# ANALOGY BETWEEN NEOCLASSICAL GROWTH MODEL OF ROBERT SOLOW AND MOLECULAR DYNAMICS PERMITS A NEW PATHWAY ON GOODS-IN-PROCESS 

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#### Abstract

In the present study, Robert Solow's neoclassical growth model is extended and is shown to have resemblances with molecular dynamics. Here the labor force growth rate is comparable with $1^{\text {st }}$ order kinetics. The natural rate of Solow due to involvement of technical progress is comparable to concurrent reaction which indicates that under competitive situation technical made product is more advanced than labor made product. The concurrent production system helps to develop a new system of production (sequential type). Mathematically the status of the labor force before the commencement of technical progress per unit volume of definite output is shown which is critically analyzed and the condition for maximization is found out.


## 1. Introduction:

The potential steady growth path for one sector model counts:

1. full employment growth rate having static equilibrium is related to hypothetical economy and is shifted towards dynamic equilibrium in neoclassical growth model. The substitution-income effect leads to the fact that savings is an increasing function of profit rate,
2. in the phase diagram for labor-intensive production function, the marginal productivity is positive but diminishing return to increase in k where it is tacitly shown that the role of laborer is insignificant,
3. for production function having constant returns to scale the exogenously determined growth rate of labor force $\left(\mathrm{g}_{\mathrm{L}}\right)$ is adjusted with technical progress factor $(\lambda)$ due to the appearance of inconsistency between neoclassical growth model and stylized fact which states that both which states that both $\dot{Q}$ and $\dot{K}$ are greater than $\dot{L}$.

The initial attempt was due to Domar and Harrod who considered capital accumulation followed by output growth but suffers from the limitation of rigidity assumption and over determination. To compensate the gap, R.Sollow introduced the concept of $\lambda$ which considers effective labor force instead of labor force itself.
R.Solow's model can explain the constancy of relative income shares without $\lambda$ which does not lead to any confusion because profit and wage are shared accordingly unlike Marx who used the same sort of production to create social instability through dialectical materialism. Thus Marx could be accused in international tribunal. With the introduction of $\lambda$, the profit rate is roughly constant but the real wage grows at a rate $\lambda$.

In developing economics, at the increasing returns to scale regarding social overhead capital, the growth rate population can bring the existence of multiple equilibrium where low level equilibrium tap is most unlikely and consequently $g_{L}$ adjusts itself accordingly up to a certain level of output which ultimately leads to the concept of convergence.

## 2. Analysis:

Robert Solow's proposition of equation (i) regarding neoclassical growth model[1-5] is comparable to equation (ii) related to $1^{\text {st }}$ order liquid/gas phase reaction

$$
\begin{equation*}
\mathrm{L}_{\mathrm{t}}=\mathrm{L}_{0} e^{g_{L} t} \tag{i}
\end{equation*}
$$

$$
\begin{equation*}
\mathrm{C}=\mathrm{C}_{0} e^{-k_{1} t} \tag{ii}
\end{equation*}
$$

where labor force at time $t \approx$ concentration of reactant at same instant, the initial labor force $\approx$ initial concentration of reactant, $\mathrm{k}_{1}(=-\mathrm{dC} / \mathrm{Cdt})=$ specific reaction rate $\approx \mathrm{g}_{\mathrm{L}}(=\mathrm{dL} / \mathrm{LdL})=$ specific rate of growth of labor force. The only difference lies in the fact that in equation (i) the exponential term carries a positive power while that in equation (ii) carries a negative power. This is due to the fact that growth rate of labor force is increasing whereas the rate of reaction in terms of reactant is decreasing.

Again, according to Solow due to introduction of technical progress the effective labor force at time $t$ can be given by

$$
\begin{equation*}
\mathrm{E}_{\mathrm{t}}=\mathrm{L}_{\mathrm{t}} \mathrm{e}^{\lambda \mathrm{t}}=\mathrm{L}_{0} e^{\left(g_{L}+\lambda\right) t} \tag{iii}
\end{equation*}
$$

Equation (iii) is comparable to equation (iv) of a concurrent reaction which yields $\mathrm{k}_{2} \approx \lambda$ and negative power of exponential term can be explained as before

$$
\begin{equation*}
\mathrm{C}=\mathrm{C}_{0} e^{-\left(k_{1}+k_{2}\right) t} \tag{iv}
\end{equation*}
$$

Equation (iv) suggests that similar concept can be extended for growth model where a firm may yield either a. technical made product b. laborer made product. For the competitive situation of similar final finished product we can write, at any moment,
Rate of growth of labor force/rate of technical progress = rate of formation of laborer made product/ rate of formation of technical made product

$$
\begin{equation*}
=\text { amount of laborer made product/amount of technical made product }=\mathrm{gL} / \lambda \tag{v}
\end{equation*}
$$

Solution of equations (iii) and (v) will give the experimental determination of $g_{L}$ and $\lambda$ wherefrom it is evident that $\mathrm{g}_{\mathrm{L}}$ acts as a adopted child only no matter whether $\lambda$ is associated with bourgeois tic culture. However, it is much more difficult to correlate between $\mathrm{g}_{\mathrm{L}}$ and $\lambda$ when the final finished products are different because in this case comparison is irrelevant. The above explanation is consistent with figure-1 where labor force or labor force equivalent (technical progress) is taken along vertical axis whereas time is taken along horizontal axis. The plot gives two different curvatures touching the origin with $g_{L}<\lambda$. The supremacy of $\lambda$ over $g_{L}$ is also indicated.


Figure 1: Variation of Labor force or labor force equivalent (technical progress) with time.
The concurrent reaction also helps to introduce a consecutive (or sequential) type of reaction in this context[614] where goods-in process may lead to final finished product. The scheme (may be a subsystem of manufacture system) is as follows:
 components in the production system

At $\mathrm{t}=0, \quad \mathrm{~L}_{0}$
At time $\mathrm{t}, \quad \mathrm{L}_{1} \mathrm{~L}_{2}$
Here, the process carries one intermediate stage (theoretically several such intermediate stages may also be possible) i.e. it involves with two consecutive steps. Every step has its own specific rates. The $1^{\text {st }}$ step is involved with exogenously determined labor force growth rate whereas the $2^{\text {nd }}$ step is exclusively dependent on technical progress factor. Both the steps are homogeneous with degree one.

At time $t$, the first step decreases the value of $L_{2}$ but the $2^{\text {nd }}$ step increases the value of $L_{2}$ (technical progress factor generally increases the labor force growth rate because the possibility of employment is shifted into the possibility of unemployment). Thus the labor force growth rate accumulation can be given by

$$
\begin{gather*}
\mathrm{dL}_{2} / \mathrm{dt}=\lambda \mathrm{L}_{2}-\mathrm{g}_{\mathrm{L}} \mathrm{~L}_{1}  \tag{vi}\\
\mathrm{dL}_{2} / \mathrm{dt}-\lambda \mathrm{L}_{2}=-\mathrm{g}_{\mathrm{L}} \mathrm{~L}_{1}=-\mathrm{g}_{\mathrm{L}} \mathrm{~L}_{0} e^{t g_{\mathrm{L}}} \tag{vii}
\end{gather*}
$$

Multiplying both sides of equation (vii) by $\mathrm{e}^{-\lambda \mathrm{t}}$, we get

$$
\begin{equation*}
\mathrm{dL}_{2} / \mathrm{dt}^{-\lambda \mathrm{t}}-\lambda \mathrm{L}_{2} \mathrm{e}^{-\lambda \mathrm{t}}=-\mathrm{g}_{\mathrm{L}} \mathrm{~L}_{0} e^{\operatorname{tg}_{\mathrm{L}}} \mathrm{e}^{-\lambda \mathrm{t}} \tag{viii}
\end{equation*}
$$

Integrating equation (viii), we get

$$
\begin{equation*}
\mathrm{L}_{2} \mathrm{e}^{-\lambda t}=-\mathrm{g}_{\mathrm{L}} \mathrm{~L}_{0} e^{t g_{L}} \mathrm{e}^{-\lambda t} /\left(\mathrm{g}_{\mathrm{L}}-\lambda\right)+\mathrm{Z} \tag{ix}
\end{equation*}
$$

where Z is integrating constant. Imposing the condition of $t=0, L_{2}=$ on equation (ix), we get

$$
\begin{equation*}
\mathrm{Z}=\mathrm{g}_{\mathrm{L}} \mathrm{~L}_{0} /\left(\mathrm{g}_{\mathrm{L}}-\lambda\right) \tag{x}
\end{equation*}
$$

From equations (ix) and (x), we have

$$
\begin{equation*}
\mathrm{L}_{2} \mathrm{e}^{-\lambda \mathrm{t}}=\mathrm{g}_{\mathrm{L}} \mathrm{~L}_{0}\left(1-e^{t g_{\mathrm{L}}} \mathrm{e}^{-\lambda \mathrm{t}}\right) /\left(\mathrm{g}_{\mathrm{L}}-\lambda\right) \tag{xi}
\end{equation*}
$$

Therefore,

$$
\begin{equation*}
\mathrm{L}_{2}=\mathrm{g}_{\mathrm{L}} \mathrm{~L}_{0}\left(\mathrm{e}^{\lambda \mathrm{t}}-e^{t g_{\mathrm{L}}}\right) /\left(\mathrm{g}_{\mathrm{L}}-\lambda\right) \tag{xii}
\end{equation*}
$$

Thus the magnitude of $L_{2}$ depends both on the values of $g_{L}$ and $\lambda$.
Situation I: condition for maximization: From equation (xii), we have $\mathrm{dL}_{2} / \mathrm{dt}=\mathrm{g}_{\mathrm{L}} \mathrm{L}_{0}\left(\lambda \mathrm{e}^{\lambda \mathrm{t}}-g_{\mathrm{L}} e^{t g_{L}}\right) /\left(\mathrm{g}_{\mathrm{L}}-\lambda\right)=0$. Therefore, $\lambda \mathrm{e}^{\lambda \mathrm{t}}=g_{L} e^{t g_{L}}$ which implies

$$
\begin{equation*}
\left(\ln \lambda-\operatorname{lng}_{\mathrm{L}}\right) /\left(\mathrm{g}_{\mathrm{L}}-\lambda\right)=\mathrm{t} \tag{xiii}
\end{equation*}
$$

Situation II: When $\mathrm{g}_{\mathrm{L}}<\lambda$ (commonly accepted), then

$$
L_{2}=g_{L} L_{0}\left(e^{\lambda t}\right) /(-\lambda)
$$

Situation III: Wheng ${ }_{\mathrm{L}}>\lambda$ (third class lever), then

$$
\begin{equation*}
\mathrm{L}_{2}=\mathrm{L}_{0}\left(-e^{t g_{L}}\right) \tag{xv}
\end{equation*}
$$

In the both the cases, $L_{2}$ is negatively related with the exponential term raised to proper power of that very specific rate which is highlighted in the two step processes. The role of opposing reaction in this context is practically insignificant because the yield of firm with progress of time is irreversible in nature. A homogeneous higher degree production function in comparable to nth order reaction ( $n>1$ ) but it is associated with complicated mathematical treatment regard.

## References:

[1] Macroeconomics, theory and policy, W.H.Branson, $3^{\text {rd }}$ edition, p 561-564,568-570,573-579,582,587-588,590-596,599-600,602.
[2] Macroeconomics, N.G.Mankiw, $5^{\text {th }}$ edition (low price edition), reprinted 2006, p 180-186,200-202,208210,220.
[3] Economics, P.A.Samuelson and W.D.Nordhaus, McGraw-Hill international edition, economic series, $133^{\text {th }}$ edition, p 855-859,861,863,872.
[4] Macroeconomics, R.Dornbusch, S.Fischer, R.Startz, Tata McGraw-Hill edition, $8^{\text {th }}$ edition, p 46-47,5357,61,73,75.
[5] Macroeconomics, theory and policy,M.R.Edgmond, $2^{\text {nd }}$ edition, p 303-313.
[6] Chemical Kinetics, K.J.Laidler, Tata McGraw-Hill publishing company limited, $2^{\text {nd }}$ edition, $p$ 322-325.
[7] Physical Chemistry, Ira N.Levine, Tata McGraw-Hill edition, $4^{\text {th }}$ edition, p 502-504.
[8] Principles of Physical Chemistry,S.H.Maron and C.F.Prutton, $4^{\text {th }}$ edition, p 567-570.
[9] Physical Chemistry, vol-2, edited by Ya.Gerasimov, MIR Publishers, Moscow, p 36-43.
[10] A textbook of Physical Chemistry,S.Glasstone, $2^{\text {nd }}$ edition, p 1075-1077,1085-1087.
[11] Physical Chemistry,P.W.Atkins, $3^{\text {rd }}$ edition, p 703-705.
[12] Physical Chemistry,W.J.Moore, $5^{\text {th }}$ edition, p 345-347.
[13] Elements of Physical Chemistry,S.Glasstone and D.Lewis, $2^{\text {nd }}$ edition, p 621-622.
[14] Physical Chemistry,G.W.Castellan, $2^{\text {nd }}$ edition, p 751.

