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Planning of power system operation for reduction of system loss using series power flow controller devices involving PSO optimization technique

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Abstract: ***This work presents a planning of power system operation for reducing total real power loss (TRPL) using series power flow controller (SPFC) in some selected transmission lines of a power system. To identify the transmission lines suitable for placements of SPFC devices, sensitivity relations between the power through lines and TRPL are used. For this purpose, power loss sensitivity factor (PLSF) is derived to relate the change in TRPL to the change in power flow (CPF) through the lines of a power system. Then the lines having significant values of PLSF are considered for the introduction of SPFC devices to reduce TRPL. Utilizing the PLSF and their relation to the change in TRPL, the necessary optimal values of CPF for the selected lines are obtained by using Particle Swarm Optimization (PSO) technique. Incorporating the optimal values of CPF for the lines considered for introduction of SPFC devices, a load flow analysis with SPFC (LFAWSPFC) is carried out, to estimate the state variables of SPFC devices, namely voltage magnitude and phase angle. To show the effectiveness of the proposed method, simulations were performed on standard IEEE 118 bus system and it is found that the TRPL has been reduced to 2.5151 p.u. from 3.0851 p.u., also improvement of bus voltages of the system are observed with the implementation of SPFC devices in the selected sensitive lines. Further, the cost analysis related to installation of SPFC devices and saving due to reduction of TRPL is also presented.***

Keywords: **Power system planning, loss reduction, sensitivity factor, series power flow controller, Particle Swarm Optimization.**

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NOMENCLATURE

n	Total number of buses in a power system.
l	Line numbers.
$P_i + jQ_i$	Complex power injection at bus-*i*.
$V_i\angle\delta_i$	Voltage in polar form at bus-*i*.
$G_{ij} + jB_{ij}$	Elements of Y_{bus} matrix.
P_L	Total Real Power Loss.

$P_i + jQ_i$	Complex power injection at bus-*i*.
$P_{km} + jQ_{km}$	Complex power flow in the line *km*.
$\Delta P_{km} + j\Delta Q_{km}$	Change in complex power flow in line *km.*
SF_{Pkm}	Sensitivity factor due to flow of real power.
SF_{Qkm}	Sensitivity factor due to flow of reactive power.
$P_l^{sch} + jQ_l^{sch}$	Schedule complex power flow in line *l.*
$V_s \angle \delta_s$	Voltage of SPFC in polar form.
Z_s	Impedance of converter transformer.

I. INTRODUCTION

Due to ever growing demand or consumption of power around the globe, a large amount of power need to be transmitted though the network. As a result, the existing network has to operate close to their limiting value and hence the power system becomes less secure. Thus, the generation as well as transmission network has to be expanded to meet the load requirement. When it comes to an interconnected power system network, random nature of power demands creates complexity in controlling or operating the power system. Moreover, with the use of Flexible AC transmission System (FACTS) devices it is possible to control and transfer more power without changing power system layout and keeping the line flow within their respective limits. FACTS devices are used in power system to enhance line power transfer capacity and increase controllability of the transmission networks. Occurrence of unavoidable disturbances in power system network may lead to outages of generators or transmission lines. This creates overloading in the other lines and voltage deviations which result in partial or complete blackout of the system. Therefore, it is important to control the line power flow. Controlling of power flow with FACTS devices reduce line overload, reduce system loss, maintain bus voltage profile and improve overall power system performance. Placements of FACTS devices in suitable locations plays significant role in the improvement of power system conditions.

Population based search techniques are very popular in power system optimization and most of them are evolution based. Several evolutionary algorithms have been used for the optimal placements of compensating devices for the reduction of transmission losses [1]-[3]. Swarm based probabilistic search technique PSO, developed by inspiring the behavior of fish schooling and bird flocking [1] is introduced by Eberhart and Kennedy in 1995. Due to implicit parallelism property of PSO it has less probability to get stuck in local optimum [1]. PSO has quick convergence towards optimum solution and may have slow convergence when it reaches near the minimum solution. R.C. Eberhart et.al. [2] have presented that the PSO algorithm superior than other optimization techniques for its robust nature and efficient computation in finding optimal control parameters. PSO is used to determine optimal values of control variables to minimize the power system loss [1]-[3].

Several research works are done on applications of FACTS devices [4-9] which control the power transmission by changing three parameters namely transmission voltage magnitude (V), impedance (Z) and phase angle (δ). Suitable location of FACTs devices can reduce congestion in the power lines which reduces power system loss as well as improve voltage profile [4]. FACTS devices are used in power systems for voltage profile improvement, power loss reduction, stability enhancement, maximize power transfer capability/ power system load-ability etc. [5]-[8]. It has been noticed that the identification of best position and selection of suitable type of FACTS device plays a significant role for improvement of power system conditions. D. Hazarika et. al. proposed a method to reduce line overload through rescheduling the power flow in the line with SPFC devices [8]. Analysis of installation cost (IC) of FACTS device is necessary while used in transmission lines for improving power system conditions. Optimal allocation of FACTS devices considering cost of installation are presented in [9].

This paper presents a model of SPFC device represented by synchronous voltage source with controllable magnitude and phase angles based on Static Synchronous Series Compensator (SSSC) device, considered in the transmission lines, possessing significant values of PLSF for the reduction of TRPL of a power system. The SPFC model used in reference [8], the SSSC was considered as a synchronous voltage source with controllable magnitude and phase angles without the impedance of converter transformer. The advantage of the model is that it represents the SPFC device as an integral part of the transmission line. As a result, the sensitivity relations between power flows through the SPFC device with any power system control variable would possess the collective or integrated effect of SPFC device and transmission line state variables. However, it is necessary to include the impedance of SSSC along with controllable voltage source to evaluate the performance of SPFC in a realistic manner. Therefore, in this paper the detailed modelling of a SPFC device with its impedance has been considered when it is incorporated in a transmission line. Further, a solution methodology has been proposed for reduction of TRPL by changing the line power flow of the selected transmission lines using SPFC devices. For this purpose, a load flow (LF) analysis without SPFC is performed for the base load condition of the system. Initially utilizing this LF results, PLSF values are calculated and highly sensitive lines are identified to introduce SPFC devices for reduction of TRPL. Then sensitivity factors (SF) and their relation to the change in system loss is used to represent an objective function for applying PSO technique to estimate the necessary optimal values of CPF in the lines considered for introduction of SPFC devices. Incorporating the optimal values of CPF for the sensitive lines obtain using PSO, LF analysis with SPFC is performed to estimate the state variables (V_s) and $(\delta_s)$of SPFC device and determine TRPL. Finally, cost analysis for installation of SPFC devices and savings due to reduction of TRPL is carried out.

II. OBJECTIVE FUNCTION FOR REDUCTION OF TRPL

The aim of this work is to reduce the TRPL of a power system network by rescheduling the line flow of the selected transmission lines using SPFC devices, also analyze the installation cost (IC) of SPFC and saving due to reduction of TRPL. The selection of transmission lines for the introduction of SPFC devices have been made based on the rank of PLSF. To achieve this objective, it is necessary to maximize the difference between TRPL obtained before and after the introduction of SPFC device in selected transmission lines. Therefore, the objective function for maximization in terms of difference between TRPL obtained before and after the introduction of SPFC device with power system operating constraints is represented as:

$$\int_{MAX} P_L = P_L^{BCLF} - P_L^{SPFC} \tag{1}$$

where,

P_L^{BCLF} is the TRPL obtained from base case load flow(BCLF)analysis without SPFC device.

P_L^{SPFC} is the TRPL obtained using LF analysis with SPFC devices in the selected sensitive lines.

The operating constraints of the network are listed below:

a) Equality constraints:

- Power balance equations are:

$$\left.\begin{aligned} P_i - \left[\sum_{j=1}^{n} |V_i||V_j|\left(G_{ij}\cos(\delta_{ij}) + B_{ij}\sin(\delta_{ij})\right)\right] = 0 \\ Q_i - \left[\sum_{j=1}^{n} |V_i||V_j|\left(G_{ij}\sin(\delta_{ij}) - B_{ij}\cos(\delta_{ij})\right)\right] = 0 \end{aligned}\right\} \tag{2}$$

b) Inequality constraints:

$$\left.\begin{aligned} P_l^{\min} \le P_l \le P_l^{\max} \\ V_i^{\min} \le V_i \le V_i^{\max} \\ \delta_i^{\min} \le \delta_i \le \delta_i^{\max} \end{aligned}\right\} \tag{3}$$

Constraints on SPFC device are:

$$\left.\begin{aligned} |V_s| \le V_s{}^{\max} \\ -\delta_s{}^{\min} \le \delta_s \le \delta_s{}^{\max} \end{aligned}\right\} \tag{4}$$

A. Identification of sensitive lines to introduce SPFC devices

The TRPL for a power system containing 'n' number of buses can be calculated as:

$$P_L = \sum_{i=1}^{n} P_i \tag{5}$$

The real power injection (P_i) at bus-i is represented as:

$$P_i = \sum_{j=1}^{n} V_i V_j \left(G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}\right) \tag{6}$$

Therefore, using (5) and (6), the TRPL can be represented as:

$$P_L = \sum_{i=1}^{n}\sum_{j=1}^{n} V_i V_j \left(G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}\right) \tag{7}$$

Considering bus number 1 as slack bus of the power system, the change in TRPL (ΔP_L) in terms of Jacobian matrix (J) and changes in real and reactive power injection (RRRI) can be expressed as:

$$\Delta P_L = [SFPL][\Delta P_2 \quad .. \quad \Delta P_n \quad \Delta Q_2 \quad ..\Delta Qn]^T \tag{8}$$

where, $\left[\frac{\partial P_L}{\partial \delta_2} \quad .. \quad .. \quad \frac{\partial P_L}{\partial \delta_n} \qquad \frac{\partial P_L}{\partial V_2} .. \quad .. \quad \frac{\partial P_L}{\partial V_n}\right][J]^{-1} = [SFPL]$

Again, ΔP_{km} in terms of J and ΔP_i & ΔQ_i for the line *km* can be expressed as:

$$\Delta P_{km} = [SLF_P][\Delta P_2 \quad .. \quad \Delta P_n \quad \Delta Q_2 \quad ..\Delta Qn]^T \tag{9}$$

where, $\left[\frac{\partial P_{km}}{\partial \delta_2} \quad .. \quad .. \quad \frac{\partial P_{km}}{\partial \delta_n} \qquad \frac{\partial P_{km}}{\partial V_2} .. \quad .. \quad \frac{\partial P_{km}}{\partial V_n}\right][J]^{-1} = [SLF_P]$

Multiply both sides of (9) by $[SLF_P]^T$

$$[SLF_P]^T \Delta P_{km} = [SLF_P][SLF_P]^T [\Delta P_2 \quad .. \quad \Delta P_n \quad \Delta Q_2 \quad ..\Delta Qn]^T$$

(10)

Divide (8) by (10),

$$\frac{\Delta P_L}{[SLF_P]^T \Delta P_{km}} = \frac{[SFPL]}{[SLF_P][SLF_P]^T}$$

$$\frac{\Delta P_L}{\Delta P_{km}} = \frac{[SFPL][SLF_P]^T}{[SLF_P][SLF_P]^T} \tag{11}$$

Using (11), SF_{Pkm} can be represented as:

$$SF_{Pkm} = \frac{\partial P_L}{\partial P_{km}} = \frac{[SFPL][SLF_P]^T}{[SLF_P][SLF_P]^T} \tag{12}$$

Similarly, ΔQ_{km} in terms of J and RRRI can be represented as

$$\Delta Q_{km} = [SLF_Q][\Delta P_2 \quad .. \quad \Delta P_n \quad \Delta Q_2 \quad ..\Delta Qn]^T \tag{13}$$

where, $\left[\frac{\partial Q_{km}}{\partial \delta_2} \quad .. \quad .. \quad \frac{\partial Q_{km}}{\partial \delta_n} \quad \quad \frac{\partial Q_{km}}{\partial V_2} .. \quad .. \quad \frac{\partial Q_{km}}{\partial V_n}\right][J]^{-1} = [SLF_Q]$

Multiply both sides of (13) by $[SLF_Q]^T$

$$[SLF_Q]^T \Delta Q_{km} = [SLF_Q][SLF_Q]^T [\Delta P_2 \quad .. \quad \Delta P_n \quad \Delta Q_2 \quad ..\Delta Qn]^T \tag{14}$$

Divide (8) by (14)

$$\frac{\Delta P_L}{[SLF_Q]^T \Delta Q_{km}} = \frac{[SFPL]}{[SLF_Q][SLF_Q]^T}$$

The above equation can be written as:

$$\frac{\Delta P_L}{\Delta Q_{km}} = \frac{[SFPL][SLF_Q]^T}{[SLF_Q][SLF_Q]^T} \tag{15}$$

Therefore, SF_{Qkm} can be represented as:

$$SF_{Qkm} = \frac{\partial P_L}{\partial Q_{km}} = \frac{[SFPL][SLF_Q]^T}{[SLF_Q][SLF_Q]^T} \tag{16}$$

Using (12) and (16), the PLSF value for the line *km* is calculated as:

$$PLSF = \sqrt{{SF_{Pkm}}^2 + {SF_{Qkm}}^2} \tag{17}$$

Using (17), PLSF values for all lines of a power system have been determined and they are ranked to select the lines for introduction SPFC device to reduce TRPL. The lines possessing significant value of PLSF in the PLSF rank list are selected for introduction of SPFC device. Now, using PSO technique the optimum values of schedule power flow for the selected lines are obtained to reduce TRPL with SPFC devices.

IIII. IMPLEMENTATION OF PSO TECHNIQUE

PSO is a swarm based search technique. Potential individuals or the particles fly through the search space and follow the current optimum particles. The potential particles make decision based on their own and other particles' experiences. The particles update or change their velocity and positions after each generation based on previous best position of their own and their neighbors'. Velocity of the particle is changed using (18) [2].

$$V_{ij}^{itr+1} = I_w \times V_{ij}^{itr} + W_1 \times R_1 \times (Pbest_{ij} - P_{ij}^{itr}) + W_2 \times R_2 \times (Gbest_{ij} - P_{ij}^{itr}) \tag{18}$$

The position of particle is changed using (19) [2].

$$P_{ij}^{(itr+1)} = P_{ij}^{(itr)} + V_{ij}^{(itr)} \tag{19}$$

where,

I_w= Inertia weight.
W_1= weighting factor (constant) related to P_{best}.
W_2= weighting factor (constant) related to G_{best}.
R_1 *and* R_2 is the random numbers varies from 0 to 1.

Proper selection of *'I_w'* provides balance between local and global search which results in less number of iteration required to get an optimal solution. The expression for I_w is calculated according to the expression given by (20)[2].

$$I_w = I_{w\max} - \frac{I_{w\max} - I_{w\min}}{itr_{\max}} \times itr \tag{20}$$

where,

I_{wmax} = initial weight (random numbers).
I_{wmin} = final weight (random numbers).
$itr_{\max}$ = maximum number of iteration.
itr = iteration count.

The basic algorithm of PSO technique is as follows:

1. Generate initial parameters of each particle randomly.
2. Calculate fitness function value at each location.
3. Determine the best function and the best location of particles.
4. Modify velocities and positions using previous best and global best.
5. Check stopping criteria if it does not satisfy the stopping criteria then go to step 2 and repeat.
6. Otherwise stop and show results.

A. Objective function for applying PSO technique

The objective function for implementation of PSO for reduction TRPL is expressed as:

$$\int_{PSO} = \xi P_L^{BCLF} + \Delta P_L \tag{21}$$

ξ is the factor that represents the targeted percentage of loss to be reduced with respect to the system loss determined from the BCLF results. ΔP_L is the change in system loss to be determined through rescheduling of power flow in the selected lines containing SPFC devices. The value of ΔP_L has to be negative to reduce TRPL of the power system. Therefore, ΔP_L is considered as positive term in the objective function configured for PSO.

Using calculus of variation, ΔP_L due to CPF through the line *km* can be represented as:

$$\Delta P_L = \left[\frac{\partial P_L}{\partial P_{km}}\Delta P_{km} + \frac{\partial P_L}{\partial Q_{km}}\Delta Q_{km}\right] \qquad (22)$$

Now, using sensitivity factors due to real and reactive power flow (RRPF) represented in (12) and (16) ΔP_L can be calculated as:

$$\Delta P_L = SF_{Pkm}\Delta P_{km} + SF_{Qkm}\Delta Q_{km} \qquad (23)$$

Therefore, for M number of lines, (23) can be expressed as:

$$\Delta P_L = \sum_{l=1}^{M} SF_{Pl}\Delta P_l + SF_{Ql}\Delta Q_l \qquad (24)$$

where, '*l*' represent the line number, connected between two buses.

The limits of the change in RRPF for the selected lines are set and the optimal values of ΔP_l and ΔQ_l of the selected lines are obtained using PSO. Thus, the schedule value of RRPF of the line is calculated as:

$$\left.\begin{aligned} P_l^{sch} &= P_l + \Delta P_l \\ Q_l^{sch} &= Q_l + \Delta Q_l \end{aligned}\right\} \qquad (25)$$

IV. IMPLEMENTATION OF LF ANALYSIS WITH SPFC DEVICE

The configuration of transmission line with a SPFC device connected in line *km* is shown in Fig. 1. The detailed modeling for LF analysis of the transmission line with controllable magnitude and phase angles based on SSSC device is presented in Appendix-I.

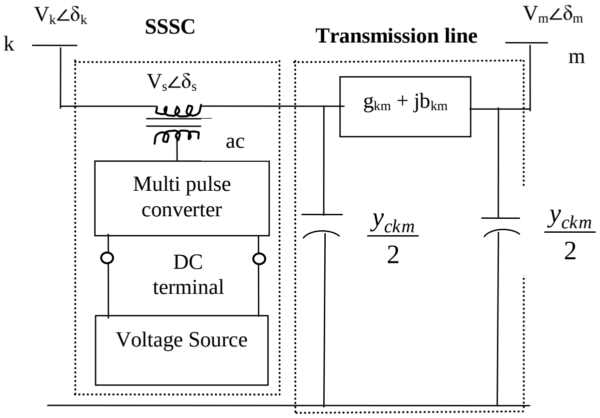

Fig. 1. Transmission line with a SPFC device in line *km*.

The expression of real and reactive power injections at bus-i without SPFC device for a power system is represented as [8]:

$$P_i = \left[\sum_{j=1}^{n} V_i V_j \left(G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}\right)\right] \qquad (26)$$

$$Q_i = \left[\sum_{j=1}^{n} V_i V_j \left(G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}\right)\right] \qquad (27)$$

As, SPFC is considered as an integral part of the transmission line, the expression for power injections where SPFC is considered will be different from the power injections represented by (26) and (27). Therefore, the expression for P_{km} with SPFC device given by (A.6) is added and subtracted by $V_k^2 g_{km}$ to represent the P_{km} through the line with SPFC device so that the expression for line flow without SPFC is also present in it.

$$\begin{aligned} P_{km} &= V_k^2 g_A - V_k V_s\left[g_A\cos\delta_{ks} + b_A\sin\delta_{ks}\right] \\ &+ V_k V_m\left[g_B\cos\delta_{km} + b_B\sin\delta_{km}\right] \\ &- V_k V_m\left[g_{km}\cos\delta_{km} + b_{km}\sin\delta_{km}\right] + V_k^2 g_{km} - V_k^2 g_{km} \end{aligned} \qquad (28)$$

The term $V_k^2 g_{km} - V_k V_m\left[g_{km}\cos\delta_{km} + b_{km}\sin\delta_{km}\right]$ represents P_{km} through the line without SPFC device and it is already incorporated in the power injection expression. Therefore, the modified expression for power injection (P_k) with SPFC device is represented as:

$$\begin{aligned} P_k &= \left[\sum_{j=1}^{n} V_k V_j\left(G_{kj}\cos\delta_{kj} + B_{kj}\sin\delta_{kj}\right)\right] + V_k^2 g_A \\ &- V_k V_s\left[g_A\cos\delta_{ks} + b_A\sin\delta_{ks}\right] \\ &+ V_k V_m\left[g_B\cos\delta_{km} + b_B\sin\delta_{km}\right] - V_k^2 g_{km} \end{aligned} \qquad (29)$$

Similarly, the expression for Q_{km} with SPFC device given by (A.7) is added and subtracted by $V_k^2\left(b_{km} + \frac{y_{ckm}}{2}\right)$ to represent the Q_{km} through the line with SPFC device so that the expression for line flow without SPFC is also present in it.

$$\begin{aligned} Q_{km} &= -V_k^2 b_A - V_k V_s\left[g_A\sin\delta_{ks} - b_A\cos\delta_{ks}\right] \\ &+ V_k V_m\left[g_B\sin\delta_{km} - b_B\cos\delta_{km}\right] + V_k^2\left(b_{km} + \frac{y_{ckm}}{2}\right) \\ &- V_k^2\left(b_{km} + \frac{y_{ckm}}{2}\right) - V_k V_m\left[g_{km}\sin\delta_{km} - b_{km}\cos\delta_{km}\right] \end{aligned} \qquad (30)$$

The term $-V_k^2\left(b_{km} + \frac{y_{ckm}}{2}\right) - V_k V_m\left[g_{km}\sin\delta_{km} - b_{km}\cos\delta_{km}\right]$ represents the Q_{km} through the line without SPFC device and it is already incorporated in the expression of the power injection expression. Therefore, the modified expression for power injection (Q_k) with SPFC device is represented as:

$$\begin{aligned} Q_k &= \left[\sum_{j=1}^{n} V_k V_j \left(G_{kj}\sin\delta_{kj} - B_{kj}\cos\delta_{kj}\right)\right] - V_k^2 b_A \\ &- V_k V_s \left[g_A \sin\delta_{ks} - b_A \cos\delta_{ks}\right] \\ &+ V_k V_m \left[g_B \sin\delta_{km} - b_B \cos\delta_{km}\right] + V_k^2\left(b_{km} + \frac{y_{ckm}}{2}\right) \end{aligned} \tag{31}$$

Again, the expression P_{mk} with SPFC device given by (A.13) is added and subtracted by $V_k V_m\left(g_{km}\cos\delta_{mk} + b_{km}\sin\delta_{mk}\right)$ to represent P_{mk} with SPFC device, so that the expression for line flow without SPFC is also present in it.

$$\begin{aligned} P_{mk} &= V_m^2 g_{km} - V_k V_m\left(g_{km}\cos\delta_{mk} + b_{km}\sin\delta_{mk}\right) - V_m^2 g'_B \\ &- V_k V_m\left(g'_A\cos\delta_{mk} + b'_A\sin\delta_{mk}\right) + V_s V_m\left(g'_A\cos\delta_{ms} + b'_A\sin\delta_{ms}\right) \\ &+ V_k V_m\left(g_{km}\cos\delta_{mk} + b_{km}\sin\delta_{mk}\right) \end{aligned} \tag{32}$$

The term $V_m^2 g_{km} - V_k V_m\left(g_{km}\cos\delta_{mk} + b_{km}\sin\delta_{mk}\right)$ represents the P_{mk} through the line without SPFC device and it is already incorporated in expression of P_m. Therefore, the modified expression for P_m with SPFC device is represented as:

$$\begin{aligned} P_m &= \left[\sum_{j=1}^{n} |V_m||V_j|\left(G_{mj}\cos\delta_{mj} + B_{mj}\sin\delta_{mj}\right)\right] - V_m^2 g'_B \\ &- V_k V_m\left(g'_A\cos\delta_{mk} + b'_A\sin\delta_{mk}\right) + V_s V_m\left(g'_A\cos\delta_{ms} + b'_A\sin\delta_{ms}\right) \\ &+ V_k V_m\left(g_{km}\cos\delta_{mk} + b_{km}\sin\delta_{mk}\right) \end{aligned} \tag{33}$$

Similarly, the expression for Q_{mk} with SPFC device given by (A.14) is added and subtracted by $V_k V_m(g_{km}\sin\delta_{mk} - b_{km}\cos\delta_{mk})$ to represent the expression of Q_{mk} with SPFC device, so that the expression for line flow without SPFC is also present in it.

$$\begin{aligned} Q_{mk} &= -V_m^2\left(b_{km} + \frac{y_{ckm}}{2}\right) - V_m V_k\left(g_{km}\sin\delta_{mk} - b_{km}\cos\delta_{mk}\right) - V_m^2 b'_B \\ &- V_k V_m\left(g'_A\sin\delta_{mk} - b'_A\cos\delta_{mk}\right) + V_s V_m\left(g'_A\sin\delta_{ms} - b'_A\cos\delta_{ms}\right) \\ &+ V_m V_k\left(g_{km}\sin\delta_{mk} - b_{km}\cos\delta_{mk}\right) \end{aligned} \tag{34}$$

The term $-V_m^2\left(b_{km} + \frac{y_{ckm}}{2}\right) - V_m V_k\left(g_{km}\sin\delta_{mk} - b_{km}\cos\delta_{mk}\right)$ represents the Q_{mk} without SPFC device and it is already incorporated in expression of Q_m. Therefore, the modified expression for Q_m with SPFC device can be represented as:

$$\begin{aligned} Q_m &= \left[\sum_{j=1}^{n} |V_m||V_j|\left(G_{mj}\sin(\delta_{mj}) - B_{mj}\cos(\delta_{mj})\right)\right] - V_m^2 b'_B \\ &- V_k V_m\left(g'_A\sin\delta_{mk} - b'_A\cos\delta_{mk}\right) + V_s V_m\left(g'_A\sin\delta_{ms} - b'_A\cos\delta_{ms}\right) \\ &+ V_m V_k\left(g_{km}\sin\delta_{mk} - b_{km}\cos\delta_{mk}\right) \end{aligned} \tag{35}$$

Using (A.5, A.6, A.13, A.14) for power flow through the line with SPFC device and power injection expressions given in (29,31,33,35), the NR based LF model with a SPFC device connected in the line *km* can be represented as:

$$\begin{bmatrix}
\frac{\partial P_2}{\partial \delta_2} & .. & \frac{\partial P_2}{\partial \delta_k} & .. & \frac{\partial P_2}{\partial \delta_m} & .. & \frac{\partial P_2}{\partial \delta_n} & \frac{\partial P_2}{\partial V_2} & .. & \frac{\partial P_2}{\partial V_k} & .. & \frac{\partial P_2}{\partial V_m} & .. & \frac{\partial P_2}{\partial V_n} & 0 & 0 \\
.. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\
\frac{\partial P_k}{\partial \delta_2} & .. & \frac{\partial P_k}{\partial \delta_k} & .. & \frac{\partial P_k}{\partial \delta_m} & .. & \frac{\partial P_k}{\partial \delta_n} & \frac{\partial P_k}{\partial V_2} & .. & \frac{\partial P_k}{\partial V_k} & .. & \frac{\partial P_k}{\partial V_m} & .. & \frac{\partial P_k}{\partial V_n} & \frac{\partial P_k}{\partial \delta_s} & \frac{\partial P_k}{\partial V_s} \\
.. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\
\frac{\partial P_m}{\partial \delta_2} & .. & \frac{\partial P_m}{\partial \delta_k} & .. & \frac{\partial P_m}{\partial \delta_m} & .. & \frac{\partial P_m}{\partial \delta_n} & \frac{\partial P_m}{\partial V_2} & .. & \frac{\partial P_m}{\partial V_k} & .. & \frac{\partial P_m}{\partial V_m} & .. & \frac{\partial P_m}{\partial V_n} & \frac{\partial P_m}{\partial \delta_s} & \frac{\partial P_m}{\partial V_s} \\
.. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\
\frac{\partial P_n}{\partial \delta_2} & .. & \frac{\partial P_n}{\partial \delta_k} & .. & \frac{\partial P_n}{\partial \delta_m} & .. & \frac{\partial P_n}{\partial \delta_n} & \frac{\partial P_n}{\partial V_2} & .. & \frac{\partial P_n}{\partial V_k} & .. & \frac{\partial P_n}{\partial V_m} & .. & \frac{\partial P_n}{\partial V_n} & 0 & 0 \\
\frac{\partial Q_2}{\partial \delta_2} & .. & \frac{\partial Q_2}{\partial \delta_k} & .. & \frac{\partial Q_2}{\partial \delta_m} & .. & \frac{\partial Q_2}{\partial \delta_n} & \frac{\partial Q_2}{\partial V_2} & .. & \frac{\partial Q_2}{\partial V_k} & .. & \frac{\partial Q_2}{\partial V_m} & .. & \frac{\partial Q_2}{\partial V_n} & 0 & 0 \\
.. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\
\frac{\partial Q_k}{\partial \delta_2} & .. & \frac{\partial Q_k}{\partial \delta_k} & .. & \frac{\partial Q_k}{\partial \delta_m} & .. & \frac{\partial Q_k}{\partial \delta_n} & \frac{\partial Q_k}{\partial V_2} & .. & \frac{\partial Q_k}{\partial V_k} & .. & \frac{\partial Q_k}{\partial V_m} & .. & \frac{\partial Q_k}{\partial V_n} & \frac{\partial Q_k}{\partial \delta_s} & \frac{\partial Q_k}{\partial V_s} \\
.. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\
\frac{\partial Q_m}{\partial \delta_2} & .. & \frac{\partial Q_m}{\partial \delta_k} & .. & \frac{\partial Q_m}{\partial \delta_m} & .. & \frac{\partial Q_m}{\partial \delta_n} & \frac{\partial Q_m}{\partial V_2} & .. & \frac{\partial Q_m}{\partial V_k} & .. & \frac{\partial Q_m}{\partial V_m} & .. & \frac{\partial Q_m}{\partial V_n} & \frac{\partial Q_m}{\partial \delta_s} & \frac{\partial Q_m}{\partial V_s} \\
.. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\
\frac{\partial Q_n}{\partial \delta_2} & .. & \frac{\partial Q_n}{\partial \delta_k} & .. & \frac{\partial Q_n}{\partial \delta_m} & .. & \frac{\partial Q_n}{\partial \delta_n} & \frac{\partial Q_n}{\partial V_2} & .. & \frac{\partial Q_n}{\partial V_k} & .. & \frac{\partial Q_n}{\partial V_m} & .. & \frac{\partial Q_n}{\partial V_n} & 0 & 0 \\
0 & .. & \frac{\partial P_{km}}{\partial \delta_k} & .. & \frac{\partial P_{km}}{\partial \delta_m} & .. & 0 & 0 & .. & \frac{\partial P_{km}}{\partial V_k} & .. & \frac{\partial P_{km}}{\partial V_m} & .. & 0 & \frac{\partial P_{km}}{\partial \delta_s} & \frac{\partial P_{km}}{\partial V_s} \\
0 & .. & \frac{\partial Q_{km}}{\partial \delta_k} & .. & \frac{\partial Q_{km}}{\partial \delta_m} & .. & 0 & 0 & .. & \frac{\partial Q_{km}}{\partial V_k} & .. & \frac{\partial Q_{km}}{\partial V_m} & .. & 0 & \frac{\partial Q_{km}}{\partial \delta_s} & \frac{\partial Q_{km}}{\partial V_s}
\end{bmatrix}
\begin{bmatrix} \Delta\delta_2 \\ .. \\ \Delta\delta_k \\ .. \\ \Delta\delta_m \\ .. \\ \Delta\delta_n \\ \Delta V_2 \\ .. \\ \Delta V_k \\ .. \\ \Delta V_m \\ .. \\ \Delta V_n \\ \Delta\delta_s \\ \Delta V_s \end{bmatrix}
=
\begin{bmatrix} \Delta P_2 \\ .. \\ \Delta P_k \\ .. \\ \Delta P_m \\ .. \\ \Delta P_n \\ \Delta Q_2 \\ .. \\ \Delta Q_k \\ .. \\ \Delta Q_m \\ .. \\ \Delta Q_n \\ \Delta P_{km} \\ \Delta Q_{km} \end{bmatrix} \tag{36}$$

Equation (36) can be rewritten in compact from as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \\ \Delta P_{km} \\ \Delta Q_{km} \end{bmatrix} = \left[J_{SPFC}\right] \begin{bmatrix} \Delta\delta \\ \Delta V \\ \Delta\delta_s \\ \Delta V_s \end{bmatrix} \tag{37}$$

The limits on SPFC parameters are taken as:

$$|V_s| \le 0.2 \tag{38}$$

$$-\pi \le \delta_s \le \pi \tag{39}$$

A. Solution procedure for reduction of TRPL

The solution process starts with a BCLF analysis of a power system to determine its initial operating condition and TRPL of the system. Using the BCLF results, PLSFs for the lines are calculated, and those are ranked according to the degree of their sensitivity values for the purpose of selecting lines, where SPFC device is to be introduced for reducing TRPL. The lines that possess significant values of PLSF in the

PLSF rank list are selected for the introduction SPFC devices. It is necessary to adjust the value of ξ given in the objective function of PSO to determine the changes in RRPF (ΔP_l and ΔQ_l) through the lines with SPFC device. It is to be noted that the reduction of TRPL is possible to a certain percentage of P_L^{BCLF}. Therefore, an iterative search technique has to be adopted with assigned limits of ξ. For this purpose, bisection method is applied to assign the value of ξ in the objective function of PSO using the upper limit $\left(\xi^{\max}\right)$ and lower limit $\left(\xi^{\min}\right)$ of ξ. With the assigned value of ξ, the optimal values of ΔP_l and ΔQ_l through the lines with SPFC device are determine using PSO technique. The modified values of schedule power flow are determined by (25). At this point it is necessary to validate the process by performing a LF analysis with SPFC device. Therefore, LFAWSPFC is carried out to examine the possible operating limit violation of SPFC parameters and other power system operating constraints. In case any violation detected, $\xi^{\max}$ is set with value of assigned ξ in the objective function of PSO, i.e., with the present assigned value of ξ, reduction of TRPL is not possible due to limit violation of power system operating constraints. Thus, the PSO solution has to be carried out using bisection method with modified restricted upper limit of $\xi^{\max}$. If no limit violation is detected, $\xi^{\min}$ is set with assigned value of ξ in the objective function of PSO, i.e., with the present value of ξ , reduction of TRPL is achieved and further reduction in TRPL is possible. It is worth mentioning that the CPF through lines with SPFC device would affect power balance of the system and to maintain the power balance, the power flow through other lines would change. The CPF through other lines which are less sensitive to the CPF would also influence the TRPL of the system. At one point of time TRPL may increase for further modification of schedule power flow through the line with SPFC. This condition is checked in the iterative process. The term $\varepsilon^{K+1} = P_L^{BCLF} - P_L^{SPFC}$ is introduced in the flow chart above, where, ε^{K+1} represent the reduction in TRPL achieved at *(K+1)*th iteration. Iterative process has to be carried out till $\varepsilon^{K+1} > \varepsilon^K$ where, ε^K represent the reduction in TRPL achieved at K^{th} iteration. Basically, the logic used for the termination of iterative process confirms the satisfaction of objective function shown in (1). The solution procure is depicted with the flow chart shown in Fig. 2.

V. IMPLEMENTATION OF THE LF ANALYSIS WITH SPFC DEVICES IN THE SENSITIVE LINES

To implement the proposed method reduction of TRPL using SPFC devices the standard IEEE 118 bus system is considered. The results obtained for both with and without considering SPFC devices in the sensitive lines of the system are presented in this section.

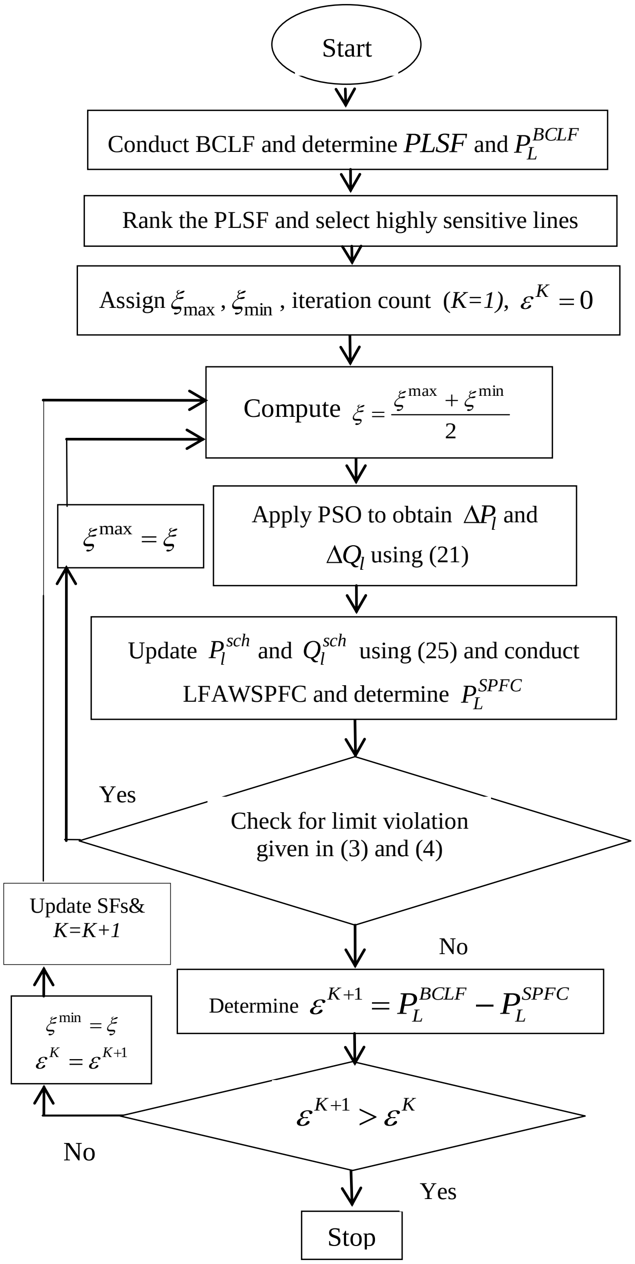

Fig. 2. Flowchart for rescheduling of power flow using SPFC device.

A. Identification of sensitive lines and reduction of TRPL using SPFC device

Initially LF analysis without SPFC device is performed on IEEE 118 bus system to determine initial operating condition of the system. Using the initial LF results, SF relation for change in TRPL (ΔP_L) with the changes in real (ΔP_l) and reactive (ΔQ_l) power flow are determined using (12) and (16). Using the values SF_{Pl} and SF_{Ql} , PLSF values for the transmission lines are calculated from (17). Fig. 3 represents the PLSFs obtained using initial LF results for IEEE 118 bus system. It is seen that the line no 111, 41, 112, 39, 116, 115 and 120 have significantly large values of PLSF than the other lines of the system. Therefore, these lines are considered for the introduction of SPFC device. As the line no 111 is connected between slack bus and PV bus, line nos. 41, 39 and 115 are connected between two PV buses, it is not possible to change the power flow through these lines as they are radially connected without any parallel path to accommodate the CPF in the lines. Therefore, these lines are not considered for introduction of SPFC devices, the remaining lines (line nos. 112 and 116 and 120) are considered for rescheduling of power flow with SPFC devices to reduce TRPL of the system.

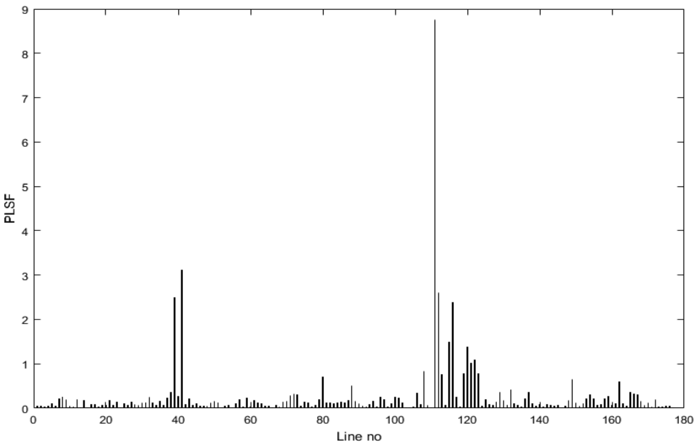

Fig.3. PLSF for IEEE 118 bus system.

TABLE I. SENSITIVITY FACTORS OF THE SELECTED LINES FOR THE INTRODUCTION OF SPFC DEVICES OBTAINED USING BCLF RESULTS

Line No	Line connected between		Sensitivity Factor	
	Bus k	*Bus m*	SF_P	SF_Q
112	69	75	0.2917	2.5859
116	70	75	0.3138	2.3577
120	75	77	-0.1036	-1.3723

While solving for optimal values for ΔP_l and ΔQ_l for the selected lines using PSO, the CPF limits are taken as -0.2 pu to +0.2 pu. The limits are restricted to narrow values of 0.2 pu to +0.2 pu, so that estimated values of ΔP_l and ΔQ_l remains relatively low during the iteration process of bisection method. It is done to avoid large change in ΔP_l and ΔQ_l in single step of bisection method (insignificantly low value of SFs for line would lead to estimation large value of ΔP_l and ΔQ_l). Large value of ΔP_l and ΔQ_l create large changes in schedule RRPF through the line with SPFC. Thus during LF analysis using SPFC devices, the LF may not converge or violates the operating limits of the SPFC device. For every bisection iteration step, ΔP_l and ΔQ_l (for *l*=1, .. , M) is determined by PSO are used to calculate modified schedule power flow through the lines with SPFC using (25) and LF analysis with SPFC is performed. For the bisection method the limits for ξ are assigned as $\xi^{\max} = 0.2$ (i.e., 20%) and $\xi^{\min} = 0.0$. The maximum reduction of TRPL is targeted as the 20% of P_L^{BCLF} . The final result after termination of iterative process, are presented in Table (II-IV).

The RRPF obtained for initial LF analysis and LF analysis with SPFC devices are given in Table II. The proposed method of LF analysis with SPFC devices at the selected lines are able to control both RRPF through the lines which results in reduction of TRPL presented in Table II. The voltage magnitudes and phase angles of the three SPFC devices connected at the line numbers 112, 116 and 120 are also presented in Table II. It can be observed that all the SPFC parameters are within their respective limits given in (38) and (39). Again, it has been observed that due to introduction SPFC in the three lines of the system voltage profile of IEEE 118 bus system has improved. The improved bus voltages of the test system are shown in Table III. The TRPL obtained with and without SPFC devices is shown in Table IV and it is found that the proposed method of LF analysis with SPFC devices is able to reduce power loss to 2.5151 p.u. from 3.0851 p.u. with final value of ξ at 0.18475 at the end of the termination of iterative process. The value of ξ appeared as 0.2 (i.e., 20%, the maximum targeted % reduction of TRPL), the bisection iterative process has to be repeated with larger value of $\xi^{\max}$, as there is room for further reduction of TRPL of the system. The values of $\xi_{\max}$ and $\xi_{\min}$ are to be chosen such way that it should not overestimate P_l^{sch} and Q_l^{sch} , otherwise it may lead to overloading of the lines with SPFC and further cause numerical instability of LF analysis and

TABLE II. POWER FLOWS IN THE SELECTED LINES BEFORE AND AFTER THE INTRODUCTION OF SPFC DEVICE AND SPFC PARAMETERS (ALL PARAMETERS ARE MENTIONED IN P.U.)

Line No	Line connected between buses		Line flows without SPFC device		Schedule line flow with SPFC device		Change in line flow		SPFC Parameters for line no (l)	
(l)	*Bus-k*	*Bus-m*	P_{km}	Q_{km}	P_{km}	Q_{km}	ΔP_{km}	ΔQ_{km}	V_s	θ_s
112	69	75	2.3912	0.8314	2.0972	1.7025	-0.294	0.8711	0.1569	2.4806
116	70	75	0.4452	0.6786	0.3173	1.593	-0.1279	0.9144	0.1825	2.4267
120	75	77	-0.7317	-0.3891	-0.5324	0.2331	-0.1993	-0.156	0.1226	3.1374

TABLE III. BUS VOLTAGE AND PHASE ANGLES OF IEEE 118 BUS SYSTEM OBTAINED USING LF ANALYSIS WITHOUT AND WITH SPFC DEVICE IN THE SELECTED TRANSMISSION LINES (ALL PARAMETERS ARE MENTIONED IN P.U.)

Bus No	Bus type	Bus voltage and phase angles			
		Without SPFC device		*With SPFC device*	
		$\|V\|$	δ	$\|V\|$	δ
69	Slack	1.035	0	1.035	0
75	PQ	0.863	-0.2868	0.9004	-0.3041
70	PV	0.984	-0.2488	0.984	-0.2404
77	PV	1.006	-0.1422	1.006	-0.1317

TABLE IV. TRPL IN P.U. OBTAINED FOR IEEE 118 BUS SYSTEM

TRPL from BCLF	TRPL from LFAWSPFC	Reduction of TRPL
3.0851	2.5151	0.57

create convergence problem. Again, immediate large CPF through lines with SPFC device would result large CPF through the other lines of the power system to maintain the power balance. This may again, increase TRPL of the system. In this work the assigned range for targeted percentage loss to be reduced is taken as (0-20) % and it has been settled down to 18% corresponding to minimum TRPL.

B. Cost analysis for installation of SPFC devices and cost benefit due to reduction of TRPL

The cost of installation of a SPFC device is quite significant and it varies in accordance to the installation capacity of the device. The cost installation of SPFC is provide as [9]

$$C = 0.0003 R_p^2 - 0.2691 R_p + 188.22 \tag{40}$$

where, C is the IC of a SPFC device and R_p is the range of operation of the SPFC device in MVAR [12]. Therefore, the value of R_p would be the difference in MVAR flow for a line with and without a SPFC device and can be represented as:

$$R_p = \left\| \left| Q_{km}^{\overset{With}{SPFC}} \right| - \left| Q_{km}^{\overset{Without}{SPFC}} \right| \right\| \tag{41}$$

Equation (40) shows that the cost of installation of a SPFC device has a fixed term and two other variable terms dependent on installation capacity of the device. The overall installation cost for a SPFC device is represented as [9]:

$$IC = C * R_p * 1000 \text{ in \$} \tag{42}$$

Calculation of cost of installation for SPFC devices to reduce TRPL of the test system is shown in Table V. The cost of energy generation in India varies from ₹4/kWhr - ₹5/kWhr throughout the day. Therefore, to compare the IC of SPFC devices and cost of saving due to reduction of system loss, the cost of energy is considered in US$.

The reduction in TRPL using SPFC devices in the selected sensitive lines of the IEEE 118 bus for the base load has been found as 0.57 p.u. (=57 MW). However, it would vary according to the loading condition of the system at different hours of the day. As such, average reduction of TRPL for a year can be represented as:

TABLE V. CALCULATION OF INSTALLATION COST OF THE SPFC DEVICES IN THREE LINES (WITH BASE MVA =100)

Line no	R_{pl}	Installation Cost of the SPFC devices $IC_l = C_l * R_{pl} * 1000$ in \$	Over all installation cost $IC = IC_{112}+IC_{116}+IC_{120}$ in \$
112	87.11	167.06*87.11*1000=14,552,597	32,614,569
116	91.44	166.12*91.44*1000=15,190,013	
120	15.6	184.10*15.6*1000=2,871,960	

Average reduction of TRPL/ year
= MF * Reduction of TRPL in MW for the base case load *1000* 8760*cost/unit

Therefore, cost of saving due to reduction of TRPL for IEEE 118 bus system can be represented as:

Cost of savings = MF * Reduction of TRPL in MW for the base case load * 1000*8760*cost/unit (in $)

It can be seen from Table VI that the cost of savings due to reduction in TRPL with two different cost per unit and MF are significant. It shows that with MF=1 and cost of power unit as ₹5/kWhr, would recover the installation cost of the three SPFCs within a year. However, the reduction in TRPL would vary according to the load variation through a day, therefore, MF may be less or more than 1 depending upon the loading condition of power system.

TABLE VI. COMPARISON OF COST FOR INSTALLATION OF SPFC AND SAVING DUE TO REDUCTION OF TRPL

Total IC of 3 SPFC devices ($)	Cost/ unit in ₹/kWhr	Cost/unit ($/kWhr) Taking conversion rate 1 $= ₹ 73.35	MF	Average reduction of TRPL/ year in $
32,614,569	4	0.054	0.5	13,481,640
			0.6	16,177,968
			0.7	18,874,296
			0.8	21,570,624
			0.9	24,266,952
			1.0	26,963,280
	5	0.068	0.5	16,976,880
			0.6	20,372,256
			0.7	23,767,632
			0.8	27,163,008
			0.9	30,558,384
			1.0	33,953,760

To present a scenario for recovery of cost of installation of three SPFCs, MF is taken as 0.7 and for this value, the savings per year for the IEEE 118 bus system would be $18,874,296 or $23,767,632/ year for unit cost of ₹4/kWhr or ₹5/kWhr respectively. Even for this scenario, the recovery of cost of installation of three SPFCs could be attained within 1 and $\frac{3}{4}$ years and 1and $\frac{5}{12}$ years for two different costs per unit. Beyond this period, they will offer cost benefit due to reduction of TRPL of the system. In addition to this, the SPFC devices would provide other useful applications, such as power transfer enhancement, alleviation of line overload in nearby overloaded line, improvement of bus voltage profile etc. for improving the performance of the power system.

VI. CONCLUSIONS

In this work a method for reduction of TRPL by CPF through the lines using SPFC devices are proposed. To identify the lines suitable for introduction of SPFC devices, PLSF for lines were determined. Lines with significant values of PLSFs were considered for introduction SPFC devices and power flow through these lines were rescheduled to reduce TRPL of a power system. PSO technique is used to obtain the optimal changes in RRPF for reduction of TRPL of a power system. To verify the effectiveness of the proposed method, LF analysis is performed in standard IEEE 118 bus and it has been observed that rescheduling of power flow through the sensitive lines using SPFC devices involving PSO technique could reduce TRPL of the system from 3.0851 p.u to 2.5151 p.u. Further, it has been observed from the cost analysis of the SPFC devices that installation cost of the SPFCs would be recovered within 1 and $\frac{3}{4}$ years, even with MF=0.7 and cost of unit as ₹4/kWhr. Further, they would also create facility for other useful applications such as power transfer enhancement, alleviation of line overload in nearby overloaded line, improvement of bus voltage profile etc.

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Appendix-I:

The electrical equivalent circuit diagram of a transmission line with a SPFC device containing SSSC controller is shown in Fig. A1. The following circuit is used to determine the RRPF through the line with SPFC device.

$\bar{I}_{km}$ = current through the line *km* i.e., from bus k to m.

$\bar{I}_{mk}$ =current through the line *mk* i.e., from bus m to k.

V_x= voltage at node *x*.

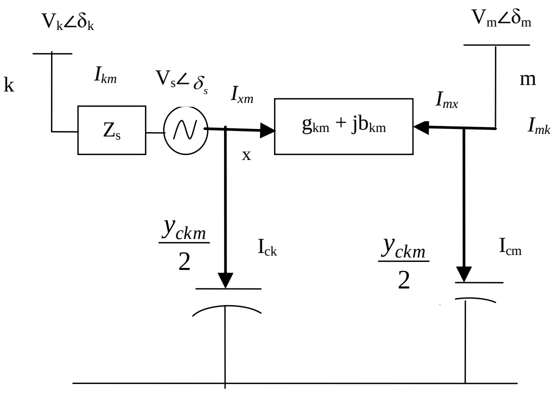

Fig. A1: The equivalent circuit diagram of a transmission line with an SPFC device containing SSSC controller.

The expression for complex power flow in line *km* is represented as:

$$P_{km} - jQ_{km} = \bar{V}_k^* \left(\bar{I}_{xm} + \bar{I}_{ck}\right) \qquad \text{(A.1)}$$

Now,

$$\bar{I}_{km} = \left(\bar{V}_k - \bar{V}_s - \bar{V}_x\right) y_s \qquad \text{(A.2)}$$

$$\bar{I}_{km} = \bar{V}_x \frac{y_{ckm}}{2} + \left(\bar{V}_x - \bar{V}_m\right) y_{km} \qquad \text{(A.3)}$$

Using (A.2) and (A.3),

$$\left(\bar{V}_k - \bar{V}_s - \bar{V}_x\right) y_s = \bar{V}_x \frac{y_{ckm}}{2} + \left(\bar{V}_x - \bar{V}_m\right) y_{km}$$

$$\left(\bar{V}_k - \bar{V}_s\right) y_s - \bar{V}_x y_s = \bar{V}_x \left[\frac{y_{ckm}}{2} + y_{km}\right] - \bar{V}_m y_{km}$$

$$\bar{V}_x \left[\frac{y_{ckm}}{2} + y_{km} + y_s\right] = \left(\bar{V}_k - \bar{V}_s\right) y_s + \bar{V}_m y_{km}$$

$$\bar{V}_x = \frac{\left(\bar{V}_k - \bar{V}_s\right) y_s}{y_T} + \frac{\bar{V}_m y_{km}}{y_T}$$

$$\bar{V}_x = \left(\bar{V}_k - \bar{V}_s\right) y_a + \bar{V}_m y_b \qquad \text{(A.4)}$$

where,

$$y_T = \left[\frac{y_{ckm}}{2} + y_{km} + y_s\right],$$

$$y_a = \frac{y_s}{y_T}$$

$$y_b = \frac{y_{km}}{y_T}$$

Power flow in line *km* is represented as:

$$P_{km} - jQ_{km} = \bar{V}_k^* \left[\bar{V}_x \frac{y_{ckm}}{2} + \left(\bar{V}_x - \bar{V}_m\right) y_{km}\right]$$

$$= \bar{V}_k^* \bar{V}_x \left[\frac{y_{ckm}}{2} + y_{km}\right] - \bar{V}_k^* \bar{V}_m y_{km}$$

Using (A.4) in the above equation,

$$= \bar{V}_k^* \left[\left(\bar{V}_k - \bar{V}_s\right) y_a + \bar{V}_m y_b\right] \left[\frac{y_{ckm}}{2} + y_{km}\right] - \bar{V}_k^* \bar{V}_m y_{km}$$

$$= \bar{V}_k^2 \left(y_a \left[\frac{y_{ckm}}{2} + y_{km}\right]\right) - \bar{V}_k^* \bar{V}_s \left(y_a \left[\frac{y_{ckm}}{2} + y_{km}\right]\right)$$

$$+ \bar{V}_k^* \bar{V}_m \left(y_b \left[\frac{y_{ckm}}{2} + y_{km}\right]\right) - \bar{V}_k^* \bar{V}_m y_{km}$$

$$= \bar{V}_k^2 y_A - \bar{V}_k^* \bar{V}_s y_A + \bar{V}_k^* V_m y_B - \bar{V}_k^* \bar{V}_m y_{km}$$

$$\begin{aligned} &= V_k^2 y_A \angle\theta_A - V_k V_s y_A \angle\left(\theta_A - \delta_{ks}\right) + \\ &V_k V_m y_B \angle\left(\theta_B - \delta_{km}\right) - V_k V_m y_{km} \angle\left(\theta_{km} - \delta_{km}\right) \end{aligned} \qquad \text{(A.5)}$$

where,

$$y_A = \left(y_a \left[\frac{y_{ckm}}{2} + y_{km}\right]\right) = g_A + jb_A$$

$$y_B = \left(y_b \left[\frac{y_{ckm}}{2} + y_{km}\right]\right) = g_B + jb_B$$

Separating real and imaginary parts of (A.5), the expressions for RRPF through the line *km* i.e., from bus k to m are calculated as:

$$\begin{aligned} P_{km} &= V_k^2 y_A \cos\theta_A - V_k V_s y_A \cos\left(\theta_A - \delta_{ks}\right) + \\ &V_k V_m y_B \cos\left(\theta_B - \delta_{km}\right) - V_k V_m y_{km} \cos\left(\theta_{km} - \delta_{km}\right) \\ &= V_k^2 g_A - V_k V_s \left[g_A \cos\delta_{ks} + b_A \sin\delta_{ks}\right] + \\ &V_k V_m \left[g_B \cos\delta_{km} + b_B \sin\delta_{km}\right] - V_k V_m \left[g_{km} \cos\delta_{km} + b_{km} \sin\delta_{km}\right] \end{aligned} \qquad \text{(A.6)}$$

$$\begin{aligned}
-Q_{km} &= V_k^2 y_A \sin\theta_A - V_k V_s y_A \sin(\theta_A - \delta_{ks}) + \\
&V_k V_m y_B \sin(\theta_B - \delta_{km}) - V_k V_m y_{km} \sin(\theta_{km} - \delta_{km}) \\
Q_{km} &= -V_k^2 b_A - V_k V_s [g_A \sin\delta_{ks} - b_A \cos\delta_{ks}] + \\
&V_k V_m [g_B \sin\delta_{km} - b_B \cos\delta_{km}] - V_k V_m [g_{km} \sin\delta_{km} - b_{km} \cos\delta_{km}]
\end{aligned} \tag{A.7}$$

where,

$$\delta_{ks} = \delta_k - \delta_s$$

$$\delta_{km} = \delta_k - \delta_m$$

$$\theta_{km} = \theta_k - \theta_m$$

The expression for complex power flow through the line *mk* i.e., from bus m to k is represented as:

$$P_{mk} - jQ_{mk} = \bar{V}_m^* (\bar{I}_{mx} + \bar{I}_{cm}) \tag{A.8}$$

From Fig.A1, the current through SPFC device can be written as:

$$\bar{I}_s = (\bar{V}_x + \bar{V}_s - \bar{V}_k) y_s \tag{A.9}$$

$$\bar{I}_s = (\bar{V}_m - \bar{V}_x) y_{km} - \bar{V}_x \frac{y_{ckm}}{2} \tag{A.10}$$

Using (A.9 and A.10),

$$(\bar{V}_x + \bar{V}_s - \bar{V}_k) y_s = (\bar{V}_m - \bar{V}_x) y_{km} - \bar{V}_x \frac{y_{ckm}}{2}$$

$$\bar{V}_m y_{km} + (\bar{V}_k - \bar{V}_s) y_s = \bar{V}_x \left[y_s + y_{km} + \frac{y_{ckm}}{2} \right]$$

$$\bar{V}_x = \frac{(\bar{V}_k - \bar{V}_s) y_s}{y_T} + \frac{\bar{V}_m y_{km}}{y_T}$$

$$\bar{V}_x = (\bar{V}_k - \bar{V}_s) y_a + \bar{V}_m y_b \tag{A.11}$$

Now, Power flow through the line *mk* i.e., from bus m to k is represented as:

$$\begin{aligned}
P_{mk} - jQ_{mk} &= \bar{V}_m^* \left[(\bar{V}_m - \bar{V}_x) y_{km} + \bar{V}_m \frac{y_{ckm}}{2} \right] \\
&= \bar{V}_m^* \left[\bar{V}_m \left(\frac{y_{ckm}}{2} + y_{km} \right) - \bar{V}_x y_{km} \right]
\end{aligned}$$

Using (A.11) in the above equation,

$$\begin{aligned}
&= \bar{V}_m^* \left[\bar{V}_m \left(y_{km} + \frac{y_{ckm}}{2} \right) - \left((\bar{V}_k - \bar{V}_s) y_a + \bar{V}_m y_b \right) y_{km} \right] \\
&= V_m^2 \left(y_{km} + \frac{y_{ckm}}{2} \right) - \bar{V}_m^* \bar{V}_k y_{km} y_a + \bar{V}_m^* \bar{V}_s y_{km} y_a - V_m^2 y_{km} y_b \\
&= V_m^2 \left(y_{km} + \frac{y_{ckm}}{2} \right) - \bar{V}_m^* \bar{V}_k y'_A + \bar{V}_m^* \bar{V}_s y'_A - V_m^2 y_{km} y'_B
\end{aligned}$$

$$\begin{aligned}
&= V_m^2 \left[y_{km} \angle\theta_{km} + j \frac{y_{ckm}}{2} \right] - V_m V_k y'_A \angle(\theta'_A - \delta_{mk}) \\
&+ V_m V_s y'_A \angle(\theta'_A - \delta_{ms}) - V_m^2 y'_B \angle\theta'_B
\end{aligned} \tag{A.12}$$

where,

$$y'_A = y_{km} y_a = g'_A + jb'_A$$

$$y'_B = y_{km} y_b = g'_B + jb'_B$$

Separating real and imaginary parts of (A.12), the expressions for RRPF from bus m to k are calculated as

$$\begin{aligned}
P_{mk} &= V_m^2 y_{km} \cos\angle\theta_{km} - V_m V_k y'_A \cos\angle(\theta'_A - \delta_{mk}) + \\
&V_m V_s y'_A \cos\angle(\theta'_A - \delta_{ms}) - V_m^2 y'_B \cos\angle\theta'_B \\
&= V_m^2 (g_{km} - g'_B) - V_k V_m (g'_A \cos\delta_{mk} + b'_A \sin\delta_{mk}) \\
&+ V_s V_m (g'_A \cos\delta_{ms} + b'_A \sin\delta_{ms})
\end{aligned} \tag{A.13}$$

$$\begin{aligned}
Q_{mk} &= -V_m^2 \left(b_{km} + \frac{y_{ckm}}{2} \right) + V_m V_k y'_A \sin\angle(\theta'_A - \delta_{mk}) \\
&- V_m V_s y'_A \sin\angle(\theta'_A - \delta_{ms}) + V_m^2 y'_B \sin\angle\theta'_B \\
&= -V_m^2 \left(b_{km} + \frac{y_{ckm}}{2} + b'_B \right) - V_k V_m (g'_A \sin\delta_{mk} - b'_A \cos\delta_{mk}) \\
&+ V_s V_m (g'_A \sin\delta_{ms} - b'_A \cos\delta_{ms})
\end{aligned} \tag{A.14}$$

where,

$$\delta_{ms} = \delta_m - \delta_s$$

$$\delta_{mk} = \delta_m - \delta_k$$

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