaped NLFM could achieve a reduction up to -18.6 dB.

Xuebo Zhang et al., in 2021[10] designed four stage and five stage NLFM waveforms which have lower peak sidelobe level by 25.62 dB with 3 dB main lobe that is also lower.

In 2019, Guodong Jin et al.,[11] developed a high-precision NLFM signal generator with the ability of predistortion compensation. The compressed signal has a peak sidelobes of -38.5 dB.

Qinyu Xie et al, 2022 [12] proposed a novel two-step progressive optimization framework for the NLFM waveform using Fourier series. By compensating the system error in the frequency domain, the achieved PSLR and ISLR are -44.6 and -28.5 dB, respectively.

Ghavamirad, Roohollah, and Mohammad Ali Sebt, 2019 [13], proposed a method implemented for six windows of Raised-Cosine, Taylor, Chebyshev, Gaussian, Poisson, and Kaiser. The results reveal that the peak sidelobe level of autocorrelation function (ACF) reduces about an average of 5 dB in the proposed method compared with the stationary phase method. PSL reduction for the Poisson window compared to other windows is significant.

Yongwei Zhang et al., 2020 [14] proposed a novel NLFM waveform with modified Chebyshev window PSD, which combines the Chebyshev with edge distortion compensation, allowing, in theory, low sidelobe level as Chebyshev window. The autocorrelation of the NLFM waveform with MChab PSD has a PSL level -39.51 dB, an improvement of 5.09 dB over Chebyshev PSD, which possesses a PSL level of -34.43 dB.

A Study on Pulsed-LFM and Pulsed-NLFM waveforms for Radar systems carried out by M.S. Kang et al.,2019 [15] made certain that the time-bandwidth product of frequency modulated pulse signal is the key characteristic for long range detection with low transmitting power and high range resolution with long pulse duration in radar systems. The peak sidelobe level could be suppressed up to 23 dB if pulsed-NLFM signal which has the β of 8 , is used. The cost for sidelobe reduction is the degradation of range resolution, which is caused by increasing the main lobe width.

In this article, LFM and three designed NLFM signals with different time-bandwidth products are compared for sidelobe reduction. Compared to LFM, designed NLFM signals are investigated for overall performance and for the signal to noise ratio (SNR) and doppler effect.

II. NLFM SIGNAL DESIGN

LFM and newly designed NLFM signals are considered for simulation. LFM is popular because of its simplicity and Doppler tolerant features. This waveform has a period which varies inversely with the bandwidth.

$$f(t) = (B/T) * t \quad 0 \leq t \leq T$$

Our new NLFM Signals:

NLFM signals are designed in such a way that the spectrum shape yields reduced sidelobe levels. In the present study, two curve shaped NLFM signals NLFM I, NLFM II and NLFM III based on cosine function have been developed. In general, NLFM chirp is defined by equation $s(t) = \exp(j\varphi(t))$, where $\varphi(t)$ is instantaneous frequency obtained using the differential of phase modulation.

NLFM I:

$$f(t) = k1 * (\cos(k*t/tau) k3) / k4$$

NLFM II:

$$f(t) = (\cos(k*t/tau) k3) / k4$$

NLFM III:

$$f(t) = k1 - k5 - k6 * \log(\text{abs}(\cos(k*t/tau)))$$

where $\tau = 1$; $k = 3.06$; $k/k1 = 4.722$; $k/k2 = 10.303$; $k/k3 = 51$; $k/k4 = 5.464$; $k/k5 = 6.891$; $k/k6 = 10.928$

III. SIMULATION

The performance of detectability and resolution is related to peak sidelobe level (PSL) and the main lobe width. Theoretical PSL is achievable with BT product. Simulations were carried out with different BT products. The results obtained for designed NLFM waveforms are compared with LFM. As noise is present in all devices and cannot be

avoided in case of radar systems, the effect of background noise is investigated. Simulations are carried out by adding Additive white gaussian noise (AWGN) before pulse compression. The SNR value is varied from -20 dB to 20 dB. The number of trials conducted are 25 . Doppler effect analysis is performed on designed NLFM waveforms. For this, two different doppler frequencies are considered. To measure the Doppler tolerance, the Doppler shift frequency (f_d) calculated is added to the centre frequency (f_c) of matched filter ($(f_c \pm f_d)$).

IV. RESULTS AND DISCUSSION

To analyse and compare newly designed waveforms, simulations are carried out for different BT products as shown in Table 1. The autocorrelation of the waveforms for BT products of 100 and 1000 are shown in figures 1 (a) and 1(b) respectively. The peak sidelobe levels achieved by NLFM I, II & III are -63.93 dB, -75.47 and -76.32 dB respectively. For high BT product ($BT = 1000$) the waveforms achieved these lowest PSL values and NLFM II exhibited narrow main lobe width compared to others. NLFM II exhibited better PSL values (for $BT = 100$) and narrowed main-lobe width compared to NLFM I, NLFM III and LFM. It is evident from figures 1(a) -1(b) and Table 1 that PSL values reduced with increase in BT product and vice versa in case of LFM. The PSL values of designed NLFM signals, NLFM I, II, III are -41.62 dB, -43.74 and -29.39 dB, lower than conventional LFM (-13.49 dB).

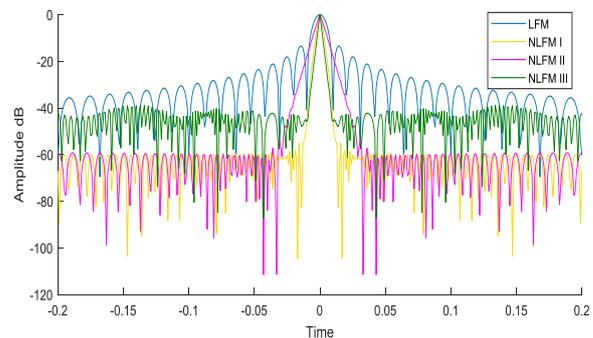


Fig.1(a). Matched Filter output for LFM, NLFM I, NLFM II & NLFM III waveforms with $\beta\tau = 100$

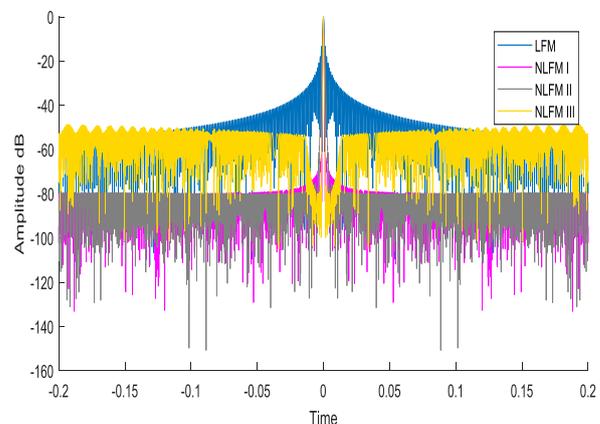


Fig. 1(b). Matched Filter output for LFM, NLFM I, NLFM II & NLFM III waveforms with $\beta\tau = 1000$

TABLE 1. Comparison of PSLR and Main-lobe width of LFM and NLFM signals

Compression Ratio $\beta\tau$	Sampling rate F_s	PSLR				Main-lobe width			
		LFM	NLFM I	NLFM II	NLFM III	LFM	NLFM I	NLFM II	NLFM III
32	10β	-	-50.14	-49.04	-32.91	0.19189	0.40625	0.12939	0.61621
100	10β	13.68	-55.11	-57.22	-42.88	0.20117	0.69678	0.14355	0.79932
1000	10β	13.47	-63.93	-75.47	-76.32	0.19972	0.44398	0.17912	1.0108

TABLE 2. Comparison of PSLR with existing literature

Compression Ratio $\beta\tau$	Sampling rate F_s	Functions	PSLR			
			Existing literature	NLFM I	NLFM II	NLFM III
100	10β	Taylor window [16]	-35.97	-55.11	-57.22	-42.88
250	10β	PSL with Phase Improvement Algorithm (Raised Cosine) [17]	-37.67	-57.69	-64.46	-62.57
32	10β	FM function [18]	-29.03	-50.14	-49.04	-32.91

The PSLR values of new NLFM signals are compared with other NLFM signals in the literature. From Table 2 it is evident that the new designed NLFM waveforms have lower side-lobe levels. Among the three, NLFM II exhibited better values for $\beta\tau$ of 100 and 250. However, for $\beta\tau=32$, NLFM I has lower PSLR value.

The effect of background noise is analysed by adding AWGN on NLFM signals. The PSL and main lobe width are measured and analysed by varying SNR from 20 dB to -20 dB. The mean value of measurements is taken after several trials and tabulated (Table 3 and Table 4). From Figure 2(a), at SNR 20 dB, the PSL value of NLFM I is -58.63 dB and raised to -36.26 dB when SNR dropped to -20 dB. In the case of NLFM II and III the PSL values are -57.43 dB, -56.90 dB when SNR is 20 dB and -36.76 & -38.61 dB respectively at -20 dB SNR. An increase in SNR causes the PSL values to decrease and vice versa. The varying SNR affected the main lobe width of designed NLFM signals. In case of NLFM I & II the main lobe width drastically reduced with an increase in SNR (Figure 2(b)). The chirp signals of LFM and designed NLFM waveforms are examined for doppler shift corresponding to two different frequencies (Figures 3(a), 3(b),3(c) & 3(d)). From the figures, it can be seen clearly that the matched filter output of designed NLFM signals is distorted and LFM is insensitive with doppler frequencies. The distortion in case of NLFM signals increased with doppler frequency. This degrades the detectability of radar systems. The proposed NLFM waveforms, like other waveforms exhibited doppler intolerance.

TABLE 3. PSL values corresponding to background noise

SNR	PSLR		
	NLFM I	NLFM II	NLFM III
-20 dB	-36.26	-36.76	-38.61
-10 dB	-38.71	-38.96	-40.57
0	-42.51	-42.12	-41.35
10 dB	-50.72	-54.20	-47.81
20 dB	-58.63	-57.43	-56.90

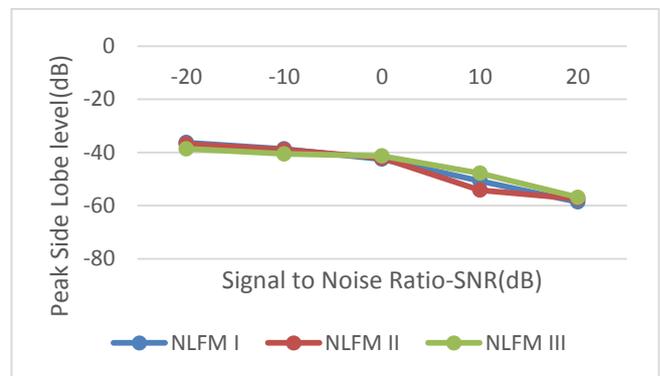


Fig.2(a). Background noise to PSL of NLFM I, NLFM II and NLFM III

TABLE 4. Main Lobe width corresponding to background noise

SNR	Main lobe width		
	NLFM I	NLFM II	NLFM III
-20 dB	2	1.9988	1.9986
-10 dB	1.9980	1.96	1.9996
0	1.9892	0.2946	1.9656
10 dB	0.5435	0.1949	1.3360
20 dB	0.4589	0.1809	1.0034

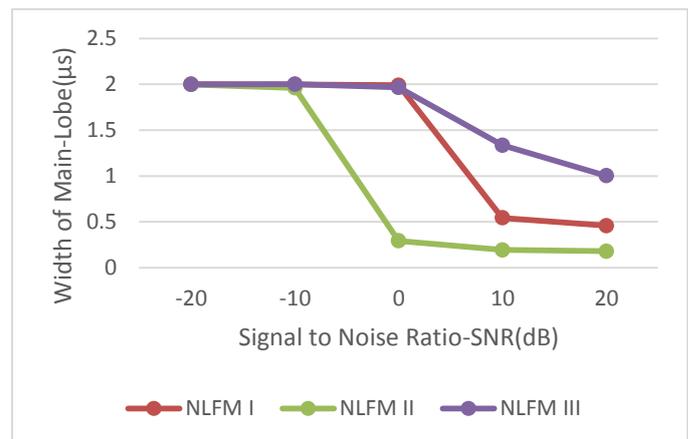


Fig.2(b). Background noise to width of main lobe of NLFM I, NLFM I and NLFM II

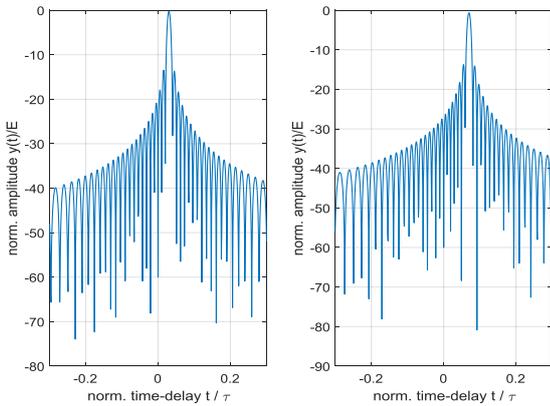


Fig.3 (a). Comparison of matched filter output with doppler shift on LFM (i) $fd = 3$ (ii) $fd = 7$

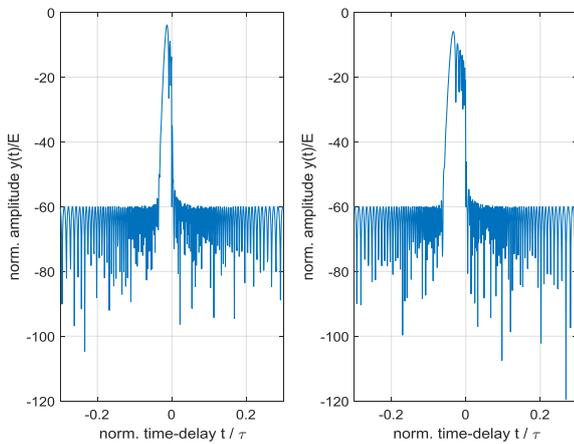


Fig. 3 (b). Comparison of matched filter output with doppler shift on NLFM I (i) $fd = 3$ (ii) $fd = 7$

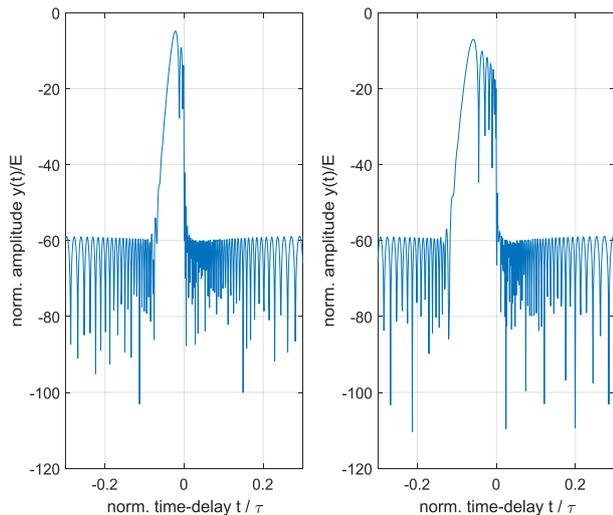


Fig. 3 (c). Comparison of matched filter output with doppler shift on NLFM II (i) $fd = 3$ (ii) $fd = 7$

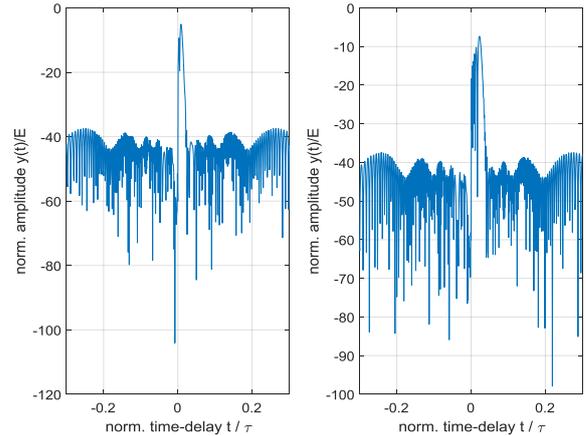


Fig. 3 (d). Comparison of matched filter output with doppler shift on NLFM III (i) $fd = 3$ (ii) $fd = 7$

V. CONCLUSIONS

The NLFM waveforms designed using cosine function suppressed side lobe levels better than LFM and other NLFM waveforms. NLFM I suppressed side lobe level up to -63.93dB , NLFM II and NLFM III achieved -75.47 dB , -76.32 dB respectively at high BT product ($BT = 1000$). The advantage of this design is that NLFM II achieved reduced PSLR without much increase in the main lobe width. Like any other NLFM, the designed waveforms exhibited Doppler intolerance. The signal is distorted by doppler shift more than LFM. The designed NLFM waveforms showed low sidelobe levels and have better detectability under background noise.

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